CHARTA LEARNING · LESSON NOTES

Discounted cash flow and growth models

CFA Level I · Equity Investments

You will learn: Value a share as the present value of its dividends or free cash flows, use the Gordon growth shortcut, build a multi-stage value, match each cash flow to its discount rate, and value a plain preferred share.

Running example: Worked examples follow one made-up company, Larkspur Foods. Dividend just paid (D0) $1.80, dividend growth 5% a year, required return 9%, market price $42.00. Every number is an example, not market data.

1. The dividend discount model and the Gordon shortcut

- Intrinsic value is what the expected cash is worth to you; the market price is what the screen says. If your value is above the price, the share may be cheap. The required return r is what you demand for the risk.
- The dividend discount model (DDM) says a share is worth the present value of every dividend it will ever pay: each dividend Dt is divided by (1 + r) raised to its year t. The list never ends, so a shortcut is needed.
- If dividends grow at one rate g for ever, the sum collapses to the Gordon growth model. The first term is D1, next year's dividend: grow the dividend just paid, D0, by (1 + g) first.
- The model suits stable firms with steady dividends; it is poor for firms that pay little or nothing or whose dividends jump around. The value V0 sits one year before D1 arrives.

V0 = D1 ÷ (r − g), with D1 = D0 × (1 + g)

LARKSPUR, GORDON MODEL (EXAMPLE 1)

Next year's dividend, D1	1.80 × 1.05	**$1.89**
The gap, r − g	0.09 − 0.05	**0.04**
Value today, V0	1.89 ÷ 0.04	**$47.25**
Market price	on the screen	**$42.00 (below value: looks cheap if the inputs are right)**

Watch out: Using D0 where the formula needs D1.

2. What the Gordon model assumes, and where growth comes from

- Dividends grow at one rate for ever, g must be strictly below r, and r is assumed stable for ever. No company can outgrow the economy for ever, so in practice g is capped near the long-run nominal growth of the economy.
- If g creeps toward r the gap shrinks to zero and value races toward infinity; if g passes r the answer turns negative, which is nonsense.
- Growth must be funded. The retention ratio b is the share of earnings kept (1 − payout ratio). Multiplying it by the return on equity (ROE, profit divided by shareholders' money) gives the sustainable growth rate. It is a starting point: competition can pull ROE down.
- Value is very sensitive to g. Analysts show a table of values, not one number, and the price can be run backwards to the growth it implies.

g = b × ROE, where b = 1 − payout ratio

LARKSPUR: EARNINGS PER SHARE $3.00, PAYOUT 60%, ROE 12.5% (EXAMPLES 2 AND 3)

Retention ratio, b	1 − 0.60	**40%**
Sustainable growth, g	0.40 × 0.125	**5%**
D1 = payout × next earnings	0.60 × (3.00 × 1.05)	**$1.89**
Value at g of 4% / 5% / 6% / 7% (D1 $1.89, r 9%)	1.89 ÷ (0.09 − g)	**$37.80 / $47.25 / $63.00 / $94.50**
Value at g = 0% and at 8%		**$21 and $189**
Growth implied by a $42 price		**4.5%**

3. Multi-stage models and the terminal value

- Few firms grow at one speed for ever. A multi-stage DDM forecasts dividends one by one through the fast stage, then uses the Gordon formula once growth settles to value everything after that point: the terminal value. Value = PV of the dividends + PV of the terminal value.
- A two-stage model drops growth abruptly; an H model lets it fade in a straight line.
- Timing trap: Gordon gives a value one year before the dividend it uses. A terminal value at the end of year 3 uses the year 4 dividend and is discounted 3 years, not 4.
- The terminal value is usually most of the answer (here about 90%), so the long-run growth rate deserves the most scrutiny.

V0 = PV of forecast dividends + TV ÷ $(1 + r)^n$, with TV at year n = D(n+1) ÷ (r − g)

LARKSPUR: GROWTH 10% FOR 3 YEARS, THEN 5% FOR EVER, R = 9% (EXAMPLES 4 AND 5)

Dividends, years 1 / 2 / 3	1.80 × 1.10, ×1.10, ×1.10	**$1.98 / $2.18 / $2.40**
Dividend, year 4	2.3958 × 1.05	**$2.52**
Terminal value, end of year 3	2.5156 ÷ 0.04	**$62.89**
PVs: D1, D2, D3, terminal	÷ 1.09, 1.09^2, 1.09^3 (TV by 1.09^3)	**$1.82, $1.83, $1.85, $48.56**
Value today	sum of the four	**$54.06 (vs $47.25 steady)**
Long-run growth 6% instead of 5%	TV 2.3958 × 1.06 ÷ 0.03 = 84.65; PV 65.37 + 5.50	**$70.87 (+$16.81, about 31%)**

Watch out: Letting g reach or pass r; wrong dividend or wrong year for the terminal value; quoting one value as if it were exact.

4. Free cash flow to equity and to the firm

- Dividends are what the board chose to pay; free cash flow to equity (FCFE) is what the firm could pay owners after costs, interest, tax, reinvestment and borrowing. Use it when dividends are zero or unrelated to capacity, or for a control view. Discount at the cost of equity r.
- Cash flow from operations (CFO) is already after interest, tax and working capital changes. Net borrowing is new debt minus debt repaid, and is negative if debt is repaid. Do not subtract debt from the value.
- Free cash flow to the firm (FCFF) belongs to lenders and owners together, so interest is added back after tax (interest × (1 − t)). Bridge: FCFE = FCFF − after-tax interest + net borrowing.
- Match the rate to the owners of the cash flow. Equity flows (dividends, FCFE) are discounted at the cost of equity and give equity value. FCFF is discounted at the weighted average cost of capital (WACC) and gives firm value; subtract debt to reach equity.
- FCFE at the WACC overstates value; FCFF at the cost of equity understates it.

FCFE = CFO − FCInv + net borrowing; FCFF = CFO + Int × (1 − t) − FCInv

LARKSPUR ($ MILLIONS; NET INCOME 60, DEPRECIATION 45, WORKING CAPITAL UP 7; 20 MILLION SHARES)

CFO	60 + 45 − 7	**98**
FCFE	98 − 70 (equipment) + (30 − 16)	**42, or $2.10 a share**
Value from FCFE (r 9%, g 5%)	2.10 × 1.05 ÷ 0.04	**$55.13 (vs $47.25 from dividends)**
Why the gap	dividends 36 ÷ FCFE 42	**86% paid out**
FCFF (interest 12, tax 25%)	98 + 12 × 0.75 − 70	**37**
Firm value (WACC 8%, g 5%)	37 × 1.05 ÷ 0.03	**1,295**
Equity value, per share	1,295 − 200 debt = 1,095; ÷ 20 million	**$54.75**

5. Residual income

- This model does not discount cash. It starts from the book value of equity and adds the present value of future residual income: profit above what owners require.
- Owners require r on opening book value B0 (the curriculum uses the book value at the start of the year). That is the equity charge; earnings above it are residual income.
- If the firm earns exactly the required return, residual income is zero and the share is worth its book value.
- Which model fits: dividends for steady payers, FCFE for cash generators with low payouts, FCFF when debt is changing, residual income when cash flows are negative or erratic. With consistent inputs the routes agree.

RI = (ROE − r) × B0; V0 = B0 + present value of future RI

LARKSPUR PER SHARE: ROE 12.5%, R = 9%, G = 5%, EARNINGS NEXT YEAR $3.15 (EXAMPLE 9)

Opening book value, B0	3.15 ÷ 0.125	**$25.20**
Equity charge	0.09 × 25.20	**$2.27**
Residual income	3.15 − 2.268	**$0.88**
PV of future RI	0.882 ÷ 0.04	**$22.05**
Value today	25.20 + 22.05	**$47.25 (the Gordon answer)**

6. Preferred shares

- A preferred share pays a fixed dividend, paid before any common dividend, usually with no growth and no vote. A plain one (neither callable nor convertible) is a perpetuity: dividend ÷ required return, using the return required on this preferred, usually lower than on the common.
- Value moves opposite to the required return, like a bond.
- A redeemable preferred is valued like a bond: the dividends as an annuity plus the present value of the redemption price.
- Features move value toward whoever holds the option. An issuer call lowers value and caps the price near the call price; a holder put (retraction), a conversion right and cumulative dividends each raise it. D ÷ r is for the plain case only.

V0 = D ÷ r

LARKSPUR PREFERRED: $100 PAR, 6% DIVIDEND = $6 A YEAR (EXAMPLE 10)

Value at 8%	6 ÷ 0.08	**$75.00**
If the return rises to 10%	6 ÷ 0.10	**$60.00 (down 20%)**
If it falls to 6%	6 ÷ 0.06	**$100.00 (up 33%)**
Redeemable at $100 in 5 years (8%)	$6 \times 3.9927 + 100 \div 1.08^5$	**$92.01**

Key takeaways

- Gordon: V0 = D1 ÷ (r − g), with D1 = D0 × (1 + g) and g strictly below r.
- Sustainable growth is retention × ROE; value is very sensitive to the gap r − g.
- Multi-stage: add the PV of forecast dividends to the PV of the terminal value, which is usually most of the answer.
- FCFE = CFO − FCInv + net borrowing, at the cost of equity; FCFF adds back after-tax interest, at the WACC, and debt is subtracted at the end.
- Residual income = (ROE − r) × opening book value; value = book value + PV of residual income.
- A plain preferred is its dividend ÷ the required return; calls lower value, puts, conversion and cumulative dividends raise it.

Quick check

1. D1 = $2.10, r = 10%, g = 4%. What is the value of the share?
   Answer: 2.10 ÷ 0.06 = $35.00
2. CFO $80m, FCInv $50m, net borrowing −$10m. What is FCFE?
   Answer: 80 − 50 − 10 = $20m
3. A plain preferred pays $4 a year; investors require 8%. Its value?
   Answer: 4 ÷ 0.08 = $50