CHARTA LEARNING · LESSON NOTES

Exotic Options

FRM Part I

1 A story: off the rack or made to measure

- Plain option: off the rack, posted prices
- Exotic option: made to measure, a dealer designs it
- Better fit for odd exposures, but harder to price
- Today: what exotics are, why they exist, how they pay

IN THE LESSON

- Think about buying a suit.
- Most people buy off the rack.
- The sizes are standard, the price is posted, and any shop will sell you the same thing.
- A few people go to a tailor.
- The suit fits perfectly, but there is no posted price, and nobody else sells the identical thing, so comparing is hard.
- Options are the same.
- A plain option is the suit off the rack, and an exotic option is the suit made to measure.
- A company with an odd exposure, say a stream of foreign currency receipts arriving every month, may find the standard contract fits badly.
- So a dealer designs one that fits, and charges for the tailoring.

2 What is a plain vanilla option?

- European or American call or put, one fixed strike
- Payoff depends only on the underlying price at exercise
- Standardized terms; quoted as implied volatilities
- Active market: easy to compare, value and hedge

IN THE LESSON

- A plain vanilla option is the standard contract.
- It is a European or American call or put with a single fixed strike.
- A call gives the right to buy at the strike, and a put gives the right to sell.
- The payoff depends only on the price of the underlying asset at exercise, and nothing else.
- The terms are standardized enough that the option trades actively and is quoted by many dealers, usually in terms of implied volatility, which is simply the volatility that, put into the pricing model, reproduces the market price.
- Because everyone knows the contract and prices are public, a vanilla option is easy to compare, to value and to hedge.

3 What is an exotic derivative?

- Departs: payoff, path, assets or exercise rights
- Mostly over the counter, designed for one client
- Little or no liquid market behind it
- Design decides, not venue: OTC plain call is vanilla

IN THE LESSON

- An exotic derivative is any product that departs from that standard.
- There are four common ways to depart.
- The payoff formula may be nonstandard.
- The payoff may depend on the path the price took, not just on its final value.
- The product may depend on several underlyings.
- Or the exercise rights may be unusual.
- Exotics are mostly traded over the counter, designed by a dealer for a particular client, with little or no liquid market.
- But notice that the distinction is about contract design, not about where the trade happens.
- A plain European call bought over the counter is still vanilla.
- A listed contract with a strange payoff would still be exotic.

4 Exercise: vanilla or exotic?

Rule: vanilla is a standard design; the venue does not matter

Contract	Vanilla or exotic?
OTC European call	Vanilla (standard design)
American put, one strike	Vanilla
Pays on the average price	Exotic (path dependent)
Disappears at a barrier	Exotic (path dependent)
Option on the better of two	Exotic (several assets)

IN THE LESSON

- Let's sort five contracts, with the rule that design decides.
- First, a European call on a stock, bought over the counter from a bank.
- Vanilla, because the contract is standard even though the venue is private.
- Second, an American put with one fixed strike.
- Vanilla.
- Third, an option that pays on the average price over the year.
- Exotic, because the payoff depends on the path.
- Fourth, an option that disappears if the price touches a barrier.
- Exotic, again path dependent.
- Fifth, an option on the better of two assets.
- Exotic, because it involves several underlyings.
- Expect the course to ask you to classify instruments in exactly this way.

5 Why exotics are riskier to hold

- No screen price: the value comes from a model
- Different models can give different answers
- Sensitivities can jump near a barrier: hard to hedge
- Few others trade it, so the position is hard to unwind

IN THE LESSON

- There is a price to pay for tailoring.
- First, exotics have more model risk.
- With no liquid market there is no screen price to check against, so the value comes from a model, and different models can give quite different answers, especially for path dependent payoffs.
- Second, exotics are harder to hedge.
- A barrier option, for example, can switch from valuable to worthless in an instant, so its sensitivity to the price jumps near the barrier, and the hedge can be hard to run.
- Third, the dealer cannot easily unwind the position, because few others trade it.
- So expect questions asking why exotics need more careful valuation and more careful risk management.

6 Why exotic products were developed

- Hedging: match an odd exposure, like monthly receipts
- Cost: give up unlikely payoffs for a lower premium
- Views: rise, but never touch a given level
- Restrictions: tax, accounting, legal, regulatory
- Dealers: higher margins, hard to compare

IN THE LESSON

- Why do exotics exist?
- Standard options do not fit every need, and there are four main reasons.
- The first is hedging.
- A treasurer with monthly foreign currency receipts can buy an average rate option, which matches the exposure better than a vanilla option on a single date.
- The second is cost.
- Many exotics have lower premiums, because the buyer gives up payoff in states thought unlikely, or because an average or a portfolio is less volatile than one price.
- The third is to express a view precisely, for instance that a price will rise but never touch a given level.
- The fourth is to work around tax, accounting, legal or regulatory restrictions.
- Dealers also like exotics, because margins are higher and the products are hard to compare.

7 Tying each reason to a product

Match the reason to a concrete product, as the course expects

Reason	Typical product
Hedge monthly receipts	Asian (average rate) option
Lower premium	Barrier option
Cheaper than many options	Basket option
Rise but never touch	Barrier option
Work around restrictions	Note with embedded option

IN THE LESSON

- The course wants each reason tied to a concrete product, so here is the pairing.
- For hedging a stream of receipts, the average rate or Asian option fits, since the average of many monthly rates is what the company actually earns.
- For a lower cost, a barrier option is cheaper than its vanilla twin, because the buyer accepts that it can vanish.
- A basket option is cheaper than a set of separate options, because a portfolio of assets that are not perfectly correlated is less volatile than the parts.
- To express a precise view, use a barrier option again.
- And to work around a restriction, a dealer may embed an unusual feature into a structure, within the rules.

8 Zero cost: change how the premium is paid

- Defer payment: pay nothing now, pay at expiry
- Deferred amount is larger: later money is worth less
- Contingent premium: pay only if in the money, so larger
- Or combine options so the premiums offset
- Zero cost is not zero risk

IN THE LESSON

- Now a neat idea.
- Any derivative can be made zero cost at inception by changing how, or whether, the premium is paid.
- The simplest route is to defer payment.
- The holder pays nothing today and pays at expiry, whether or not the option ends up being exercised.
- But the deferred payment must be bigger than the original premium, because money paid later is worth less than money paid today.
- A variant is the contingent premium option.
- It charges the premium at expiry only if the option finishes in the money, so the charge must be larger still.
- The second route is to combine options so that the premiums offset one another.
- Zero cost is never zero risk.

9 The deferred and contingent premium

Deferred payment = c × e^{rT}
Contingent payment = c × e^{rT} ÷ Q

Premium, c	Premium if paid today
Compounding, e^{rT}	Grows at the risk-free rate
Probability, Q	Chance of finishing in the money

IN THE LESSON

- Here are the two formulas.
- The deferred payment equals the premium, compounded at the risk free interest rate over the life of the option.
- That is, the premium times the exponential of the rate times the time.
- This is what the premium would grow to if you had paid it today and left it earning interest.
- The contingent payment is the same compounded amount divided by Q, where Q is the risk neutral probability that the option finishes in the money.
- Because Q is below one, dividing by it makes the payment larger.
- The usual mistake is forgetting to compound the deferred premium, and just charging the original amount at expiry.

10 Example 1: deferred and contingent premiums

Illustration: premium 4.00, rate 5%, one year

Paid up front	the original premium	**4.00**
Deferred to expiry	4.00 × e^{0.05}	**4.21**
Contingent, Q = 0.5	4.205 ÷ 0.5	**8.41**
Contingent, Q = 0.4	4.205 ÷ 0.4	**10.51**

IN THE LESSON

- Take a premium of four dollars today, an interest rate of five per cent and a life of one year.
- Paid up front, the cost is four dollars.
- Deferred to expiry, the payment is four times the exponential of point zero five, which is four dollars and twenty one cents, rounded.
- The extra twenty one cents is simply the interest.
- Now make the premium contingent.
- If the chance of finishing in the money is one half, divide the unrounded amount, four dollars and twenty and a half cents, by point five, and the payment becomes eight dollars and forty one cents, but only if the option pays off.
- If the chance is only point four, the payment rises to ten dollars and fifty one cents.
- The less likely the payoff, the larger the charge.

11 Example 2: a zero-cost collar

Illustration: asset at 100, put strike 95 bought, call strike 108 sold

Premium paid, put 95	bought	**−2.40**
Premium received, call 108	sold	**+2.40**
Net premium	−2.40 + 2.40	**0.00**
Asset falls to 85	put pays 95 − 85	**+10**
Asset rises to 115	call costs 115 − 108	**−7**

IN THE LESSON

- The second route is a zero cost collar, also called a range forward.
- Suppose a holder of an asset priced at one hundred buys a put with a strike of ninety five, and sells a call with a strike of one hundred and eight.
- Choose the strikes so the two premiums match, say two dollars and forty cents each.
- The premium paid for the put is two forty, the premium received for the call is two forty, and the net premium is zero.
- But it is not free of risk.
- If the price falls to eighty five, the put pays ten.
- If it rises to one hundred and fifteen, the sold call costs seven, so the upside above one hundred and eight is given up.

12 Nonstandard American options

- Bermudan: exercise only on specified dates
- Lock-out: no exercise during an initial period
- Strike can step up or down over time
- Built from a standard American by restricting exercise
- Restricted: value between European and American

IN THE LESSON

- A standard American option can be exercised at any time up to expiry at one fixed strike.
- Over the counter, and in employee warrants, the rules are often changed.
- A Bermudan option can be exercised only on specified dates, so it sits between a European option, with one date, and an American option, with all dates.
- A lock out period forbids exercise during an initial part of the life.
- The strike can also step up or down over time.
- All of these are engineered from the standard American option by restricting where early exercise is allowed, or by changing the strike schedule.
- When the changes only restrict exercise, the value lies between the European value and the unrestricted American value.

13 Example 3: the exercise test on a tree

Put K 100. End prices 144, 96, 64 pay 0, 4, 36. q = 0.6

Up node (120): continue	0.4 × 4 ÷ 1.04	**1.54**
Up node: exercise	max(100 − 120, 0)	**0**
Down node (80): continue	(0.6×4 + 0.4×36) ÷ 1.04	**16.15**
Down node: exercise	100 − 80	**20**
Root: American value	(0.6×1.54 + 0.4×20) ÷ 1.04	**8.58**

IN THE LESSON

- In a binomial tree this is easy to build.
- Take a put with strike one hundred on an asset at one hundred.
- Each step the price rises by twenty per cent or falls by twenty per cent, with a rate of four per cent a step and two steps.
- At expiry the prices are one hundred forty four, ninety six and sixty four, so the put pays zero, four and thirty six.
- The risk neutral probability of an up move is point six.
- Look at the up node, at one hundred and twenty.
- Continuing is worth one point five four, while exercising is worth nothing.
- At the down node, at eighty, continuing is worth sixteen point one five, but exercising gives twenty.
- So exercise there.
- Folding back to the root gives an American value of eight point five eight.

14 Example 4: restricting the exercise rights

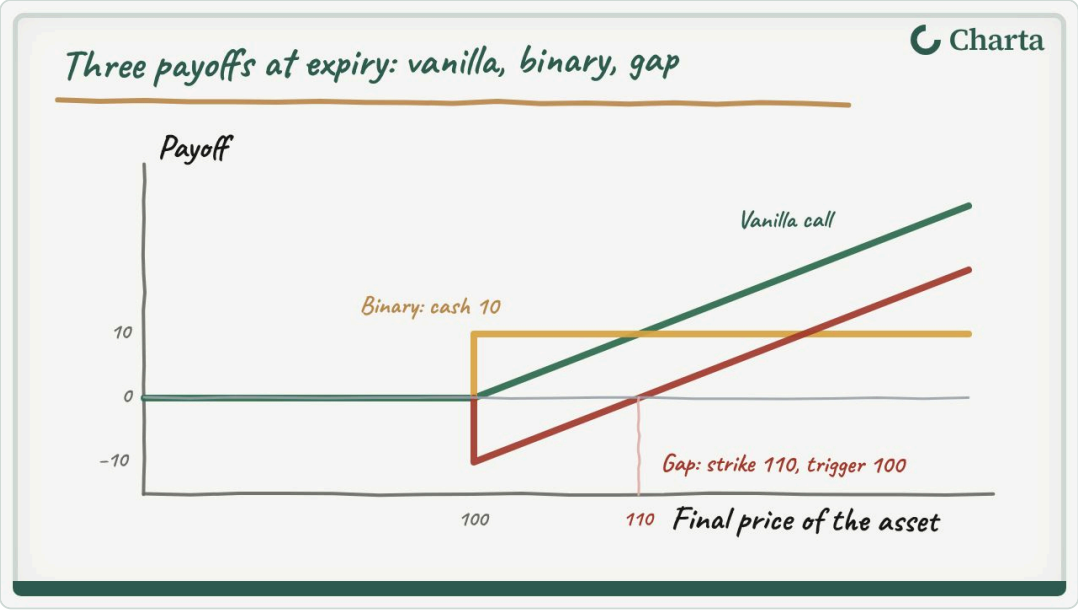
Same tree. The exercise test is applied only where exercise is allowed

Lock-out until expiry	no early exercise: European	**7.10**
Early strike 98	down node: exercise at 18	**7.81**
Early strike 100	down node: exercise at 20	**8.58**
Ordering of the values	7.10 ≤ 7.81 ≤ 8.58	**Holds**

IN THE LESSON

- Now restrict the rights on the same tree.
- With a lock out that lasts until expiry, there is no early exercise at all, so the down node is worth its continuation value of sixteen point one five, and the root is worth seven point one zero.
- That is the European value.
- Next, suppose early exercise is allowed at the first date, but the strike in force is ninety eight, stepping back to one hundred at expiry.
- At the down node, exercising pays eighteen, which beats sixteen point one five, so the value there is eighteen and the root is seven point eight one.
- With the full strike of one hundred, we had eight point five eight.
- So seven point one zero is below seven point eight one, which is below eight point five eight.

15 Three payoffs at expiry: vanilla, binary, gap



IN THE LESSON

- Before the list, a picture of three payoffs at expiry.
- Along the bottom is the final price of the asset, and up the side is the payoff.
- The green line is the vanilla call with a strike of one hundred.
- It pays nothing up to the strike and then rises one for one.
- The amber line is a binary option.
- It pays a fixed ten dollars of cash if the final price is above one hundred, and nothing otherwise, however far above.
- So the payoff jumps.
- The red line is a gap option, with a payoff strike of one hundred and ten and a trigger of one hundred.
- Just above the trigger it pays minus ten, then climbs, so the payoff can be negative.

16 Basket, exchange and Asian options

- Basket: portfolio payoff, cheaper (correlation < 1)
- Exchange: swap asset U for asset V, max(V − U, 0)
- Asian: pays on the average price, max(average − K, 0)
- Averaging smooths the price, so it costs less

IN THE LESSON

- Now the list, in three groups.
- A basket option pays on a portfolio of assets, for a call the maximum of the basket value minus the strike, or zero.
- It is cheaper than separate options on each asset, because correlation is below one, so the basket is less volatile than its parts.
- An exchange option gives the right to swap one asset for another, with a payoff of the maximum of the second asset's price minus the first's, or zero.
- Notice there is no cash strike.
- An Asian option pays on the average price rather than the final price.
- For a call, the payoff is the maximum of the average minus the strike, or zero.
- Averaging smooths the price, so Asians are cheaper than vanilla.

17 Lookback, binary and gap options

- Lookback floating call: final price − lowest price seen
- Lookback fixed call: max(highest price − K, 0)
- Binary: fixed cash or the asset if beyond K
- Gap: S − strike if S passes trigger; can be negative

IN THE LESSON

- A lookback option pays based on the best price seen during the life.
- A floating strike lookback call pays the final price minus the lowest price seen, so the holder always buys at the bottom.
- A fixed strike lookback call pays the maximum of the highest price seen minus the strike, or zero.
- Because they hand over hindsight, lookbacks are expensive.
- A binary option pays a fixed cash amount, or the asset itself, if the final price is beyond the strike.
- A gap option pays the final price minus one strike, but only if the final price passes a different trigger.
- Since the two levels differ, the payoff can be negative, as we saw in the chart.

18 Barrier, chooser, compound, forward start

- Barrier: knock-in or knock-out when touched
- Knock-in + knock-out (same terms) = the vanilla option
- Chooser: pick call or put at a later date
- Compound: option on an option, two strikes, two dates
- Forward start: starts later, strike at the money

IN THE LESSON

- A barrier option comes into existence, a knock in, or ceases to exist, a knock out, if the price touches a barrier.
- An important fact: a knock in plus a knock out with the same terms equals the vanilla option, because exactly one of them is alive at the end.
- A chooser option lets the holder pick, at a later date, whether it becomes a call or a put.
- A compound option is an option on an option, so it has two strikes and two dates.
- A forward start option starts at a later date, with its strike set at the money on that date.
- So chooser, compound and forward start options are all about timing and choice, not about the final price alone.

19 Example 5: one price path, five payoffs

Prices 104, 112, 96, 108. Strike 100, barrier 110 (illustration)

Vanilla call	108 − 100	**8**
Asian call (average 105)	105 − 100	**5**
Floating lookback call	108 − lowest 96	**12**
Fixed lookback, strike 105	highest 112 − 105	**7**
Up-and-out, barrier 110	112 touched: out (in pays 8)	**0**

IN THE LESSON

- Let's price five payoffs on one price path, the vanilla call as the comparison and the rest path dependent.
- The asset is observed at four dates, with prices one hundred and four, one hundred and twelve, ninety six and one hundred and eight.
- The strike is one hundred.
- The vanilla call uses only the final price: one hundred and eight minus one hundred, which is eight.
- The Asian call uses the average, one hundred and five, so the payoff is five.
- The floating lookback call is the final price minus the lowest price, one hundred and eight minus ninety six, which is twelve.
- A fixed lookback with strike one hundred and five uses the highest price, one hundred and twelve, so it pays seven.
- Finally an up and out call with barrier one hundred and ten was knocked out at one hundred and twelve, so it pays zero.
- The matching up and in call pays eight.

20 Example 6: payoffs that use the final price

Final prices: A 108, B 103, C 96. Strike 100 (illustration)

Cash binary call, pays 10	108 > 100	**10**
Asset-or-nothing call	pays the asset	**108**
Gap: strike 110, trigger 100	108 − 110	**−2**
Exchange: give B, get A	max(108 − 103, 0)	**5**
Basket call, A and C 50/50	avg 102; 102 − 100	**2**

IN THE LESSON

- Now five payoffs that use final prices only.
- Asset A ends at one hundred and eight, asset B at one hundred and three, and asset C at ninety six, with a strike of one hundred.
- A cash binary call paying ten dollars pays ten, because one hundred and eight is above one hundred.
- The asset or nothing version pays the asset itself, one hundred and eight.
- A gap call with payoff strike one hundred and ten and trigger one hundred has passed its trigger, so it pays one hundred and eight minus one hundred and ten, which is minus two.
- An exchange option, giving up B to get A, pays one hundred and eight minus one hundred and three, which is five.
- A fifty fifty basket of A and C has value one hundred and two, so the call pays two.

21 Which of these are path dependent?

Path dependent: the whole price history matters, not just the end

Option	Path dependent?
Asian	Yes: the average
Lookback	Yes: highest or lowest
Barrier	Yes: was it touched?
Binary and gap	No: final price only
Basket and exchange	No: final prices only

IN THE LESSON

- The course will ask which of these depend on the path.
- Three do.
- The Asian option depends on the average of the prices along the way.
- The lookback depends on the highest or lowest price seen.
- The barrier option depends on whether the barrier was touched at any time.
- Everything else on our list depends only on the final price.
- The binary and gap options look unusual, but they use the final price alone, so they are not path dependent.
- The basket and exchange options use the final prices of several assets.
- Path dependence makes an option harder to value, and often harder to hedge.

22 Volatility swaps against variance swaps

- Vol swap: realized volatility against a fixed strike
- Variance swap: realized vs strike variance
- Vega notional is quoted per volatility point
- Variance payoff is convex: long gains more, loses less

IN THE LESSON

- Now two products that let you trade volatility itself.
- A volatility swap exchanges realized volatility for a fixed strike volatility at maturity.
- The payoff is the vega notional times realized volatility minus strike volatility.
- The vega notional is quoted per volatility point, which is one percentage point of volatility.
- A variance swap exchanges realized variance for a strike variance, with a payoff of the variance notional times realized variance minus strike variance, where variance is measured in squared volatility points.
- For small moves around the strike the two are equivalent.
- But the variance swap payoff is convex in volatility, so the long gains more when volatility rises and loses less when it falls.

23 Payoffs and the conversion between notionals

Vol swap = vega notional × (σ_real − σ_strike)
Var swap = var notional × (σ_real² − σ_strike²)

Vega notional	Dollars per volatility point
Volatility swap	Linear in realized volatility
Variance swap	Convex: squared volatility
Variance notional	= Vega notional ÷ (2 × σ_strike)

IN THE LESSON

- Here are the formulas.
- The volatility swap pays the vega notional times the difference between realized volatility and the strike volatility.
- The variance swap pays the variance notional times the difference between realized variance and the strike variance, both measured in volatility points squared.
- To make the two equivalent near the strike, convert the notionals.
- The variance notional equals the vega notional divided by two times the strike volatility.
- The factor of two comes from the slope of the square: a small change in volatility changes variance by two times the volatility.
- The course likes conversions between the notionals and payoffs, so practice them.

24 Example 7: the two swaps in numbers

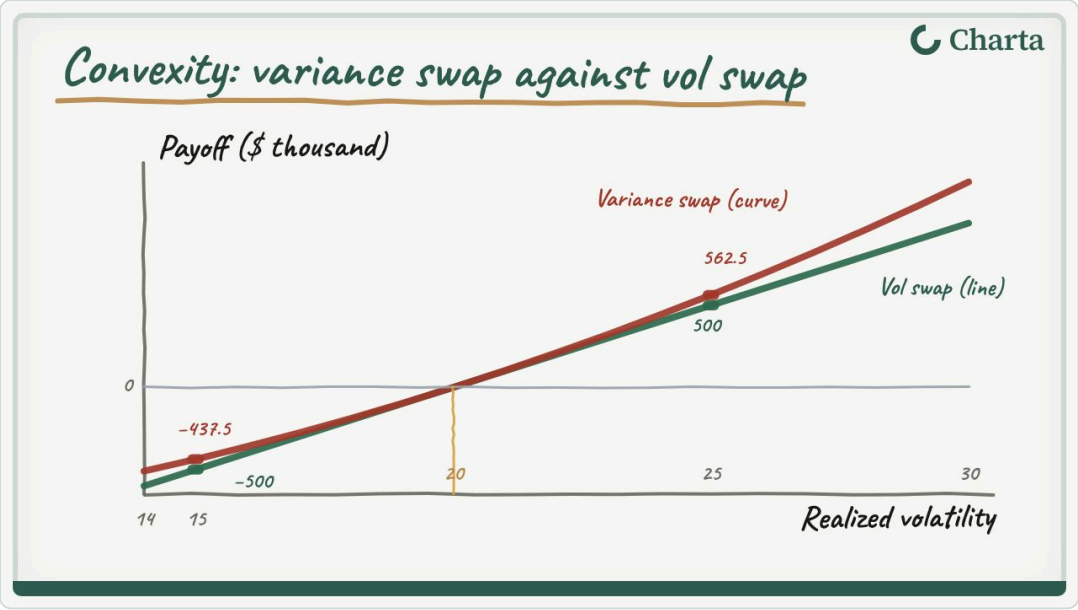
Vega notional $100,000 per point, strike vol 20 (illustration)

Variance notional	100,000 ÷ (2 × 20)	**2,500**
Vol 25: vol swap	100,000 × (25 − 20)	**+$500,000**
Vol 25: variance swap	2,500 × (625 − 400)	**+$562,500**
Vol 15: vol swap	100,000 × (15 − 20)	**−$500,000**
Vol 15: variance swap	2,500 × (225 − 400)	**−$437,500**

IN THE LESSON

- Take a volatility swap with a vega notional of one hundred thousand dollars per volatility point and a strike of twenty.
- The equivalent variance notional is one hundred thousand divided by two times twenty, which is two thousand five hundred.
- Now suppose realized volatility turns out to be twenty five.
- The volatility swap pays one hundred thousand times five, which is five hundred thousand dollars.
- The variance swap pays two thousand five hundred times six hundred twenty five minus four hundred, which is five hundred sixty two thousand five hundred dollars.
- If realized volatility is fifteen instead, the volatility swap loses five hundred thousand, while the variance swap loses two thousand five hundred times minus one hundred seventy five, or four hundred thirty seven thousand five hundred.

25 Convexity: variance swap against vol swap



IN THE LESSON

- This picture shows the convexity.
- Along the bottom is realized volatility, from fourteen to thirty.
- Up the side is the payoff, in thousands of dollars.
- The green straight line is the volatility swap, a line through zero at the strike of twenty.
- The red curve is the variance swap, built with the equivalent notional, so it touches the line at the strike and has the same slope there.
- Away from the strike it bends.
- At twenty five, the variance swap sits above the line, at five hundred sixty two point five against five hundred.
- At fifteen, it sits above again, losing less.
- That is why the fair variance strike is higher than the square of the fair volatility strike.
- The long pays for the convexity, through that higher strike.

26 Static option replication: the reasoning

- Delta hedging: trade constantly, hard near a barrier
- Static: hold a fixed portfolio of vanilla options
- Match the exotic on the boundary and at expiry
- If the boundary is hit, sell at the same value: no cost
- Benefit: no delta hedging; limit: depends on the model

IN THE LESSON

- Now the last idea: how to hedge an exotic.
- Delta hedging means constantly trading the underlying, which is costly and hard near a barrier.
- Static replication offers an alternative.
- Hold a fixed portfolio of vanilla options and never rebalance it.
- The portfolio is chosen so that its value equals the exotic's value everywhere on a boundary, for instance the barrier.
- Its payoff must also equal the exotic's at expiry, in the region where the exotic survives.
- Then if the boundary is hit, the portfolio is sold for the same value the exotic would have had, so the hedge is unwound at no cost.
- The benefit is avoiding delta hedging near a barrier, but the method relies on model assumptions.

27 Example 8: hedging a down-and-out call

Rate = yield = 3%; vol 20%, one year (illustration)

Long call, strike K	strike 100	**1 call**
Short puts, number	K ÷ H = 100 ÷ 80	**1.25**
Short puts, strike	$H^2 \div K$ = 6400 ÷ 100	**64**
Puts if price stays above 80	max(64 − S, 0) = 0	**0**
Value at barrier, net	call 1.15 − puts 1.15	**0.00**

IN THE LESSON

- A famous example uses put call symmetry, which holds when the interest rate equals the dividend yield.
- Take a down and out call with strike K of one hundred and a barrier H of eighty.
- Replicate it by buying one ordinary call with strike one hundred, and selling K divided by H puts, which is one point two five puts, each with strike H squared over K, which is sixty four.
- Check the survival region.
- While the price stays above eighty, the puts struck at sixty four expire worthless, so the portfolio pays the call payoff, as the exotic does.
- Check the barrier.
- With twenty per cent volatility, a rate and dividend yield of three per cent each, and one year, the call is worth one point one five and the puts, one point two five times point nine two, also one point one five.
- The net is zero.

28 Example 9: strips and call spreads

Weights relative to the strike-100 option. Binary pays 10 above 100

Weight at strike 80	$(100 \div 80)^2$	**1.5625**
Weight at strike 100	$(100 \div 100)^2$	**1.00**
Weight at strike 125	$(100 \div 125)^2$	**0.64**
Binary 10: call spread	long 5 at 99, short 5 at 101	**5 spreads**
Spread maximum payoff	5 × (101 − 99)	**10**

IN THE LESSON

- Two more replications.
- A variance swap is replicated by a strip of out of the money options with weights proportional to one over the strike squared.
- Measured against the option at strike one hundred, the weight on the option at strike eighty is one hundred over eighty, squared, which is one point five six two five.
- The weight at one hundred is one.
- The weight at one hundred twenty five is point six four.
- Low strikes get more weight.
- A binary option is hedged by a call spread.
- To replicate a binary paying ten dollars above a trigger of one hundred, buy five calls at ninety nine and sell five calls at one hundred and one.
- The maximum payoff is five times two, which is ten.

29 Four mistakes to avoid

- Not compounding the deferred premium: use c × e^{rT}
- Testing exercise at every node: only where allowed
- Mixing vol and variance units: ÷ (2 × strike vol)
- Judging by venue: design decides vanilla or exotic

IN THE LESSON

- Mistake one: forgetting to compound the deferred premium.
- It must be the premium times the exponential of the rate times the time, and remember that zero cost is not zero risk.
- Mistake two: applying the early exercise test at every node of the tree.
- Apply it only where exercise is permitted, using the strike in force on that date, and always include the terminal payoff.
- Mistake three: mixing up the units of volatility and variance swaps.
- Convert the notional by dividing by two times the strike volatility, and remember variance is in squared points.
- Mistake four: deciding vanilla or exotic by venue.
- It is contract design that decides, not whether the trade was over the counter.

30 Recap: the main ideas

- Vanilla: standard, final price. Exotic: departs
- Exotics: hedging, cost, views, restrictions; model risk
- Zero cost: defer, make contingent, or offset premiums
- Ten payoffs; Asian, lookback, barrier: path dependent
- Variance swap convex; static hedge uses vanillas

IN THE LESSON

- Let's pull it together.
- A vanilla option is a standard contract with a single strike and a payoff that depends only on the final price.
- An exotic departs from that in its payoff, its path, its assets or its exercise rights, and brings more model risk and harder hedging.
- Exotics exist for hedging, cost, views and restrictions.
- Any derivative can be made zero cost by deferring or contingent payment, or by combining options.
- Nonstandard American options come from restricting exercise.
- Know the ten payoffs, and which are path dependent.
- Variance swaps are convex in volatility.
- And static replication hedges an exotic with a fixed portfolio of vanilla options.

31 Try three quick questions

1. Premium 3 deferred 2 years at 4%. Payment at expiry?
   Answer: 3 × e^{0.08} = 3.25.
2. Vega notional 50,000, strike vol 25. Variance notional?
   Answer: 50,000 ÷ (2 × 25) = 1,000.
3. Down-and-out call, K 100, H 80, r = q. Replicate?
   Answer: Long 1 call; short 1.25 puts, strike 64.

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- A premium of three dollars is deferred for two years, at a rate of four per cent.
- What is the payment at expiry?
- Three times the exponential of point zero eight, about three dollars and twenty five cents.
- Question two.
- A volatility swap has a vega notional of fifty thousand dollars per point and a strike of twenty five.
- What variance notional is equivalent?
- Fifty thousand divided by fifty, one thousand.
- Question three.
- A down and out call has strike one hundred and a barrier of eighty, with the rate equal to the dividend yield.
- How is it statically replicated?
- Long one call, and short one point two five puts with strike sixty four.
- The answers are on the board.