Probability and Market-Making Maths

Interview preparation summary · HF track

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INT.HM.1.a · Fair value and information

Price a bet or a simple contract at its expected value and decide whether to trade at a quoted price.

Expected value is each possible payoff multiplied by its probability, added up. The fair price of a bet or contract is its expected payoff, so you buy below it and sell above it. Check that the probabilities add to one, list the payoffs, weight and add, then compare with the quoted price to find your edge per unit and multiply by the size. For games with a choice, such as a re-roll, work backwards: value the last step first, then decide what you would do at each earlier step, because the option to stop is worth something. Expected value ignores the spread of outcomes, so also say how large the swings are compared with the edge. A payoff such as the distance from a level is not zero just because the average level equals it. Your probability is itself an estimate, so ask how wrong it can be before the edge disappears. Interview point: say the fair price first, then the edge, then the risk.

WORKED EXAMPLE: PRICING A DIE GAME AND CHECKING THE EDGE

A desk game rolls a fair die. A six pays 12, a three, four or five pays 4, and a one or two pays nothing. A colleague offers you the game at 3.50 a play, to be played 200 times. Ignore risk preferences and costs. Is it worth taking, and what do you expect to make?

Answer: The game is worth 4.00 a play, so at 3.50 you expect to make 0.50 a play, about 100 over 200 plays, with a standard deviation of about 57.

> **Interview tip:** Give the fair price first, then the edge against the quoted price, then say how big the swings are compared with the edge. The interviewer listens for whether you check that the probabilities add to one before you multiply.

INT.HM.1.b · Fair value and information

Update a probability with new information, such as an informed counterparty or a news item, using natural frequencies.

New information changes a probability by an amount that depends on how much more likely the information is under one explanation than the other. The least error-prone method in an interview is natural frequencies: imagine a population of 1,000 or 10,000 cases, split it by the prior, apply the chance of seeing the information in each group, then ask what share of the cases that show it belong to the explanation you care about. The same logic handles an informed counterparty, because a trade that an informed person makes only in one state is more likely in that state, and a news item such as a price jump after rumours. Do not confuse the chance of the evidence given the explanation with the chance of the explanation given the evidence. Independent repeated signals can be applied one after another by multiplying the odds. Interview point: say the population size you are using before you start.

WORKED EXAMPLE: READING A STOCK SCREEN WITH NATURAL FREQUENCIES

In a universe of 2,000 stocks, 5% have an accounting problem. A forensic screen flags 80% of the stocks with a problem and 10% of the clean ones. A stock has been flagged. What is the probability that it has a problem, and what if a second, independent screen with the same rates also flags it?

Answer: After one flag the chance of a problem is 80 out of 270, about 29.6%; after two independent flags it is 64 out of 83, about 77.1%.

> **Interview tip:** Say the population size first, then count each group aloud so the interviewer can follow. The point they listen for is that you answer the chance of a problem given the flag, not the screen's 80% hit rate.

INT.HM.2.a · Making markets and sizing bets

Make a two-sided market, widen it for adverse selection and move it after a trade.

A market maker quotes a bid at which they will buy and an offer at which they will sell, centred on their estimate of fair value. The spread pays for providing the service, but it is not free money: some counterparties know more than the quoter, so a trade against you carries information. That is adverse selection. To find the break-even quote, work out the expected value of the asset given that the counterparty has bought or sold, mixing informed and uninformed flow, and quote at least that far from the original fair value. Widen the spread when information risk is high, for example before a scheduled announcement or after repeated trades in one direction, and reduce the size you show. After a trade, update your fair value for what the trade told you and skew both quotes against your inventory: a short position calls for higher quotes, a long one for lower. Interview point: say what you think the trade tells you before you say what you will quote.

WORKED EXAMPLE: SETTING A BREAK-EVEN OFFER AGAINST INFORMED FLOW

You make a market in a share that will be worth 210 or 190 tomorrow, equally likely. 20% of counterparties are informed: they buy only if it will be 210 and sell only if it will be 190. The other 80% buy or sell at random, half each way, whatever the value. Set quotes with at least 0.50 of expected edge on each side, and say what you do after being lifted.

Answer: Quote 197.50 bid and 202.50 offer; after a lift, re-centre on a fair value of about 202 and skew up while short.

> **Interview tip:** Say what the trade tells you before you say what you will quote. The interviewer listens for you to work out the value given the trade, not to quote around the old fair value.

INT.HM.2.b · Making markets and sizing bets

Size a repeated bet from its edge and odds using the Kelly idea, and judge the risk of ruin.

A bet's edge is the expected profit per unit staked: the win probability times the amount won, less the loss probability times the amount lost. When bets repeat, the size matters as much as the edge. The Kelly idea is to stake the fraction of the bankroll that maximises long-run growth, which for odds of b to 1 is (b × p − q) ÷ b, where p is the win probability and q is 1 − p; at even money it is simply p − q. Growth rises to a peak at the Kelly stake and falls beyond it, and staking double Kelly gives roughly zero growth with large swings. Because probabilities are estimated with error, practitioners often stake half of Kelly or less, which keeps most of the growth. Ruin remains possible even with an edge if the bankroll is small relative to the stake. Interview point: give the edge, then the stake, then say why you would shade it down.

WORKED EXAMPLE: SIZING A 2 TO 1 BET WITH THE KELLY IDEA

A bet pays 2 to 1 (you win 2 for each 1 staked and lose the stake otherwise) and wins 45% of the time. Your bankroll is 400. How much of it would a full Kelly bettor stake, and why might you stake less? Ignore costs and assume the bet can be repeated many times.

Answer: Full Kelly is 17.5% of the bankroll, a stake of 70; I would stake about half of that, 35, which keeps roughly 76% of the growth.

> **Interview tip:** Give the edge, then the stake in cash, then say why you would shade it down. The interviewer listens for the point that your probability is uncertain, so betting the full Kelly figure is riskier than it looks.

INT.HM.3.a · Counting and speed

Count outcomes in card and dice problems using combinations, complements and conditioning.

Counting problems reward a short, careful set-up. Decide whether order matters and whether items are replaced. For dice, list the 36 equally likely ordered outcomes of two dice, remembering that sums are not equally likely: seven can be made six ways, two only one way. For cards, a hand of k from 52 is counted with combinations, 52 choose 2 being 1,326, and a probability is favourable hands over all hands, or a chain of fractions such as 4/52 × 3/51 for two aces. When the question says at least one, count the complement, none, and subtract from one, because adding probabilities of overlapping events overcounts. For conditional questions, shrink the sample space to the cases that match the information, then count within it: being told that at least one die is a six leaves 11 outcomes, not 36. Interview point: say what you are counting before you count it.

WORKED EXAMPLE: COUNTING AT LEAST ONE HEART IN THREE CARDS

Three cards are dealt from a standard 52-card deck without replacement. What is the probability that at least one of them is a heart? Say how you would check the answer, and work in fractions a candidate could handle without a calculator.

Answer: The probability of at least one heart in three cards is 1 − 0.4135 = 0.5865, about 58.7%.

> **Interview tip:** Say that you are counting the complement and why, then give the fraction chain. The interviewer listens for the fact that you removed cards from the deck rather than treating the draws as independent.

INT.HM.3.b · Counting and speed

Do fast mental arithmetic and a sound estimate under time pressure, speaking your steps aloud.

Interviewers use mental arithmetic to see how you work when pressed, so speak your steps. Break percentages into pieces you can do easily: 10% by moving the decimal point, 5% as half of that, 1% as a hundredth. Multiply by rounding to an easy number and correcting, so 48 × 25 is 100 × 12. Use the rule of 72 for compounding, dividing 72 by the rate in percent to get the doubling time, and remember that a fall of x% needs a rise of x ÷ (1 − x) to recover, so a 30% fall needs about 43%. For estimates, write numbers in round form with powers of ten, keep units visible and check the size of the answer. Compounding growth is more than the simple sum of the rates. Accuracy within a few percent is usually enough, so say when you are rounding. Interview point: state the answer, then the check that tells you it is sensible.

WORKED EXAMPLE: ESTIMATING A FUND'S FEES BY HAND

A fund has 2,400 million of assets. It charges a 1.5% management fee on assets and 20% of a gross profit of 8% of assets. Without a calculator, roughly what total fee does it earn in the year? Ignore hurdles, high-water marks and any interaction between the two fees, and state your rounding.

Answer: The fund earns about 74 million in total fees: 36 million of management fee plus 38.4 million of performance fee, or 3.1% of assets.

> **Interview tip:** Say each piece aloud as you do it and state the rounding you are using. The interviewer listens for the sanity check at the end, such as turning the total back into a percentage of assets.

Summary of the Charta interview-preparation topic. Companies and figures in the examples are invented. Draft content, not yet expert reviewed.