CHARTA LEARNING · LESSON NOTES

Duration: modified, Macaulay, money duration and PVBP

CFA Level I · Fixed Income

You will learn: Estimate a bond's price change with modified duration, tell Macaulay, modified and money duration apart, say what makes duration longer or shorter, and find the price value of a basis point.

Running example: Worked examples follow one made-up company, Dunmore Utilities. Bond B: face value 100, 5% coupon paid twice a year, 10 years, yield 5%, price 100, modified duration 7.79. Bond A: 2 years, 6% coupon, yield 5%.

1. Duration as price sensitivity

- Yields up, prices down; yields down, prices up. The coupon never changes; what changes is the rate used to value the promised payments, and a higher rate makes them worth less today. Duration measures how far the price moves.
- Modified duration is the approximate percentage change in price for a one percentage point change in yield. Multiply it by the yield change and put a minus sign in front. One basis point is 0.01%, so 25 bp is 0.25%, not 25.
- Duration is a slope, a straight line against a curve, so it works well only for small moves. The line lies below the curve on both sides: it understates a gain and overstates a loss. The gap is convexity.

%ΔPrice ≈ −ModDur × ΔYield

BOND B, MODIFIED DURATION 7.79

Yield +0.50%: estimated change	−7.79 × 0.50%	**−3.9% (price about 96.1)**
Exact price at 5.50%	repriced in full	**96.19**
Yield −0.25%: estimated change	+7.79 × 0.25%	**+1.95% (price about 101.95)**
Exact price at 4.75%	repriced in full	**101.97**
Yield +3.00%: estimate; exact at 8%	7.79 × 3.00% = 23.37%	**−20.39% (price 79.61)**
Yield −3.00%: exact at 2%	bigger than estimated	**+27.07% (price 127.07)**

Watch out: Dropping the minus sign; entering 25 bp as 25; trusting the estimate for a huge move.

2. Macaulay and modified duration

- Macaulay duration is the weighted average time to the cash, in years. Each payment's weight is its present value divided by the price, so the weights add up to one. A zero-coupon bond has a single payment, so its Macaulay duration equals its maturity.
- Modified duration is Macaulay duration divided by one plus the yield per period. For semiannual coupons divide by 1 + y/2, not 1 + y. It is always a bit smaller than Macaulay.
- Units differ: Macaulay is a time in years; modified is a sensitivity, the percentage price change for a one percentage point yield change.

MacDur = Σ [t × (PV of flow t ÷ Price)]; ModDur = MacDur ÷ (1 + y/m)

BOND A: 2 YEARS, 6% COUPON TWICE A YEAR, YIELD 5%

Year 0.5: pays 3, PV 2.93	weight 2.9%	**0.014**
Year 1.0: pays 3, PV 2.86	weight 2.8%	**0.028**
Year 1.5: pays 3, PV 2.79	weight 2.7%	**0.041**
Year 2.0: pays 103, PV 93.31	weight 91.6%	**1.832**
Price; Macaulay duration	2.93 + 2.86 + 2.79 + 93.31; sum of last column	**101.88; 1.92 years**
Modified duration	1.92 ÷ (1 + 0.05/2)	**1.87**

Watch out: Treating Macaulay duration as a percentage; dividing by 1 + the full annual yield for a semiannual bond.

3. Money duration

- A 100 bond and a 100 million holding can share the same modified duration, yet one loses far more cash. Money duration (dollar duration) fixes that.
- It is modified duration times the full price of the position (market value including accrued interest, the interest earned since the last coupon and not yet paid). It gives the change in value, in currency, for a one percentage point change in yield.

> **MoneyDur = ModDur × Full price of the position**

DUNMORE POSITIONS ($ MILLIONS)		
Bond A position value (20m par)	20 × 101.88 ÷ 100	**20.38**
Bond A money duration	1.87 × 20.38	**38.11**
Loss if yield +1%	38.11 × 1%	**−0.381**
Bond B money duration (10m par)	7.79 × 10	**77.9**
Loss if yield +1%	77.9 × 1%	**−0.779**

4. What makes duration longer or shorter

- Think of duration as the balance point of the payments. Longer maturity raises duration. A higher coupon lowers it, because coupons bring value back sooner. A higher yield lowers it, because discounting shrinks distant payments most.
- A zero-coupon bond sits at its maturity. A coupon bond has a Macaulay duration below its maturity, so a longer answer is an error. For very long deep-discount bonds duration can peak before maturity; for ordinary exam bonds the rules hold.
- If you expect yields to fall, choose the higher duration.

TEN-YEAR BONDS AT A 5% YIELD (MACAULAY DURATION, YEARS)		
No coupon; 2%; 5%; 8% coupon		**10.00; 8.95; 7.99; 7.39**
5% coupon at yield 3%; 5%; 7%	prices 117.17; 100; 85.79	**8.17; 7.99; 7.8**
Zero coupon, ModDur	10.00 ÷ 1.025	**9.76**
estimated gain, yields −0.50%	9.76 × 0.50%	**+4.88%**
8% coupon, ModDur	7.39 ÷ 1.025	**7.21**
estimated gain, yields −0.50%	7.21 × 0.50%	**+3.61%**

5. Price value of a basis point (PVBP)

- PVBP is the money change for a one basis point move in yield (also called the dollar value of an 01). Because it is in money, it scales with the position and adds across a portfolio, which makes it the working measure for hedging.
- Method one: reprice at a yield one basis point lower and one higher, and halve the gap. It uses only a pricing model, so it works for unusual bonds, and gives currency per 100 of face value to scale up.
- Method two: PVBP is money duration scaled down by ten thousand. It is approximate, but the error for one basis point is tiny.
- PVBP shrinks as yields rise, because price and duration both fall, so a hedge needs adjusting after yields move.

> **PVBP = (Price, yield down − Price, yield up) ÷ 2 ≈ ModDur × Full price × 0.0001**

BOND B, $10 MILLION FACE VALUE		
Price at 4.99%; at 5.01%	yield down 1 bp; up 1 bp	**100.0780; 99.9221**
PVBP per 100 face value	(100.0780 − 99.9221) ÷ 2	**0.07795**
PVBP of the position	0.07795 × 100,000	**$7,795**
Check by duration	7.7946 × 10m × 0.0001	**$7,795**
Hedge bond $450 per $1m: face value to short	7,795 ÷ 450	**$17.32m**
PVBP per $1m of Bond B at yield 3%; 5%; 7%		**$943; $779; $646**

Key takeaways

- %ΔPrice ≈ −ModDur × ΔYield, good for small moves only; keep the minus sign and 25 bp = 0.25%.
- Macaulay duration is a weighted average time in years; ModDur = MacDur ÷ (1 + y/m).
- Money duration = ModDur × full price, in currency per one percentage point.
- Duration rises with maturity and falls with a higher coupon or yield; a zero sits at its maturity.
- PVBP is the money change for 1 bp: (P down − P up) ÷ 2, or about money duration × 0.0001.
- Duration understates gains and overstates losses for big moves (convexity).

Quick check

1. Mod. duration 6.00; yield up 25 bp. Estimated price change?
   Answer: −6.00 × 0.25% = −1.5%.
2. Semiannual bond, MacDur 7.00 years, yield 6%. Modified duration?
   Answer: 7.00 ÷ 1.03 = 6.80.
3. Position $4m, modified duration 5.0. PVBP?
   Answer: 5.0 × 4,000,000 × 0.0001 = $2,000.

Summary notes for the Charta whiteboard lesson. Examples use made-up figures. Watch the full lesson at charta-learning.com.