Commodity Forwards and Futures

FRM Part I

1 A story: the bakery and the wheat

- A baker can buy wheat now or agree a price for later
- Buying now brings rent, insurance and tied-up cash
- But a full store room keeps the ovens running
- Stocks have none of these; commodities have all of them

IN THE LESSON

- Picture a baker who needs flour every day.
- She could buy wheat now and keep it in a store room, or she could agree today on a price for wheat to be delivered in three months.
- If she buys now, she pays for the sack, but she also pays rent for the store room, insurance against fire and rats, and the interest on the cash she tied up.
- On the other hand, having wheat in the room means the ovens never stop, even if a delivery fails.
- A share of stock has none of these costs or benefits.
- A commodity has all of them.
- Those costs and benefits are exactly what separate commodity pricing from financial pricing.

2 How commodities differ from financial assets

- Stocks and bonds: held for investment, cash-flow benefit
- Commodities are consumption assets: used up in production
- Storage and transport cost money; power cannot be stored
- Seasonal, weather-driven prices; grade and location matter

IN THE LESSON

- Start with a stock or a bond.
- It is held for investment, it costs nothing to store, it is easy to sell short, and its benefit comes as a cash flow such as a dividend or a coupon.
- Many commodities differ on every count.
- They are consumption assets, used up in production, so a holder may value the physical item itself.
- They cost money to store and to transport, and some, like electricity, cannot be stored at all.
- Supply and demand are often seasonal or driven by weather, so prices swing and fall into patterns.
- Quality grade and delivery location also change the price of the very same commodity.

3 Financial assets versus commodities

Why commodity pricing is different

Feature	Stock or bond | Commodity
Held for	Investment | Use in production
Storage	Free | Costly or impossible
Short selling	Easy | Hard
Seasonality	None | Weather, harvests
Grade and place	Identical | Change the price

IN THE LESSON

- Here is the comparison in one table.
- On what the asset is held for, a stock is held for investment, while a commodity is often held to be used.
- On storage, a stock is free to hold, while a commodity costs money, or cannot be stored.
- On short selling, a stock is easy to borrow and sell, but physical owners rarely lend what they need to run their plants, so shorting a commodity is hard.
- On seasonality, a stock has no calendar, while crops and heating fuel follow the seasons and the weather.
- And on quality, one share is the same as any other, while grade and delivery location move the commodity price.
- Remember all five rows, because the pricing formulas rest on them.

4 Arbitrage bounds the forward from above only

- Forward too high: buy, store, sell forward. This works
- Forward too low: short the commodity. This often fails
- Owners of a consumed commodity will not lend or sell it
- So there is an upper bound, but no firm lower bound

IN THE LESSON

- Here is the consequence that the course stresses.
- For a financial asset, arbitrage can hold the forward price in place from both sides.
- If the forward is too high, you buy the asset and sell the forward.
- If it is too low, you sell the asset short and buy the forward.
- For a commodity, the first trade works, because anyone can buy the commodity, store it and deliver it.
- The second trade often does not work, because the owners of a consumption commodity will not lend it or sell it just to capture a small gap.
- So arbitrage puts a firm upper bound on a commodity forward, but no equally firm lower bound.

5 Storage costs and carry markets

- Storage costs: warehousing, insurance and handling
- They raise the forward price
- Carry market: forward = spot + interest + storage costs
- The forward curve slopes upward

IN THE LESSON

- Now the first key term.
- Storage costs are the warehousing, insurance and handling expenses of holding a physical commodity.
- Anyone who buys the commodity today to deliver later has to pay them, so they raise the forward price.
- A carry market is one where the forward price exceeds the spot price by the interest on the money tied up, plus the storage costs.
- In a carry market the forward curve slopes upward, with each later delivery date priced higher than the one before.
- Think of the price difference between two dates as the bill for carrying the goods from the earlier date to the later one.
- The bill is interest plus storage.

6 Convenience yield and lease rate

- Convenience yield: the benefit of holding the physical item
- High when inventories are low; it lowers the forward price
- Lease rate: what a lender earns, paid in the commodity
- It works like a dividend yield, as with gold

IN THE LESSON

- Two more terms, and the course likes to test the difference.
- The convenience yield is the benefit of owning the physical commodity, such as keeping a plant running or avoiding a stockout.
- The holder of a forward does not get it, because the forward holder has no goods in the building.
- It is high when inventories are low, and it lowers the forward price.
- The lease rate is different.
- It is what a lender of the commodity earns, paid in units of the commodity, as with gold.
- It works like a dividend yield on a stock.
- A bank that holds gold can lend it out, collect the lease rate and still own the metal.

7 Lease rate versus convenience yield

Do not use one where the other belongs

	Lease rate | Convenience yield
Nature	Market payment | Implied benefit
Paid to	The lender | Nobody
Paid in	Units of the item | Not paid
Mainly seen in	Gold, silver | Consumed goods
Effect on forward	Lowers it | Lowers it

IN THE LESSON

- The usual mistake is to confuse these two, so let us set them side by side.
- The lease rate is a market payment.
- A borrower really pays it to a lender, in units of the commodity, and you can look it up.
- The convenience yield is an implied benefit.
- Nobody pays it to you.
- It is what you work out from the gap between the prices, and it shows how much holders value having the goods on hand.
- The lease rate belongs mainly to gold and silver, which central banks and others will lend.
- The convenience yield belongs to commodities that are consumed, and it rises as stocks run low.
- Both of them lower the forward price, but they are different ideas.

8 What moves commodity prices

Know the drivers by commodity group

Group	Main drivers
Agricultural	Weather, yields, stocks, policy
Industrial metals	Construction, mines, recycling
Precious metals	Inflation, real rates, dollar
Energy	Producers, geopolitics, stocks
Electricity	Cannot be stored: local spikes

IN THE LESSON

- Now the influences on prices, group by group.
- Agricultural prices respond to weather, planting and harvest cycles, yields, stock levels relative to use, and policy such as biofuel mandates and trade rules.
- Industrial metals such as copper follow global industrial and construction demand, mine output and disruptions, and recycling.
- Precious metals also react to inflation, real interest rates and the dollar.
- Energy prices depend on producer decisions, geopolitics, inventories, refining and pipeline capacity, and seasonal demand.
- Electricity is special.
- It cannot be stored, so prices spike locally when supply is short.
- Low inventories magnify supply shocks in every group, so a small shortage can move prices a long way.

9 Weather derivatives

- Pay on an index such as heating degree days (HDD)
- Daily HDD = max(65 − average temp., 0), in °F
- Summed over the period; cold days add the most
- The index cannot be stored or traded: no replication price

IN THE LESSON

- Weather derivatives are a different kind of contract.
- They pay on an index of the weather, for example heating degree days, summed over a period.
- On each day, heating degree days equal the larger of sixty five minus the average temperature, and zero.
- Temperature is in Fahrenheit.
- A cold day adds many heating degree days, and a warm day adds none.
- A heating company, for example, loses when winter is mild, so it could take a position that pays when the total is low.
- The underlying index cannot be traded or stored, so there is no replication argument and no cost of carry price.
- The contract is priced by other methods.
- Remember that point.

10 Example 1: counting heating degree days

Illustration: five days, threshold 65°F, $20 per HDD above a strike of 40

Days 1 and 2: 60°F, 50°F	65−60 = 5; 65−50 = 15	**5 + 15**
Day 3: 66°F	max(65−66, 0)	**0**
Days 4 and 5: 40°F, 70°F	65−40 = 25; max(−5, 0)	**25 + 0**
Total HDD	5 + 15 + 0 + 25 + 0	**45**
Payoff	$20 × (45 − 40)	**$100**

IN THE LESSON

- Let's count heating degree days over five days, with made up temperatures.
- On day one the average is sixty degrees, so sixty five minus sixty gives five.
- On day two it is fifty, which gives fifteen.
- On day three it is sixty six, warmer than the threshold, so the answer is zero, not a negative number.
- On day four it is forty, which gives twenty five.
- On day five it is seventy, so zero again.
- The total is forty five heating degree days.
- Suppose a contract pays twenty dollars for every heating degree day above a strike of forty.
- The payoff is twenty times five, which is one hundred dollars.

11 The cost of carry formula

$F_0 = S_0 \times e^{((r + \lambda - c) \times T)}$

Spot price	Spot price today
Risk-free rate	Interest on tied-up money
Storage cost rate	Cost of warehousing: adds
Convenience yield	Benefit of holding: subtracts
Time in years	Years to expiry

IN THE LESSON

- Now the core pricing result.
- All rates are continuously compounded, and time is measured in years.
- The forward price equals the spot price times the exponential of the risk free rate, plus the storage cost rate, minus the convenience yield, all times the time to expiry.
- In words, start with spot.
- Grow it at the risk free rate, because buying costs you the interest on your money.
- Add the storage cost rate, because you must pay to hold the goods.
- Subtract the convenience yield, the benefit that physical holders enjoy and a forward holder does not.
- The three rates are combined in one exponent, and the net rate is often called the cost of carry.

12 Variants and common slips

- Lease form: $F = S \times e^{((r - \delta)T)}$
- Fixed cash storage: $F = S \times e^{(rT)}$ + FV of storage
- Slips: subtracting storage, adding c, months for years
- Two-way arbitrage holds best for storable, lendable goods

IN THE LESSON

- There are two variants to know.
- First, if the commodity can be lent out at a lease rate, the formula becomes the spot price times the exponential of the risk free rate minus the lease rate, times time.
- Second, if storage costs are fixed cash amounts instead of a rate, grow the spot price at the risk free rate and then add the future value of the storage costs at delivery.
- Now the common slips.
- Do not subtract storage costs.
- Do not add the convenience yield.
- And do not forget that time must be in years, so six months is point five.
- The formula also assumes arbitrage works in both directions, which holds best for commodities that can be stored and lent.

13 Example 2: one forward, three yields

Illustration: spot 100, r 5%, storage 3%, T = 1 year

No convenience yield	100 × e^(0.05 + 0.03)	**108.33**
Convenience yield 2%	100 × e^(0.05 + 0.03 − 0.02)	**106.18**
Convenience yield 10%	100 × e^(0.05 + 0.03 − 0.10)	**98.02**

IN THE LESSON

- Let's price a forward with made up numbers.
- Spot is one hundred.
- The risk free rate is five per cent, the storage cost rate is three per cent, and time is one year.
- With no convenience yield, the net rate is five plus three, eight per cent, and the forward is one hundred times the exponential of point zero eight, which is one hundred and eight point three three.
- Now add a convenience yield of two per cent.
- The net rate falls to six per cent, and the forward is one hundred and six point one eight.
- Finally, suppose stocks are very low and the convenience yield jumps to ten per cent.
- The net rate is minus two per cent, and the forward is ninety eight point zero two, below spot.

14 The cash-and-carry arbitrage

- Forward above full cost of carry? Borrow spot price
- Buy and store the commodity; sell it forward
- At expiry: deliver, collect F, repay loan, pay storage
- Profit = $F_0 - S_0e^{rT}$ − FV of storage, per unit

IN THE LESSON

- Why must the forward obey the formula?
- Because of arbitrage.
- A cash and carry arbitrage works when the forward price is above the full cost of carry, which is the spot price compounded at the risk free rate, plus the storage costs carried to expiry.
- Here are the steps.
- Borrow the spot price.
- Buy the commodity and store it.
- Sell it forward at no cost.
- At expiry, deliver the commodity and collect the forward price.
- Then repay the loan with interest and pay for the storage.
- What is left over is a riskless profit, earned with no money of your own.
- If storage is a continuous rate, the carry value is one exponential: the spot price times the exponential of the risk free rate plus storage, times time.

15 Example 3: a cash-and-carry profit

Illustration: spot 80, r 5%, T = 1, storage 2 paid at expiry, forward 90, 1,000 units

Loan repaid	80 × e^0.05	**84.10**
Full cost of carry	84.10 + 2 storage	**86.10**
Profit per unit	90 − 86.10	**3.90**
Profit at expiry	3.8983 × 1,000	**$3,898**
Present value of profit	3,898 × e^(−0.05)	**$3,708**

IN THE LESSON

- Let's run the trade with made up numbers.
- The spot price is eighty per unit, the risk free rate is five per cent, time is one year, and storage costs two per unit, paid at expiry, so it is simply added.
- The loan grows to eighty times the exponential of point zero five, which is eighty four point one zero.
- Add storage of two, and the full cost of carry is eighty six point one zero.
- The forward is quoted at ninety.
- The profit per unit is ninety minus eighty six point one zero, which is three point nine zero.
- The contract covers one thousand units, so the profit at expiry is about three thousand nine hundred dollars.
- Its present value, discounted at five per cent, is about three thousand seven hundred and eight dollars.

16 The reverse trade, and a warning

- Forward too low and the item can be lent: do the reverse
- Short it, lend the cash, buy forward, return it at expiry
- Often impossible for consumed commodities
- Check: value at expiry, or present value?

IN THE LESSON

- What if the forward is too low?
- If the commodity can be borrowed, do the reverse.
- Sell the commodity short, lend the proceeds at the risk free rate, and buy it forward.
- At expiry you take delivery and return the commodity.
- But think back to the table of differences.
- For a consumed commodity, the owner often will not lend it, so this reverse trade may not be available.
- That is why the forward can sit well below the cost of carry for a long time without any arbitrage.
- Two usual errors in calculations: leaving out storage or lease fees, and mixing up present value with the value at expiry.
- Always check which one the question asks for.

17 The lease rate and the forward

F = S × e^((r − δ) × T)
δ = r − ln(F ÷ S) ÷ T

Forward price	No-arbitrage forward
Spot price	Spot price
Risk-free rate	Risk-free rate
Lease rate	Lease rate: lowers the forward
Time in years	Years: annualize the log

IN THE LESSON

- Now the lease rate, for commodities such as gold that can be lent.
- With a lease rate, the no arbitrage forward equals the spot price times the exponential of the risk free rate minus the lease rate, times time.
- A higher lease rate lowers the forward price, just as a higher dividend yield lowers a stock forward.
- Turned around, the same relation lets you back out the lease rate from market prices: the lease rate equals the risk free rate minus the natural log of forward over spot, divided by time.
- If the quoted forward is above the no arbitrage value, buy the commodity, lend it out and sell forward.
- If it is below, borrow it, sell it, invest the cash, and buy forward.

18 Example 4: gold and the lease rate

Illustration: gold spot 2,000, r 5%, T = 1 year

Forward with lease rate 1%	2,000 × e^(0.05 − 0.01)	**2,081.62**
Quoted forward	Market price	**2,060**
Implied lease rate	0.05 − ln(2,060 ÷ 2,000)	**2.04%**

IN THE LESSON

- Let's try gold, with made up figures.
- Spot is two thousand dollars an ounce.
- The risk free rate is five per cent and the lease rate is one per cent, for one year.
- The forward must be two thousand times the exponential of point zero four, which is two thousand and eighty one point six two.
- Now suppose the market quotes a forward of two thousand and sixty.
- It is below the no arbitrage value, so the market is implying a higher lease rate.
- The ratio of forward to spot is one point zero three, and its natural log is point zero two nine six.
- The implied lease rate is point zero five minus point zero two nine six, which is two point zero four per cent.

19 The upper bound and the cost of carry model

- More storage cost: forward rises
- More convenience yield: forward falls, curve tilts down
- Upper bound: Fmax = S × e^((r + λ)T), by arbitrage
- No firm lower bound: that gap is the convenience yield

IN THE LESSON

- Now we put the pieces together in the cost of carry model.
- The forward price is spot times the exponential of risk free rate plus storage cost rate minus convenience yield, times time.
- A higher storage cost raises the forward.
- A higher convenience yield lowers it, so the whole curve shifts toward backwardation.
- Cash and carry arbitrage gives an upper bound: the spot price times the exponential of the risk free rate plus storage, times time.
- If the forward is above that bound, arbitrage profits exist.
- There is no equally firm lower bound, because owners of the physical commodity will not sell it just to capture a gap.
- That is exactly why the convenience yield appears in the model.

20 Example 5: implied convenience yield

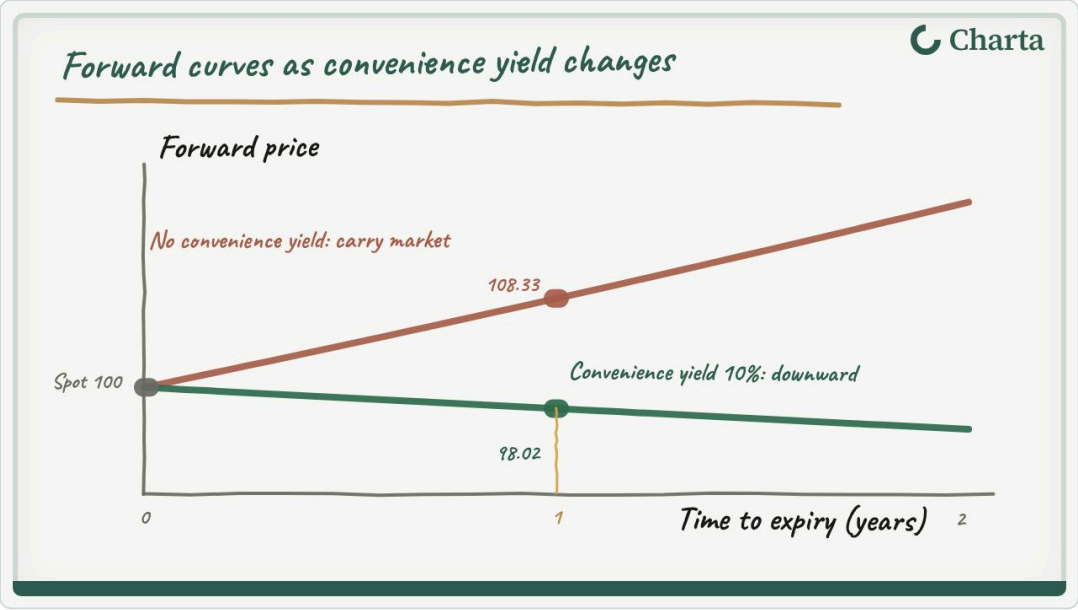
Illustration: spot 100, r 5%, storage 3%, T = 0.5, forward 101

Upper bound	100 × e^(0.08 × 0.5)	**104.08**
Annualized log of F ÷ S	ln(1.01) ÷ 0.5	**1.99%**
Implied convenience yield	5% + 3% − 1.99%	**6.01%**

IN THE LESSON

- Let's read the convenience yield from a quoted price, using made up numbers.
- Spot is one hundred, the risk free rate is five per cent, storage is three per cent, and the contract expires in six months, so time is point five.
- First, the upper bound: one hundred times the exponential of point zero eight times point five, which is one hundred and four point zero eight.
- The market forward is only one hundred and one, so it is well inside the bound.
- Now turn the model around.
- The natural log of one hundred and one over one hundred is point zero one zero zero, divided by point five gives point zero one nine nine.
- The convenience yield is point zero eight minus point zero one nine nine, about six point zero one per cent.
- It is positive, as it should be.

21 Forward curves as convenience yield changes



IN THE LESSON

- Here is the cost of carry formula as a picture.
- Along the bottom is time to expiry, from zero to two years.
- Up the side is the forward price, with spot at one hundred.
- The upper curve has no convenience yield, so the net rate is eight per cent, and the curve slopes upward: a carry market.
- At one year it reaches one hundred and eight point three three.
- The lower curve has a convenience yield of ten per cent, so the net rate is minus two per cent, and the curve slopes downward.
- At one year it is ninety eight point zero two.
- A bigger convenience yield tilts the curve toward backwardation, a smaller one toward contango.
- Same spot price, very different shapes.

22 Two ways to treat storage costs

- Proportional rate: F = S × e^((r + λ)T)
- Fixed cash payments: F = S × e^(rT) + FV of storage
- FV of storage: each payment compounded to expiry
- Early payment compounds longer; one at expiry, not at all

IN THE LESSON

- Let's focus on storage itself.
- When storing a commodity costs money, the forward price must carry those costs, because an arbitrageur who buys and holds the commodity has to pay them.
- There are two ways to describe the cost.
- With storage as a continuous proportional cost, a rate per year, the forward is the spot price times the exponential of the risk free rate plus the storage rate, times time.
- With storage paid as fixed cash amounts, the forward is spot times the exponential of the risk free rate times time, plus the future value of the storage.
- The future value is the sum of the payments, each compounded at the risk free rate from its payment date to expiry.

23 Example 6: fixed cash storage payments

Illustration: spot 100, r 6%, T = 1; storage of 2 at start and 2 at six months (unrounded)

Spot grown at r	100 × e^0.06	**106.18**
Payment at start, grown	2 × e^0.06	**2.12**
Payment at six months, grown	2 × e^(0.06 × 0.5)	**2.06**
Forward price	106.18 + FV of storage 4.18	**110.37**

IN THE LESSON

- Let's price a forward when storage is paid in fixed cash amounts.
- Spot is one hundred, the risk free rate is six per cent, and time is one year.
- The warehouse is paid two dollars at the start and two dollars after six months.
- First the spot price grown at six per cent: one hundred times the exponential of point zero six, which is one hundred and six point one eight.
- The first payment is held for the full year: two times the exponential of point zero six, about two point one two.
- The second is held for half a year: two times the exponential of point zero three, about two point zero six.
- Their future value is about four point one eight.
- The forward is therefore about one hundred and ten point three seven, rounded.

24 The synthetic commodity position

Cost = F × e^(−rT) = S
so F = S × e^(rT)

Cost of the bond	Price of the zero-coupon bond
Forward price	Forward price, paid at expiry
Risk-free rate	Risk-free rate
Time in years	Years to expiry
Spot price	Spot: what the same payoff costs

IN THE LESSON

- A synthetic commodity position copies owning the commodity without buying or storing it.
- Buy the commodity forward, and buy a zero coupon bond that pays the forward price at expiry.
- At expiry the bond pays for the forward purchase and you hold the commodity, so the payoff is the spot price on that date.
- The cost today is the price of the bond, the present value of the forward price at the continuously compounded risk free rate.
- If holding the commodity costs nothing and brings no lease income or convenience yield, that cost must equal today's spot price.
- That gives the forward price as spot times the exponential of the risk free rate times time.

25 Example 7: forward versus expected spot

Illustration: spot 100, r 5%, expected return 8%, T = 1 year

Forward price	100 × e^0.05	**105.13**
Expected spot price	100 × e^0.08	**108.33**
Forward ÷ expected spot	e^(−(0.08 − 0.05))	**0.970**
Check	108.33 × e^(−0.03)	**105.13**

IN THE LESSON

- Now link the forward to what the spot price is expected to be.
- The synthetic position earns the commodity's expected return, which we write as mu.
- With continuous compounding, the expected spot price is spot times the exponential of mu times time.
- The forward price grows from spot at the risk free rate instead.
- Let's use made up numbers.
- Spot is one hundred, the risk free rate is five per cent, mu is eight per cent, and time is one year.
- The forward is one hundred times the exponential of point zero five, which is one hundred and five point one three.
- The expected spot price is one hundred times the exponential of point zero eight, which is one hundred and eight point three three.
- The forward is below the expected spot price, because mu exceeds the risk free rate.

26 Systematic risk sets the expected return

- μ = r + β × (E(Rm) − r): CAPM, not regression alpha
- S = E(S) × e^(−μT); F = E(S) × e^(−(μ − r)T)
- Beta > 0: forward below E(S). Beta < 0: forward above
- Beta = 0: forward = E(S)

IN THE LESSON

- Why would the expected return of a commodity differ from the risk free rate?
- Because of risk.
- The required expected return, written mu, is set by systematic risk, which is the beta of the commodity against the market portfolio.
- This mu is the required return from the capital asset pricing model, not the alpha of a regression.
- Mu equals the risk free rate plus beta times the market risk premium.
- Today's spot price is the expected spot price discounted at the full required return.
- The forward price is the expected spot price times the exponential of minus mu minus the risk free rate, times time.
- So positive beta puts the forward below expected spot, negative beta puts it above, and zero beta makes them equal.

27 Example 8: two betas, two forwards

Illustration: r 3%, market 8%, spot 100, T = 1 year

Beta 0.6: required return	3% + 0.6 × (8% − 3%)	**6%**
Expected spot, forward	106.18, then × e^(−0.03)	**103.05**
Beta −0.4: required return	3% + (−0.4) × 5%	**1%**
Expected spot, forward	101.01, then × e^(+0.02)	**103.05**

IN THE LESSON

- Let's work two cases with made up numbers.
- The risk free rate is three per cent, the market return is eight per cent, and spot is one hundred, for one year.
- First, a commodity with a beta of point six.
- The required return is three plus point six times five, which is six per cent.
- The expected spot is one hundred and six point one eight.
- The forward is that times the exponential of minus point zero three, which is one hundred and three point zero five, below the expected spot.
- Second, a commodity with a beta of minus point four.
- The required return is three minus two, which is one per cent.
- The expected spot is one hundred and one point zero one, and the forward is one hundred and three point zero five, which is above expected spot.

28 Systematic and nonsystematic risk

- Arbitrage pins the forward to spot: F = S × e^(rT)
- Beta moves the expected spot price, not the forward
- Farm-level weather risk can be diversified away
- No premium for it, however volatile the price is

IN THE LESSON

- Both forwards in our example came out at one hundred and three point zero five, which is spot times the exponential of the risk free rate.
- Of course: the forward is pinned to spot by arbitrage, whatever the beta.
- It is the expected spot price that moves with risk.
- Now, what about nonsystematic risk, such as the weather on one farm?
- It can be diversified away, so it earns no premium.
- It does not move the forward relative to expected spot, however volatile the price is.
- The usual mistake is to link higher total volatility to a larger risk premium.
- Only systematic risk, beta, is rewarded.
- A very volatile commodity with zero beta has a forward equal to its expected spot.

29 Two meanings of backwardation and contango

- Keynes: normal backwardation = F below expected spot
- Producers sell futures; speculators go long and are paid
- Keynes contango = futures above expected spot (beta < 0)
- Practitioners: the words describe the shape of the curve

IN THE LESSON

- Now the last topic, and a trap.
- The words backwardation and contango have two usages, and they answer different questions.
- In Keynes's sense, normal backwardation means the futures price is below the expected future spot price.
- Producers hedge by selling futures, so speculators take the long side and earn a positive expected return as payment for bearing the risk.
- This goes with a positive beta.
- Contango in that sense means the futures price is above expected spot, so the speculator is short and earns the premium, which goes with a negative beta.
- Practitioners use the same words for the shape of the curve.
- Check which usage the question is asking about.

30 Backwardation and contango compared

Two usages: ask which one the question means

Term	Meaning
Normal backwardation	F below E(S); beta positive
Contango (Keynes)	F above E(S); beta negative
Backwardation (curve)	Downward; c > r + λ
Contango (curve)	Upward; r + λ > c

IN THE LESSON

- Let's set both usages in a table.
- In Keynes's sense, normal backwardation means the futures price is below expected spot, with a positive beta, and the long speculator is paid.
- Contango means the futures price is above expected spot, with a negative beta, and the short speculator is paid.
- In the curve sense, backwardation is a downward sloping curve, with spot above futures.
- Contango is an upward sloping curve.
- Under the cost of carry model, contango occurs when the risk free rate plus storage exceeds the convenience yield, and backwardation occurs when the convenience yield is larger.
- Our earlier chart showed both: the upper curve was contango, and the lower curve was backwardation.

31 Four mistakes to avoid

- Consumed good: only the upper bound holds
- Storage adds, convenience yield subtracts: check the signs
- Lease rate is paid, c is implied; time in years
- Forward is not expected spot; volatility is not beta

IN THE LESSON

- Here are four mistakes to avoid.
- First, treating a consumed commodity like a financial asset.
- Arbitrage bounds the forward from above, not below, so do not apply a two sided bound.
- Second, getting the signs wrong.
- Storage costs add to the forward, and the convenience yield subtracts.
- Third, confusing the lease rate, a market payment, with the convenience yield, an implied benefit, and forgetting that time must be in years.
- Fourth, equating the forward price with the expected spot price, or linking higher total volatility to a larger risk premium.
- The forward is below expected spot when beta is positive, and only systematic risk earns a premium.

32 Recap

- Consumption assets: upper bound only, storage and seasons
- F = S × e^((r + λ − c)T); lease form F = S × e^((r − δ)T)
- Cash-and-carry profit = F − full cost of carry
- F = E(S) × e^(−(μ − r)T); two meanings of contango

IN THE LESSON

- Let's recap.
- Commodities are consumption assets with storage costs, seasonal patterns and hard short selling, and arbitrage bounds a forward only from above.
- Storage costs raise the forward, the convenience yield lowers it, and the lease rate works like a dividend yield.
- Prices follow drivers that differ by group, and weather derivatives pay on an index that cannot be stored.
- The cost of carry formula uses the risk free rate plus storage minus convenience yield.
- Cash and carry arbitrage earns the gap above the full cost of carry.
- Forwards relate to expected spot through beta.
- And backwardation and contango have two usages: relative to expected spot, and the shape of the curve.

33 Try three quick questions

1. S 50, r 4%, storage 2%, T 1 yr, no c. Upper bound?
   Answer: 50 × e^(0.06) = 53.09.
2. Zero beta: forward versus expected spot?
   Answer: Equal: μ = r, so F = E(S).
3. When is the curve in backwardation?
   Answer: When c > r + λ: curve slopes down.

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- Spot is fifty, the risk free rate is four per cent, the storage cost rate is two per cent, there is no convenience yield, and the contract runs for one year.
- What is the upper bound for the forward?
- Fifty times the exponential of point zero six, which is fifty three point zero nine.
- Question two.
- A commodity has a zero beta.
- How does the forward compare with the expected future spot price?
- They are equal.
- Question three.
- In which case is the curve in backwardation, in the practitioner sense?
- When the convenience yield exceeds the risk free rate plus storage costs.