CHARTA LEARNING · LESSON NOTES

What drives option value, and put-call parity

CFA Level I · Derivatives

You will learn: Say how each of the six drivers moves a call and a put, split an option's price into intrinsic and time value, and use put-call parity (with a share or a forward) to find prices, build synthetics and spot arbitrage.

Running example: Examples follow Harbor Foods, a made-up company whose share trades at $100 (later $103), with three-month options, a risk-free rate of 4% a year and ordinary yearly compounding. Every price is an example.

1. The six drivers of an option's value

- A call is the right to buy at a fixed exercise price (strike), a put the right to sell. We use European options, usable only at expiry. The premium is the option price. The buyer has the right, the writer the obligation.
- Six drivers: underlying price and exercise price (they set the payoff), time to expiry, risk-free rate, volatility, and income on the underlying such as dividends.
- Higher underlying price: call up, put down. Higher exercise price: call down, put up. Higher volatility: both up, since it is the width of swings, not direction. More time: both up, usually. Higher risk-free rate: call up, put down. Higher dividends: call down, put up.

WHY VOLATILITY HELPS BOTH, AND WHY RATE AND DIVIDEND MATTER

Strike 100; calm ends at 90 or 110: average call payoff	0 and 10, equal chance	**5 (put also 5)**
Wild ends at 80 or 120: average call payoff	0 and 20, equal chance	**10 (put also 10)**
Strike 100 paid in a year: value today at 3% / 5%	100 ÷ 1.03 / 100 ÷ 1.05	**97.09 / 95.24**
Strike looks cheaper by	97.09 − 95.24	**1.85 (helps call, hurts put)**
Dividend of 5 paid before expiry	100 − 5	**share 95, option holder not paid**

Watch out: Saying a higher rate helps both options (it helps the call and hurts the put); reading volatility as a bet on direction.

2. Intrinsic value and time value

- Intrinsic value is what exercising now would give, never below zero. Time value is the option price minus intrinsic value, the price of the chance that things improve before expiry. It is never negative for these options.
- In the money: exercising now would pay. At the money: share price equals strike. Out of the money: it would pay nothing. Out of the money means zero intrinsic value, not worthless.
- Time value peaks at the money, where the outcome is hardest to predict. It is a wasting asset and does not fade in a straight line: it fades faster near expiry.

Call intrinsic = max(S − X, 0); put intrinsic = max(X − S, 0); option price = intrinsic + time value

HARBOR FOODS AT $100, THREE MONTHS TO EXPIRY

Strike 90 call	12.06 − 10	**time value 2.06**
Strike 100 call	5.60 − 0	**time value 5.60**
Strike 110 call	2.06 − 0	**time value 2.06**
Strike 110 put	10.99 − 10	**time value 0.99**
At-the-money time value (illustration)	12 months, 9 months, 3 months, 1 month	**10.00, 8.66, 5.00, 2.89**

Watch out: Treating intrinsic value as able to go negative; thinking time value is biggest for deep in-the-money options; assuming time value fades in a straight line.

3. Put-call parity with a share

- Two packages with the same European options (same underlying, strike, expiry). A protective put is the share plus a put. A fiduciary call is a call plus a risk-free bond paying the exercise price at expiry, costing X ÷ (1 + r)^T today.
- At expiry both are worth S if the share ends above X, and X if it ends below. No probabilities are needed. Packages with the same payoff in every case must cost the same today.
- That is put-call parity, a no-arbitrage rule for European options on an underlying with no income. If prices disagree someone can lock in a riskless profit. Exam questions give three of the four prices and ask for the fourth.

c + X ÷ (1 + r)^T = p + S

FINDING THE PUT (SHARE 103, STRIKE 104, ONE YEAR, RATE 4%, CALL 9)		
Present value of strike	104 ÷ 1.04	**100.00**
Call plus bond	9 + 100	**109**
Put = 109 − share	109 − 103	**6**
Check: put plus share	6 + 103	**109**

Watch out: Using the strike undiscounted; applying parity to options with different strikes or expiries; mixing up the two packages.

4. Synthetics and arbitrage

- Rearranging the identity builds any of the four items from the other three. Synthetic call = put + share − borrowing. Synthetic put = call + lending − share. Synthetic share = call − put + lending. Synthetic bond = put + share − call.
- Synthetics have the same payoffs at expiry, so they must cost the same. If the call is dear, sell the call, buy the put and share, and borrow the present value of the strike: cheap side in, dear side out.

PUT 6, SHARE 103, CALL 9, PRESENT VALUE OF STRIKE 100		
Synthetic call	6 + 103 − 100	**9**
Synthetic put	9 + 100 − 103	**6**
Synthetic share	9 − 6 + 100	**103**
Synthetic bond	6 + 103 − 9	**100**
Arbitrage if the call is quoted at 10: sell call, buy put and share, borrow	10 − 109 + 100	**+1 riskless today**

5. Parity with a dividend

- A share owner receives a dividend; an option holder does not. So take the present value of the dividend off the share price before using parity.
- Dividends cut calls and lift puts. The present value of a dividend is the dividend divided by (1 + r) raised to the years until it is paid.

Call = put + (share − PV of dividend) − PV of strike

DIVIDEND OF 2 IN SIX MONTHS; PUT 6, SHARE 103, STRIKE 104, RATE 4%		
Value today of the dividend	2 ÷ 1.04^0.5	**1.96**
Share price less dividend	103 − 1.96	**101.04**
Call	6 + 101.04 − 100	**7.04**
Call without a dividend	earlier example	**9.00**

Watch out: Forgetting dividends.

6. Put-call-forward parity and the synthetic forward

- A forward fixes a price today to trade later and costs nothing to enter at the fair forward price. For a share with no income the forward price is the share price grown at the risk-free rate.
- In the forward version the share is replaced by the present value of the forward price. Discount the forward price, not the share price and not the strike on both sides.
- A long call plus a short put at the same strike and expiry builds a long forward. It is an obligation, not a choice. If the strike equals the forward price, the call and the put cost the same. If the quoted forward is not fair, there is an arbitrage.

c + X ÷ (1 + r)^T = p + F ÷ (1 + r)^T, with F = S × (1 + r)^T when there is no income

SHARE 103, ONE YEAR, RATE 4%, STRIKE 104, CALL 9, PUT 6

Forward price; its present value	103 × 1.04; 107.12 ÷ 1.04	**107.12; 103.00**
Call side / put side	9 + 100 / 6 + 103	**109 / 109**
Cost of call minus put	9 − 6	**3.00**
Forward at 104 worth	(107.12 − 104) ÷ 1.04	**3.00**
Forward quoted at 108: lock in at expiry	108 − 104	**+4.00**
Value today minus cost; riskless gain	4 ÷ 1.04 = 3.85; 3.85 − 3.00	**0.85**

Key takeaways

- Higher volatility lifts calls and puts; a higher rate favours the call; higher dividends and a higher strike favour the put.
- Option price = intrinsic value + time value. Intrinsic is floored at zero; time value peaks at the money and fades faster near expiry.
- Put-call parity: c + X ÷ (1 + r)^T = p + S, for European options with the same strike and expiry.
- Rearranging parity builds synthetic calls, puts, shares and bonds; a mismatch is an arbitrage.
- With dividends, subtract their present value from the share price.
- Forward version: c + PV(X) = p + PV(F); long call plus short put at the strike is a long forward.

Quick check

1. Call, strike 100, share 108, price 11. Intrinsic and time value?
   Answer: Intrinsic 8. Time value 11 − 8 = 3.
2. Share 60, call 5, present value of strike 58. Put price?
   Answer: Put = 5 + 58 − 60 = 3.
3. Which two changes raise a call and cut a put?
   Answer: A higher share price, and a higher risk-free rate.