Probability and Mental Maths

Interview preparation summary · PE track

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INT.PM.1.a · Probability and decisions

Calculate an expected value and use it to make and defend a decision under uncertainty.

Expected value is the sum of each outcome multiplied by its probability, less any cost of taking part. A positive expected value after costs favours acting for someone indifferent to risk. List the outcomes, check the probabilities sum to one, weight and add. In decision trees work backwards: value the final branches first, then take the expected value at each chance node and the best choice at each decision node. An option to stop or abandon has value equal to the difference between the expected outcome with and without it. Expected value ignores the spread of outcomes, so a sensible answer also mentions risk tolerance, the size of the stake relative to wealth and the chance of ruin, since a single large loss may be unaffordable even when the average is attractive. Interview point: give the number first, then one sentence on whether you would take the bet and why.

WORKED EXAMPLE: DILIGENCE SPEND AND A WALK-AWAY OPTION

A sponsor pays 1m for diligence on a possible 20m purchase. There is a 40% chance the report finds a problem, in which case the sponsor walks away and loses only the 1m. Otherwise it buys, and the business then has a 60% chance of selling for 60m and a 40% chance of selling for 10m. Find the expected profit of the plan, whether to run it, and the main risk.

Answer: The plan has an expected profit of 11m, with a 24% chance of losing 11m.

> **Interview tip:** Give the number first, then say whether you would do it and what the bad case costs. The interviewer is listening for a view on risk and stake size, not only the arithmetic.

INT.PM.1.b · Probability and decisions

Apply conditional probability and Bayes' rule to interview-sized problems using natural frequencies.

Conditional probability is the chance of one event given another: P(A given B) is P(A and B) divided by P(B). Bayes' rule reverses a conditional: the chance of a cause given an observation equals the prior chance of the cause times the chance of the observation under it, divided by the overall chance of the observation. The least error-prone method in an interview is natural frequencies: imagine a population of 1,000 or 10,000, split it by the prior, apply each test rate, then count the people with the observation and ask what share of them have the cause. The classic trap is to confuse the test's accuracy with the chance of the condition given a positive result; when the condition is rare, most positives are false. State the convention for independence and for sampling with or without replacement. Interview point: say the population size you are using before you start.

WORKED EXAMPLE: A SCREEN THAT FLAGS A RARE PROBLEM

2% of the companies a sponsor screens have a hidden accounting problem. The screen flags 90% of the companies that have a problem and also flags 10% of those that do not. One company is flagged. What is the chance it really has a problem? Use natural frequencies and state the population you use.

Answer: A flagged company has about a 15.5% chance of really having a problem.

> **Interview tip:** Say 'take 10,000 companies' before you calculate, then walk through the counts. The interviewer is listening for you to avoid answering 90%.

INT.PM.2.c · Counting and games

Count arrangements and selections using permutations and combinations, including cases with repeats and symmetry.

Counting questions turn on one decision: does order matter? If it does, count permutations: n factorial arrangements of n distinct items, or n times (n minus 1) and so on for choosing and ordering k of them. If it does not, count combinations, written n choose k, which is n factorial over k factorial times (n minus k) factorial. Repeated items reduce the count: divide by the factorial of each repeat. For a circle, fix one person to remove the rotations, which gives (n minus 1) factorial. Use complements when a condition such as at least one is awkward: count all selections and subtract those with none. When splitting a group into unlabelled teams, divide by the number of ways to order the teams. Interview point: say aloud whether order matters before computing, because most wrong answers come from that choice.

WORKED EXAMPLE: TEAMS, ROLES AND A ROUND TABLE

A deal team has 8 associates. An interviewer asks four counting questions: how many ways to pick a team of 3; how many ways to name a lead, a modeller and a reviewer, all different people; how many 3-person teams include at least one of two named seniors; and how many seatings of 5 people at a round table if rotations count as the same.

Answer: There are 56 teams, 336 role assignments, 36 teams with at least one senior, and 24 round-table seatings.

> **Interview tip:** Say whether order matters before you calculate each part. The interviewer is listening for that decision more than for the final number.

INT.PM.2.d · Counting and games

Solve dice, coin and card probability problems by listing equally likely outcomes or using complements.

Dice, coins and cards are interview staples because outcomes are equally likely and countable. For two dice there are 36 equally likely ordered outcomes, so the chance of a total of 7 is 6 in 36. For coins, n flips give 2 to the power n sequences and the number with exactly k heads is n choose k. When the question says at least one, use the complement: one minus the probability of none. Cards drawn without replacement need conditional probabilities, so the chance of two aces is 4 over 52 times 3 over 51. For waiting-time questions, set up equations on states: the expected flips to see two heads in a row is 6. State whether the dice or coins are fair and whether sampling is with replacement. Interview point: count outcomes explicitly, as it makes the answer checkable.

WORKED EXAMPLE: TWO DICE, TOTALS AND AT LEAST ONE SIX

Two fair six-sided dice are rolled. An interviewer asks: what is the chance the total is at least 10, what is the chance at least one die shows a six, and how can you check the second answer a different way?

Answer: The chance of a total of at least 10 is 1/6 (about 16.7%), and the chance of at least one six is 11/36 (about 30.6%).

> **Interview tip:** Say that there are 36 equally likely outcomes, then count. The interviewer is listening for a method that can be checked, such as the complement confirmed by a direct count.

INT.PM.3.e · Mental maths and estimation

Do fast mental arithmetic with percentages, fractions, multiplication and compounding shortcuts.

Mental arithmetic in interviews rewards a small toolkit used reliably. Know the common fractions as percentages: one half is 50%, one third 33.3%, one quarter 25%, one fifth 20%, one sixth 16.7%, one seventh 14.3%, one eighth 12.5%, one ninth 11.1% and one tenth 10%. To find a percentage, split it: 15% is 10% plus half of 10%. Multiply by splitting: 17 times 24 is 17 times 20 plus 17 times 4. Multiply by 25 by dividing by 4 and multiplying by 100. To undo a discount, divide by one minus the discount, not add it back. For small compounding use one plus n times r plus a small correction, so 1.08^3 is about 1.26. Interview point: keep your working visible, round to sensible precision and check the answer against a rough estimate before you say it.

WORKED EXAMPLE: FOUR PIECES OF MENTAL ARITHMETIC

An interviewer asks four quick questions, to be answered aloud with your steps. What is a 1.5% fee on 480m? What is 17 x 24, as the revenue in millions of 17 companies with 24m each? A stake was bought for 150m at 25% below its full value; what was the full value? And roughly what is 100m after three years at 8%?

Answer: The fee is 7.2m, the revenue is 408m, the full value is 200m and the compounded value is about 126m.

> **Interview tip:** Say each step and the running number aloud, then check the answer is the right size. The interviewer is listening for a reliable method and for the discount being divided out, not added back.

INT.PM.3.f · Mental maths and estimation

Estimate quantities to the right order of magnitude and sanity-check numbers before stating them.

Estimation questions are marked on structure and sense, not on precision. Break the quantity into factors you can estimate (for example people, purchases per person and price), round each to one significant figure, multiply, and state the answer with a rough range. Sanity checks catch the common slips: an inverted ratio, a misplaced factor of ten, equity value that exceeds enterprise value when there is net debt, or an IRR that cannot be right given the multiple and the hold. Use anchors you trust, such as the rule of 72 for compounding, or simple bounds, such as rounding up and down to bracket the answer. Say your assumptions aloud so the interviewer can follow and correct them. Interview point: after you reach an answer, ask whether it is the right size, and say what you would check.

WORKED EXAMPLE: ESTIMATING A GYM CHAIN'S REVENUE

A sponsor is looking at a chain of 60 gyms. An interviewer asks you to estimate the chain's annual revenue to the nearest order of magnitude, give a range, and sanity-check it against the likely value of the business. State your assumptions as you go, and assume an operating margin of 20% and a multiple of 8 times profit.

Answer: Revenue is roughly 40m a year (range 20m to 70m), implying an enterprise value of about 70m, or around 1.2m per gym.

> **Interview tip:** State each assumption as you use it and say the range at the end. The interviewer is listening for a clear structure and a sanity check on the final size, not for precision.

Summary of the Charta interview-preparation topic. Companies and figures in the examples are invented. Draft content, not yet expert reviewed.