CHARTA LEARNING · LESSON NOTES

Bond valuation: prices, yields and matrix pricing

CFA Level I · Fixed Income

You will learn: Price a bond as the present value of its payments, explain premium, discount and the pull to par, separate full price, flat price and accrued interest, and estimate a price by matrix pricing.

Running example: Worked examples follow one made-up issuer, Marlow Water: a 3-year bond with face value 100, 4% coupon paid twice a year (coupon 2 each period, 6 periods). Prices are quoted per 100 of face value.

1. Price is the present value of the promises

- A bond is a loan packaged so it can be traded. The face value (par) is repaid at the end, the coupons are the regular payments and the yield is the return buyers demand. The price is the present value of the promised payments, discounted at the yield.
- A yield is quoted per year. With coupons twice a year, halve the annual yield and double the years: 5% a year is 2.5% a period. Use the coupon per period.
- On the calculator enter N (periods, not years), I/Y (per period, in per cent), PMT and FV, then compute PV. The negative sign means cash paid out. The common mistake is keying annual numbers.

PV = PMT/(1+r) + PMT/(1+r)² + … + (PMT + FV)/(1+r)^N

MARLOW: 3 YEARS, 4% COUPON, FACE VALUE 100, YIELD 5% (2.5% PER PERIOD)

Period 1 and 2 coupons	2 ÷ 1.025; 2 ÷ 1.025^2	**1.95; 1.90**
Periods 3 to 5, and period 6	three coupons; 102 ÷ 1.025^6	**5.44; 87.95**
Price per 100	sum, before rounding	**97.25**
PV of the six coupons	2 × 5.5081	**11.02**
PV of the face value	100 ÷ 1.025^6	**86.23**
Price (shortcut)	11.02 + 86.23	**97.25**
Calculator	N 6, I/Y 2.5, PMT 2, FV 100	**PV −97.25**

Watch out: Keying annual yield and years with semiannual coupons.

2. Premium, discount and the shape of price against yield

- The coupon rate is fixed in advance; the yield is what the market demands today. Yield equals coupon rate: par. Yield above coupon rate: discount (price below 100). Yield below coupon rate: premium (above 100).
- Price and yield move in opposite directions, because the coupons are fixed and a higher discount rate shrinks their present value. What matters is the coupon compared with the yield, not the coupon alone.
- The price curve bends (later called convexity): a fall in yield lifts the price more than the same rise lowers it, which favours the holder.
- Longer maturity gives a bigger price swing, because discounting bites harder on distant money. A lower coupon gives a bigger swing, because more value sits in the distant face value.

MARLOW AT FOUR YIELDS, AND A 10-YEAR 4% BOND

Yield 3% (premium): coupons	2 × 5.6972	**11.39**
Yield 3%: face value	100 ÷ 1.015^6	**91.45, price 102.85**
Yield 4% (par); 5%; 6%		**100.00; 97.25; 94.58**
10-year 4% bond, yield up to 5%	100 − 92.21	**−7.79**
10-year 4% bond, yield down to 3%	108.58 − 100	**+8.58**
Fall in price, yield 4% to 5%	3-yr 4%; 10-yr 8%; 10-yr 4%; 10-yr zero	**2.75%; 7.02%; 7.79%; 9.32%**

Watch out: Judging by coupon alone, not coupon versus yield; forgetting the bend.

3. The pull to par

- At maturity the issuer repays exactly the face value. A bond above par must fall towards par and a bond below par must rise towards par, even if the yield never moves.
- The pull to par assumes the yield and credit quality stay the same. It is not a loss: the premium bond paid a bigger coupon in the meantime.
- Over a period with an unchanged yield, the return (coupon plus price change, divided by the starting price) equals the yield per period. A discount bond earns more than its coupon rate, a premium bond less.

MARLOW, SIX MONTHS LATER, YIELD UNCHANGED, 5 PERIODS LEFT		
Discount bond (5%): price 97.25 now, then	5 periods left	**97.68 (+0.43)**
Return over the half-year	(2 + 0.43) ÷ 97.25	**2.50%**
Premium bond (3%): price 102.85 now, then	5 periods left	**102.39 (−0.46)**
Return over the half-year	(2 − 0.46) ÷ 102.85	**1.50%**

Watch out: Reading the pull to par as a loss; assuming the yield changes nothing else.

4. Full price, flat price and accrued interest

- Coupons are paid on set dates but interest builds daily. Accrued interest is the seller's share of the coupon. The flat (clean) price is the quote on screens, without accrued interest. The full (dirty or invoice) price is what the buyer pays.
- Accrued interest uses the days since the last coupon over the days in the coupon period. This lesson uses the 30/360 day count (every month 30 days, half-year 180 days). Government bonds often use actual days.
- The full price jumps down by about the coupon on the coupon date, a sawtooth. The flat price has no sawtooth and shows whether the market has really moved, so it is quoted.

> **Full price = Flat price + AI; AI = PMT × t ÷ T**

MARLOW, COUPONS 15 MARCH AND 15 SEPTEMBER, BOUGHT 15 JUNE, YIELD 5%		
Accrued interest	2 × 90 ÷ 180	**1.00**
Full price on 15 June	97.25 × 1.025^0.5	**98.45**
Flat price	98.45 − 1.00	**97.45**
14 Sep: accrued; full price	179 days of 180; flat 97.67	**1.99; 99.66**
16 Sep: accrued; full price	1 day of 180; flat 97.68	**0.01; 97.69**
Fall in the full price	99.66 − 97.69	**1.97**

5. Matrix pricing

- Many bonds, especially corporate and municipal, trade rarely. Matrix pricing finds comparable bonds that did trade, with the same credit quality and sector, estimates the missing yield, then discounts the bond's own cash flows at that yield.
- Linear interpolation assumes a straight line between two traded yields. Use the same quoting basis for all yields.
- Uses: value a thin bond, and set the coupon on a new issue near par. Risks: weak comparables (rating, sector, liquidity), and a straight line ignores the bend of the curve, so it is only an estimate.

> **Yield = Ys + (x ÷ y) × (Yl − Ys), where x = years from short bond to your bond, y = years from short to long bond**

MARLOW 5-YEAR BOND, COUPON 4.5%; BBB UTILITIES: 3 YEARS 4.2%, 7 YEARS 5.4%		
Weight	(5 − 3) ÷ (7 − 3)	**0.5**
Estimated yield	4.2% + 0.5 × 1.2%	**4.8%**
Periods and rate per period	5 × 2; 4.8% ÷ 2	**10; 2.4%**
PV of the ten coupons	2.25 × 8.7975	**19.79**
PV of the face value	100 ÷ 1.024^{10}	**78.89**
Estimated price per 100	19.79 + 78.89	**98.68**

Key takeaways

- Price = present value of coupons and face value at the yield per period; halve the yield and double the years for semiannual coupons.
- Price and yield move opposite ways: par when coupon rate equals yield, premium when yield is lower, discount when higher.
- The price curve bends: yield falls gain more than yield rises lose. Longer maturity and lower coupon mean bigger swings.
- With the yield unchanged, premium and discount prices are pulled to par.
- Full price = flat price + accrued interest; the flat price is quoted.
- Matrix pricing interpolates a yield from comparable bonds, then discounts the bond's cash flows at it.

Quick check

1. Two years, 6% coupon twice a year, yield 8%. Premium or discount? Price?
   Answer: Discount. $3 \times 3.6299 + 100 \div 1.04^4 = 96.37$
2. Flat price 98.50, coupon 4 a period, 45 of 180 days passed. Full price?
   Answer: Accrued = $4 \times 45 \div 180 = 1.00$. Full = 99.50
3. Comparables: 3.0% at 2 years, 4.2% at 6 years. Yield at 4 years?
   Answer: $3.0\% + (2 \div 4) \times 1.2\% = 3.6\%$

Summary notes for the Charta whiteboard lesson. Examples use made-up figures. Watch the full lesson at charta-learning.com.