CHARTA LEARNING · LESSON NOTES

Swaps, and the uses of derivatives

CFA Level I · Derivatives

You will learn: Price and value an interest rate swap, explain the benefits and risks of derivatives, and tell an issuer's hedge from an investor's reshaping of exposure.

Running example: Examples follow Alder Mills, a made-up small manufacturer with a $10m floating-rate loan and a swap on a notional of $10m. Rates and figures are made up for teaching.

1. How an interest rate swap works

- In a plain vanilla interest rate swap one party pays a fixed rate (the swap rate) and the other a floating rate, on the same notional, which is never paid. The floating rate follows a published reference rate, set at the start of each period and paid at its end.
- Each payment date the two amounts are netted, so only the difference moves. The fixed payer is also the floating receiver.
- A swap fixes a floating borrower's cost: the loan stays floating, but the swap pays the floating rate and Alder pays fixed, so the net cost is the fixed rate whatever the floating rate does.

ALDER PAYS FIXED 4%, RECEIVES FLOATING, NOTIONAL $10M

Fixed amount Alder pays	4% × $10m	**$400,000**
Floating at 3% / 4% / 5%: net	pays 400,000 gets 300,000 / equal / gets 500,000	**pays $100,000 / $0 / gets $100,000**
Loan floats at 5%: interest to lender, swap received, swap fixed paid	−500,000 + 500,000 − 400,000	**net cost $400,000 = 4.00%**
Same at 3% floating	−300,000 − 100,000	**net cost $400,000**

2. A swap is a string of forwards: the swap rate

- A forward fixes a price today for a future date. Each payment date of a swap is a forward on the floating rate for that period, so a three-year swap with yearly payments is three forwards in one contract, with the same single fixed rate on every date.
- That fixed rate is set so the whole strip is worth exactly zero at the start, which is why a swap costs nothing on day one.
- A discount factor is what $1 on a future date is worth today, built from the forward rates up to that date. The swap rate is not a plain average of the forward rates: earlier payments are worth more today, so they get more weight.

Discount factor = $1 \div [(1 + f_1)(1 + f_2)\ldots(1 + f_t)]$; swap rate = (1 − final discount factor) ÷ sum of discount factors

TWO-YEAR SWAP, FORWARDS 3% AND 4%

Discount factor, year 1	1 ÷ 1.03	**0.97087**
Discount factor, year 2	1 ÷ (1.03 × 1.04)	**0.93353**
Floating side per $1	3% × 0.97087 + 4% × 0.93353	**0.06647**
Fixed rate	0.06647 ÷ 1.90440	**3.49%**
Check: discounted gaps	+0.4902 × 0.97087; −0.5098 × 0.93353	**+0.4759; −0.4759; net 0**
Three-year swap, factors 0.9709, 0.9335, 0.8890	(1 − 0.8890) ÷ 2.7934	**3.97%**

3. Price against value after the swap starts

- The price is the fixed rate, agreed once and never changed. The value is what the swap is worth today: zero at the start, then drifting as market rates move.
- To value it, find today's market swap rate for the remaining life, compare it with the old fixed rate, and multiply the gap by the notional and by the sum of the remaining discount factors. The fixed receiver holds the mirror image; the two values sum to zero.
- If market rates rise, the fixed payer gains and the receiver loses. If they fall, the fixed payer loses and the receiver gains. The same gap is worth less as the swap ages, as fewer payments remain.

Value to fixed payer ≈ (new swap rate − old fixed rate) × notional × sum of remaining discount factors

ALDER'S THREE-YEAR SWAP, ONE YEAR LATER (TWO PAYMENTS LEFT, FACTORS 0.9569 AND 0.9070)

New two-year swap rate	(1 − 0.9070) ÷ 1.8639	**4.99%**
Gap to the old fixed rate	4.99% − 3.97%	**about 1.02 points**
Value to Alder (fixed payer)	gap × $10m × 1.8639	**about $189,000**
One point of gap: 3 payments left / 2 left	1% × $10m × 2.7934 / × 1.8639	**$279,340 / $186,390**
Rate falls to 3.00%: Alder	(3.00 − 3.97)% × $10m × 1.8639	**about −$181,000**

Watch out: Mixing up price and value; thinking the notional moves; valuing with all the discount factors instead of only the remaining ones.

4. Benefits and risks of derivatives

- Benefits: risk moves from those who do not want it to those better placed to hold it (hedger, derivative, risk bearer); trading costs are far lower than in the cash market; and some exposures are awkward to hold directly.
- Price discovery: derivatives often trade when the cash market is shut, so news shows up in prices quickly.
- Leverage is exposure divided by margin. It magnifies gains and losses alike, and a margin call may force quick action.
- Basis risk: the hedge is on something related but not identical, so a gap remains between the hedge and the exposure.
- Benefit and risk are two faces of the same feature: cheap exposure and leverage; hedging with a cousin and basis risk; private, flexible deals (over the counter) and opacity; daily settlement and margin calls.

COST, LEVERAGE AND BASIS

$10m index exposure, three months: shares vs futures round trip	0.20% × 2 × 10m vs 0.02% × 2 × 10m	**$40,000 vs $4,000**
Cash tied up: shares vs futures (margin 8%)	full price vs 8% × $10m	**$10,000,000 vs $800,000**
Leverage; index falls 5%	10m ÷ 800,000; 500,000 ÷ 800,000	**12.5 times; loss 62.5% of margin**
Diesel bill $3m rises 25%, crude futures gain 20%	750,000 − 600,000	**gap $150,000 = 5% of the bill**
If crude fell 20%	20% × $3,000,000	**margin call $600,000**

5. Issuers hedge, investors reshape

- The same contract can be hedging or taking a position. An issuer, usually a company, uses derivatives to reduce the variability of cash flows from its own operations (debt, sales, input costs). An investor uses them to gain, shed or reshape portfolio exposure, often without trading the underlying.
- The test is whether there was an exposure to begin with. Alder, with floating debt, paying fixed lowers its risk. A fund with no debt entering the identical swap raises its risk from nothing. Hedge means the trade offsets an exposure held.
- Hedge accounting follows the same logic: the derivative's gain or loss is recorded alongside the hedged item only if shown from the start to offset a real exposure.

AN ISSUER'S CURRENCY FORWARD AND AN INVESTOR'S FUTURES

Sell €2,000,000 forward at $1.10	2,000,000 × 1.10	**locks in $2,200,000**
Euro ends at $1.00	sale 2,000,000 + forward (1.10 − 1.00) × 2,000,000	**$2,200,000**
Euro ends at $1.20	sale 2,400,000 − forward loss 200,000	**$2,200,000**
Pinecrest: $50m shares, sells futures on $20m; index falls 10%	−5 + 2	**−$3m = −6% of the fund**
Exposure now	(50 − 20) ÷ 50	**60%**

Key takeaways

- A swap is a string of forwards on the floating rate with one fixed rate, set so the strip is worth zero at the start.
- Swap rate = (1 − final discount factor) ÷ sum of discount factors; it is not a plain average of the forwards.
- The price (fixed rate) never changes; the value starts at zero and moves with market rates. Rates up: fixed payer gains.
- Value ≈ (new − old rate) × notional × sum of remaining discount factors.
- Benefits and risks are two faces of one feature: cheap exposure is leverage, hedging with a cousin is basis risk.
- Issuers hedge exposures they already have; investors reshape portfolio exposure. Hedge means offsetting a real exposure.

Quick check

1. Discount factors 0.95, 0.90, 0.85. Swap rate?
   Answer: (1 − 0.85) ÷ 2.70 = 5.56%.
2. Discount factors sum to 2.0, notional $10m, market 5%, fixed 4%. Value to payer?
   Answer: 1% × $10m × 2.0 = +$200,000.
3. $40m of shares; sells futures on $10m. New exposure?
   Answer: (40 − 10) ÷ 40 = 75%.