Markets, Probability and Mental Maths

Interview preparation summary · IB track

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INT.BQ.1.a · Mental maths and estimation

Do fast mental arithmetic with percentages, fractions, multiplication and compounding, including the rule of 72, and say how accurate each shortcut is.

Interviewers use mental maths to see whether you are accurate under pressure and whether you sense-check a number before saying it. Build percentages from easy blocks: find 10% by moving the decimal point, halve it for 5%, and add blocks, so 35% is 30% plus 5%. Swap the two numbers when it helps, because 8% of 625 is the same as 625% of 8. Learn the fractions that recur in finance: 12.5% is one eighth, 25% is one quarter, 33.3% is one third and 37.5% is three eighths. For products, split one factor into round parts, so 19 x 24 is 20 x 24 less 24. For compounding, the rule of 72 says money doubles in 72 divided by the annual rate in years, and it is most reliable between about 6% and 10%. Percentage moves are not symmetric: a fall of 20% needs a rise of 25% to recover. The usual mistakes are decimal slips and adding percentages instead of compounding them. Say your method aloud and give the answer with its units.

WORKED EXAMPLE: A RAPID-FIRE ROUND OF FOUR QUESTIONS

The interviewer asks four questions in a row with no calculator. First, what is 15% of 360? Second, what is 19 x 24? Third, roughly how long does 100 take to grow to 400 at 6% a year? Fourth, what is three sixteenths of 480? Say the method each time and sense-check the answer.

Answer: 15% of 360 is 54, 19 x 24 is 456, about 24 years to quadruple at 6%, and three sixteenths of 480 is 90.

Interview tip: Say the method before the answer, for example 'ten percent is 36, half of that is 18, so 54'. If you slip, say so and restart the last step rather than going silent. A stated sense check shows the interviewer you would catch the error in a model.

INT.BQ.1.b · Mental maths and estimation

Do banking arithmetic quickly: margins, multiples, the equity bridge, per-share values and basis points, keeping percentages and percentage points apart.

Banking mental maths is a chain of small steps in which units matter more than difficulty. Margin is profit divided by revenue, so revenue times margin gives EBITDA. Enterprise value (EV) is EBITDA times the multiple, and equity value is EV less net debt and other debt-like claims, which divided by the share count gives a price per share. Price to earnings (P/E) is the share price divided by earnings per share, and its inverse is the earnings yield. Keep units straight: 1bn is 1,000m, and a basis point (bp) is 0.01%, so 50bp is 0.5%. A rise in margin from 15% to 16.5% is 1.5 percentage points, or 150bp, but a 10% relative increase, and mixing these up is the most common error. Round early to friendly numbers, state any convention such as ignoring tax, and say the result as a sentence. If the answer feels wrong, test it against the multiple you started from. Leverage makes the equity move by more than the EV does.

WORKED EXAMPLE: FROM MARGIN TO A PRICE PER SHARE AT HARLOW BRIDGE FOODS

Harlow Bridge Foods is an invented company with revenue of 840m, an EBITDA margin of 15%, net debt of 270m and 72m shares. A bidder values it at 9x EBITDA. What is the value per share? What if the multiple is 10x? What does a margin rise to 16.5% on the same revenue mean in percentage points, basis points and relative terms? Ignore tax and other claims.

Answer: Value per share is 12.00 at 9x and 13.75 at 10x. The margin rise is 1.5 percentage points (150bp), a 10% relative improvement, worth 12.6m of EBITDA.

Interview tip: Say the units at each step ('126 million of EBITDA, 1.1 billion of EV') and use the words 'percentage points' for margin changes. Add the leverage point unprompted: a small move in the multiple moves the equity by more, because the debt does not change.

INT.BQ.1.c · Mental maths and estimation

Size a market with a structured estimate: define it, build it from round-number drivers, cross-check it a second way and say how wrong it could be.

A market-sizing question tests structure and judgement, not recall. Start by defining what is being sized: units or revenue, which geography, which year and which customers. Then choose between bottom-up (customers times frequency times price) and top-down (a larger known total times a share), and prefer bottom-up because each assumption can be challenged. Use round, defensible numbers, say each one aloud and keep the arithmetic easy. Triangulate with a second, independent route, such as supply side (outlets times sales per outlet) against demand side, and treat agreement within about 20% to 30% as good. Errors multiply: three inputs each uncertain by 20% give a product that can be roughly half to 1.7 times the central figure, so give a range. For a banker the live use is testing an issuer's market claim: separate the total market from the part the company can serve, and be wary of a headline that adds adjacent categories. Finish with the number, the range and the assumption that matters most.

WORKED EXAMPLE: SIZING TAKEAWAY COFFEE SPEND IN A COUNTRY OF 68M PEOPLE

Estimate the yearly spend on takeaway coffee bought in shops in a country of about 68m people, in £. Treat every input as your own assumption. State the definition, build it from the demand side, cross-check it from the supply side and say how confident you are.

Answer: About 6.5bn a year, from 6.0bn demand side and 6.75bn supply side, with a realistic range of roughly 3bn to 11bn and the share of regular buyers as the assumption that matters most.

> **Interview tip:** Say the definition first, write the formula as customers times frequency times price, and keep every number round. Offer the cross-check yourself and close with the range, because an honest range is what a senior banker wants to hear.

INT.BQ.2.a · Probability and games

Calculate an expected value, price a bet or game from it, and say when expected value alone is not enough to decide.

Expected value (EV) is the sum of each outcome times its probability, and it is the fair price of a bet: pay less and you have an edge, pay more and you do not. Convert odds carefully, because odds against of 4 to 1 mean a probability of 1 in 5, which is 20%, not 25%. Break a game into states and use linearity: the expected total of several rolls is the sum of the expected rolls, so two dice average 7. For waiting puzzles, the expected number of tries to get an outcome of probability p is 1 divided by p, so a six takes 6 rolls on average. Market makers quote around the EV with a spread, and the spread pays for risk. A positive edge is not enough on its own, because the standard deviation of a bet can be large against its edge, and a one-off bet you cannot afford to lose differs from a thousand independent repeats. State the EV, then the spread of outcomes, then the decision.

WORKED EXAMPLE: A CARD GAME AT A CHARITY EVENING

A stall offers a card game. You pay 8 to draw one card from a standard 52-card deck. An ace pays 40, a jack, queen or king pays 12, and any other card pays 2. What is the fair price of the game, is it worth playing, and what happens over 100 plays?

Answer: Fair price 7.23 against a stake of 8, so minus 0.77 a play and about minus 77 over 100 plays, with a standard deviation of about 103 over 100 plays.

> **Interview tip:** Give the EV first, then the edge against the price, then one sentence on variance. Mention that a short run can still go the other way. That shows you know a positive or negative EV is a long-run statement.

INT.BQ.2.b · Probability and games

Use conditional probability and Bayes' rule with natural frequencies, and avoid confusing the chance of a signal given a fact with the chance of the fact given the signal.

Conditional probability is the chance of one thing given another, and it is not symmetric: most defaulted bonds were once low-rated, but most low-rated bonds do not default. Bayes' rule updates a prior belief with evidence. The reliable way to do it aloud is natural frequencies: imagine a round number of cases such as 10,000, split them by the base rate, apply the hit rate and the false-alarm rate, then divide the true alarms by all alarms. This exposes the base rate fallacy, where a test that is right 90% of the time still produces mostly false alarms when the event is rare. For independent events multiply the probabilities,

for at least one event use the complement, and for sequential evidence multiply the odds by each likelihood ratio. Interview classics include the two-children problem, the three doors and the biased coin. State assumptions such as independence of two tests, because the answer depends on them.

WORKED EXAMPLE: A FRAUD SCREEN AT AN INVENTED PAYMENTS FIRM

A payments firm screens 10,000 card transactions. 1% are fraudulent. The screen flags 90% of fraudulent transactions and, wrongly, 5% of genuine ones. A transaction has been flagged. What is the probability it is fraudulent? Then suppose a second, independent screen of the same accuracy also flags it.

Answer: A single flag means a 15.4% chance of fraud, and two independent flags raise it to 76.6%.

> **Interview tip:** Say 'imagine 10,000 transactions' and speak in counts, not formulas. State the independence assumption for the second screen and note that if both screens share an error source, the true figure is lower than 76.6%.

INT.BQ.2.c · Probability and games

Estimate a bond price move, a rough at-the-money option value and a put-call relationship without a calculator, and say how good each approximation is.

Interviewers ask for 'roughly' because a desk needs a sense of size before a screen is open. A bond price is the present value of its coupons and principal, so it falls when the yield rises and equals par when the coupon equals the yield. Modified duration estimates the percentage price change for a 1% change in yield, so a duration of 7 and a rise of 25bp is about a 1.75% fall. Convexity means the true move is slightly kinder than that, so duration overstates losses and understates gains. For an at-the-money option, value is roughly 0.4 times spot times volatility times the square root of time in years, and delta is about 0.5. Put-call parity links calls and puts: call less put equals spot less the discounted strike, so with zero rates an at-the-money call and put are worth the same. The usual mistakes are forgetting the square root of time and quoting a duration move in the wrong units.

WORKED EXAMPLE: A FIVE-YEAR BOND AND A ONE-YEAR OPTION

A five-year bond pays a 4% annual coupon and yields 5%. Estimate its price and the price if the yield rises by 50bp. Separately, a share trades at 200 with 30% annual volatility. Estimate the value of a one-year at-the-money call and of a three-month one, and say what the matching put is worth. Assume zero interest rates for the option.

Answer: Bond about 95.67, falling to about 93.57 on a 50bp rise (exact 93.59). The one-year call is about 24.0, the three-month call about 12.0 and the put matches the call.

> **Interview tip:** Say that these are approximations and name the one you would refine: duration ignores convexity, and the option rule assumes at-the-money and zero rates. Showing the limits of a rule of thumb is part of the answer.

Summary of the Charta interview-preparation topic. Companies and figures in the examples are invented. Draft content, not yet expert reviewed.