CHARTA LEARNING · LESSON NOTES

The term structure: spot, par and forward curves, and spreads

CFA Level I · Fixed Income

You will learn: Tell spot, par and forward rates apart, calculate each from the others, price a bond with spot rates, and measure credit spreads with the I-spread, Z-spread and option-adjusted spread.

Running example: Examples use a made-up market with annual coupons and annual compounding. Spot rates: 1 year 3.0%, 2 years 4.0%, 3 years 5.0%, 4 years 5.8%. Marlow's 3-year bond has a 6% coupon.

1. Spot rates and discount factors

- The term structure is yield plotted against time to maturity. There is more than one way to measure the rate, so there is more than one curve.
- A spot rate is the yield on a single payment, as on a zero-coupon bond that pays once at maturity. Every date has its own spot rate; the set is the spot (zero) curve.
- The discount factor is the value today of 1 paid at date t. It is below one when rates are positive and shrinks with distance. Multiply it by a payment to get the present value.

$DF = 1 \div (1 + z)^t$

ZERO-COUPON BONDS PAYING 100 (PRICE = 100 × DF)		
1 year at 3.0%	100 ÷ 1.03	**97.09**
2 years at 4.0%	100 ÷ 1.04^2	**92.46**
3 years at 5.0%	100 ÷ 1.05^3	**86.38**
4 years at 5.8%	100 ÷ 1.058^4	**79.81**

2. The par curve

- The par rate is the coupon that makes a new bond sell at its face value of 100. At par, the coupon rate and the yield to maturity are the same number.
- A spot rate belongs to one payment. A par rate belongs to a whole coupon bond, whose coupons are each discounted at their own spot rate, so it is a kind of average. It can be calculated from the spot curve.
- On a rising curve the order is forward on top, spot in the middle, par underneath, all equal at the first year. If the curve were flat, all three would be equal; on a falling curve the order reverses.

Par rate = (1 − DF at maturity) ÷ (sum of DFs, year 1 to maturity)

THREE-YEAR PAR RATE, SPOTS 3.0%, 4.0%, 5.0%		
Sum of the three DFs	0.9709 + 0.9246 + 0.8638	**2.7593**
One minus the final DF	1 − 0.8638	**0.1362**
Three-year par rate	0.1362 ÷ 2.7593	**4.93%**
Check: price of that bond	4.93 × 2.7593 + 100 × 0.8638	**100.00**
Rates at year three	par; spot; forward (year three only)	**4.93%; 5.00%; 7.03%**

Watch out: Mixing up spot, par and forward rates; they are not interchangeable.

3. Forward rates

- A forward rate is agreed today for a loan that starts later. 1y1y is a 1-year rate starting in 1 year; 2y1y starts in 2 years; 1y2y is a 2-year rate starting in 1 year. First number: time until the start; second: length.
- A forward rate is not a forecast. It is the rate today's prices imply, and it is a breakeven. If next year's rate will be below the forward rate, favour the longer bond.
- It comes from no-arbitrage: the long spot compounded must equal the short spot compounded times the forward compounded. Divide the growth factors, then take the K-th root. Do not subtract spot rates.

> **(1 + zN)^N = (1 + zM)^M × (1 + f)^K, with N = M + K**

FORWARD RATES FROM SPOTS 3.0%, 4.0%, 5.0%

1y1y	$1.04^2 \div 1.03 = 1.0816 \div 1.03 = 1.0501$	**5.01%**
2y1y	$1.1576 \div 1.0816 - 1$	**7.03%**
1y2y	$(1.1576 \div 1.03)^{\wedge}½ - 1$	**6.01%**
Two-year bond grows to	100×1.04^2	**108.16**
Roll at next rate 6.00% / 4.00% / 5.01%	103×1.06; 103×1.04; 103×1.0501	**109.18; 107.12; 108.16**

Watch out: Reading a forward rate as a forecast; subtracting spot rates instead of dividing growth factors.

4. Pricing a bond with spot rates

- A coupon bond is a bundle of zero-coupon bonds, each payment with its own date and spot rate. Discounting every payment at one yield to maturity quietly assumes a flat curve.
- The arbitrage-free price discounts each cash flow at the spot rate for its own date. Stripping a bond into pieces and rebuilding it must give the same price.
- The yield to maturity is a weighted average of the spot rates. A big coupon puts value in the early years, which drags the average down. A zero-coupon bond's yield is its spot rate.
- If the market price is below the arbitrage-free price, buy the bond, strip it and sell the pieces for a risk-free profit. Buying pushes the price up until the gap closes.

> **Price = Σ CFt ÷ $(1 + zt)^t$**

THREE-YEAR 6% ANNUAL-COUPON BOND

Year 1: 6 at 3.0%	$6 \div 1.03$	**5.825**
Year 2: 6 at 4.0%	$6 \div 1.04^2$	**5.547**
Year 3: 106 at 5.0%	$106 \div 1.05^3$	**91.567**
Price	sum	**102.94**
Yield: 6% bond; 2% bond (price 91.90); zero	single rate giving the same price	**4.92%; 4.97%; 5.00%**
Market price 101.50	$102.94 - 101.50$	**arbitrage profit 1.44**

Watch out: Using one yield for every cash flow.

5. Benchmarks and the I-spread

- A spread is the extra yield over a benchmark curve, quoted in basis points (hundredths of 1%). Government yields can be distorted by scarcity, regulation or central bank buying, so many markets use the swap rate curve.
- In an interest rate swap one side pays fixed and the other floating, and only the net is paid. The swap rate is the fixed rate that makes the swap fair. The swap curve has a rate for each maturity and reflects bank funding, not a government's credit.
- The I-spread (interpolated) is the bond's yield to maturity minus the swap rate at the same maturity, interpolating between the two nearest swaps if needed. It compares one yield with one rate, ignores the curve's shape and ignores when cash flows arrive.

I-spread = Bond yield − Swap rate (same maturity)

TWO I-SPREADS

4-year swap rate (3-year 4.50%, 5-year 5.50%)	halfway between	**5.00%**
Bond yield 6.20%: I-spread	6.20% − 5.00%	**120bp**
Marlow 6% bond, price 98.75: yield	solved from the price	**6.47%**
3-year swap rate	= 3-year par rate	**4.93%**
I-spread	6.47% − 4.93%	**154bp**

6. Z-spread and option-adjusted spread

- The Z-spread (zero volatility) adds the same constant to every spot rate on the benchmark curve so that the present value of the cash flows equals the market price. It uses the whole curve and cash flow timing. It is found by trial and error.
- The Z-spread mixes in the value of any embedded option. The option-adjusted spread (OAS) removes it, using an interest rate model.
- Callable: the option belongs to the issuer, its value is positive, so OAS is below the Z-spread. Putable: the option belongs to the investor, its value is treated as negative, so OAS is above the Z-spread. Only OAS makes bonds with and without options comparable.

Market price = Σ CFt ÷ (1 + zt + Z)^t; OAS = Z-spread − option value

MARLOW 6% BOND, PRICE 98.75; AND THREE BONDS FROM ONE ISSUER

Add 0.00% / 1.00% / 2.00% to every rate	price on the curve	**102.94 / 100.21 / 97.58**
Add 1.55% to every rate	repeated halving	**98.75, Z-spread 155bp**
Option-free bond	Z 150, no option	**OAS 150bp**
Callable bond	215 − 75	**OAS 140bp**
Putable bond	95 − (−50)	**OAS 145bp**

Key takeaways

- Spot rate: one payment. Par rate: the coupon that prices a bond at 100. Forward rate: a rate for a future period.
- Discount factor = 1 ÷ (1 + z)^t; par rate = (1 − final DF) ÷ sum of DFs.
- Forward rate: divide growth factors, then take the root. It is a breakeven, not a forecast.
- The arbitrage-free price discounts each cash flow at its own spot rate; yield to maturity is an average.
- I-spread = bond yield − swap rate; Z-spread adds one constant to every spot rate.
- OAS = Z-spread − option value: below the Z-spread for a callable bond, above for a putable one.

Quick check

1. Spot: 1 year 2%, 2 years 3%. What is the 1y1y forward rate?
   Answer: $1.03^2 \div 1.02 - 1 = 4.01\%$
2. Same spots. Price of a 2-year bond, 4% annual coupon?
   Answer: $4 \div 1.02 + 104 \div 1.03^2 = 101.95$
3. Callable bond: Z-spread 180bp, call worth 60bp. OAS?
   Answer: $180 - 60 = 120$bp, below the Z-spread

Summary notes for the Charta whiteboard lesson. Examples use made-up figures. Watch the full lesson at charta-learning.com.