CHARTA LEARNING · LESSON NOTES

Interest Rate Futures

FRM Part I

1 A story: two lenders, one loan

- Same loan, same rate, two ways to count the year
- The count changes the interest paid
- Every market has its own custom
- Learn the rules first; the rest is arithmetic

IN THE LESSON

- Picture two lenders who each lend one million dollars for ninety days at five per cent.
- One of them counts the year as three hundred and sixty days, the other as three hundred and sixty five.
- They will not receive the same interest.
- The first gets a little more, because each day counts for a slightly larger slice of the year.
- Neither is wrong; they follow different conventions, and every market has its own custom.
- Interest rate futures sit on top of those customs: bills, bonds and the contracts that settle against them all use particular counting rules, quoting rules and delivery rules.
- Learn the rules first, and the rest becomes arithmetic.

2 Day count conventions: the idea

- Convention = days elapsed and length of the year
- Interest = principal × rate × day count fraction
- Treasury bonds: Actual/Actual
- Corporate and municipal bonds: 30/360
- Bills, commercial paper: Actual/360

IN THE LESSON

- A day count convention answers two questions at once: how many days of interest have accrued between two dates, and how long is a year.
- Interest is then the principal, times the rate, times a day count fraction.
- In United States practice, Treasury bonds use actual over actual: the actual days elapsed, divided by the actual days in the coupon period.
- Corporate and municipal bonds use thirty over three hundred and sixty, where every month counts as thirty days and the year as three hundred and sixty.
- Money market instruments, such as bills and commercial paper, use actual over three hundred and sixty.
- The usual error is applying one convention to every instrument.

3 Which convention for which market

Match the convention to the market before calculating

Market	Convention
US Treasury bonds	Actual/Actual
Corporate, municipal bonds	30/360
Bills, commercial paper	Actual/360
Actual/360 vs Actual/365	360 pays slightly more

IN THE LESSON

- Here are the three conventions side by side.
- Treasury bonds count actual days over the actual days in the coupon period, so the fraction depends on the real calendar.
- Corporate and municipal bonds count thirty over three hundred and sixty: each month is thirty days, each year is three hundred and sixty, whatever the calendar says.
- Money market instruments count actual days over three hundred and sixty.
- Notice the practical consequence.
- At the same quoted rate, actual over three hundred and sixty pays slightly more than actual over three hundred and sixty five, because it divides by a smaller year.
- So before any calculation, match the convention to the market.

4 Example 1: the same loan, two day counts

Illustration: $1,000,000, 5%, 90 days

Actual/360 interest	1,000,000 × 5% × 90/360	**$12,500.00**
Actual/365 interest	1,000,000 × 5% × 90/365	**$12,328.77**
Difference	12,500.00 − 12,328.77	**$171.23**

IN THE LESSON

- Let's make the opening story exact.
- The principal is one million dollars, the rate is five per cent, and the period is ninety days.
- This is an illustration.
- On actual over three hundred and sixty, the fraction is ninety over three hundred and sixty, which is point two five, and the interest is twelve thousand five hundred dollars.
- On actual over three hundred and sixty five, the fraction is ninety over three hundred and sixty five, about point two four six six, and the interest is twelve thousand three hundred and twenty eight dollars and seventy seven cents.
- The difference is one hundred and seventy one dollars and twenty three cents, small per loan but large across a big book.

5 Example 2: counting days on 30/360

Days = 360×ΔY + 30×ΔM + ΔD. Illustration: 6% on $1,000,000

Jan 31 to Mar 31	30×2 + (30−30)	**60 days**
Interest, 60 days	1,000,000 × 6% × 60/360	**$10,000**
Feb 15 to Aug 31	30×6 + (31−15)	**196 days**
Interest, 196 days	1,000,000 × 6% × 196/360	**$32,666.67**

IN THE LESSON

- The thirty over three hundred and sixty count has a formula.
- Days equal three hundred and sixty times the change in year, plus thirty times the change in month, plus the change in day.
- First adjust the dates.
- A start day of thirty one becomes thirty.
- An end day of thirty one becomes thirty only if the start day is thirty or thirty one.
- Take thirty first of January to thirty first of March.
- The start becomes thirty, so the end becomes thirty too.
- Days are thirty times two, plus zero, which is sixty, and interest at six per cent on one million is ten thousand dollars.
- Now fifteenth of February to thirty first of August.
- The start is fifteen, so the end stays thirty one.
- Days are thirty times six, plus sixteen, which is one hundred and ninety six.
- Interest on that is one million times six per cent times one hundred and ninety six over three hundred and sixty, thirty two thousand six hundred and sixty six dollars and sixty seven cents.

6 Treasury bills: a price below face value

- No coupon: buy below face, receive face at maturity
- Quoted as a discount rate, not as a price
- Measured on face value, Actual/360
- Translate the quote into a price before paying
- The discount rate understates the real return

IN THE LESSON

- Now to prices.
- A Treasury bill pays no coupon.
- You buy it below face value and receive the face value at maturity; the gain is the difference.
- Bills are quoted not on price but on a discount rate.
- The discount rate measures the discount from face value as an annualized percentage of face value, using actual over three hundred and sixty.
- That is the money market convention again.
- Think of it as a price tag written in an unusual language: the quote must be translated into a price before you can pay.
- And because the quote is measured on face value rather than on the amount you pay, and uses a short year, it understates your true return.

7 From discount rate to bill price

Price = 100 − d × n ÷ 360
d = (100 − Price) × 360 ÷ n

Bill price	Per 100 of face value
Discount rate	Annualized, in percent
Days to maturity	Actual days to maturity
360-day year	Money market year

IN THE LESSON

- The translation is short.
- Let d be the discount rate in per cent and n the actual number of days to maturity.
- The price per one hundred of face value equals one hundred minus d times n over three hundred and sixty.
- Read it as follows.
- The discount rate is a yearly figure.
- Multiply by n over three hundred and sixty to scale it down to the life of this particular bill.
- Subtract from face value, and you have the price.
- The three common errors are leaving out the n over three hundred and sixty scaling, using three hundred and sixty five in the price formula, and treating the discount rate as a yield on the price paid.

8 Example 3: pricing a 91-day bill

Illustration: 91-day bill at d = 4%; 182-day bill at price 98.50

Discount in price	4 × 91 ÷ 360	**1.01**
Price of 91-day bill	100 − 1.01	**98.99**
Discount, 182-day bill	100 − 98.50	**1.50**
Implied discount rate	1.50 × 360 ÷ 182	**2.97%**

IN THE LESSON

- Take a ninety one day bill quoted at a discount rate of four per cent.
- This is an illustration.
- The discount is four times ninety one over three hundred and sixty, which is one point zero one.
- So the price is one hundred minus one point zero one, or ninety eight point nine nine.
- Now reverse it.
- A different bill, with one hundred and eighty two days left, trades at ninety eight point five.
- The discount from face value is one point five.
- Multiply by three hundred and sixty and divide by one hundred and eighty two, and the discount rate implied by that price is two point nine seven per cent.
- Prices go down when the discount rate goes up, just like yields on any bond.

9 Bond-equivalent yield: the real return

Illustration: the 91-day bill at price 98.99

Gain over the price paid	1.01 ÷ 98.99	**1.02%**
Annualize on 365 days	× 365 ÷ 91	**×4.01**
Bond-equivalent yield	1.02% × 4.01	**4.09%**
Quoted discount rate	for comparison	**4.00%**

IN THE LESSON

- The discount rate is not the return you actually earn.
- The bond equivalent yield, or B E Y, corrects both distortions.
- It is a simple yield on the price paid, not on face value, over a three hundred and sixty five day year.
- Using our ninety one day bill at ninety eight point nine nine: the gain is one point zero one, divided by the price paid, ninety eight point nine nine, which is about one point zero two per cent.
- Scale it up by three hundred and sixty five over ninety one, and the B E Y is four point zero nine per cent.
- That is above the four per cent discount rate quoted.
- Always compare bills and bonds on a yield, not on the discount rate.

10 Clean price, dirty price, accrued interest

Dirty = Clean + Accrued
Accrued = coupon × days since ÷ days in period

Dirty price	Cash paid at settlement
Clean price	The quoted price
Accrued interest	Seller's share of coupon
Semiannual coupon	Half the annual coupon

IN THE LESSON

- Treasury bonds are different.
- A bond is quoted at its clean price, which leaves out the interest accrued since the last coupon.
- The buyer actually pays the dirty price, also called the cash or invoice price.
- Dirty equals clean plus accrued interest.
- Accrued interest is the semiannual coupon times the days since the last coupon, over the days in the coupon period, counted actual over actual.
- Think of selling a flat you rent out: the tenant pays rent on the first, and the buyer who takes over on the tenth should pay you for the nine days that belong to you.
- The seller has held the bond, and is owed his share of the coupon.

11 Example 4: cash paid for a bond

Illustration: 5% bond, 90 days into a 181-day period, quote 98-16

Semiannual coupon	5% ÷ 2 per 100	**2.50**
Accrued interest	2.50 × 90 ÷ 181	**1.24**
Clean price	98 and 16/32	**98.50**
Dirty price	98.50 + 1.24	**99.74**
Cash on $100,000 face	99.74% × 100,000	**$99,740**

IN THE LESSON

- A Treasury bond has a five per cent annual coupon, so the semiannual coupon is two point five per one hundred.
- The last coupon was paid on the fifteenth of January and the next is on the fifteenth of July, a coupon period of one hundred and eighty one days.
- Settlement is the fifteenth of April, ninety days after the last coupon.
- This is an illustration.
- The accrued interest is two point five times ninety over one hundred and eighty one, which is one point two four.
- The quote is ninety eight and sixteen thirty seconds, which is ninety eight point five.
- The dirty price is ninety eight point five plus one point two four, or ninety nine point seven four.
- On one hundred thousand dollars of face value, you pay about ninety nine thousand seven hundred and forty dollars.

12 Reading quotes in thirty-seconds

Quote = whole number + 32nds; it is not a decimal

Quote	Decimal price
98-16	98.50
98-08	98.25
98-24	98.75
98-04	98.125

IN THE LESSON

- Treasury bond quotes use thirty seconds, not decimals.
- A quote of ninety eight dash sixteen means ninety eight and sixteen thirty seconds, which is ninety eight point five.
- Ninety eight dash oh eight is ninety eight and a quarter, and ninety eight dash twenty four is ninety eight and three quarters.
- Reading the part after the dash as a decimal is a classic error.
- Why quote clean?
- Because the dirty price rises every day as interest accrues, then drops by the whole coupon on the payment date, a saw tooth.
- The clean price strips that out, so it moves mainly with yields.
- Other typical errors are using the annual coupon, or subtracting the accrued interest instead of adding it.

13 Why futures need a conversion factor

- Many bonds can be delivered, at different prices
- CF = price per 1 of face at a 6% yield
- Coupon above 6%: CF above 1; below: CF below 1
- Exactly 6% and whole half-years: CF = 1
- CF is fixed in advance; it ignores today's yields

IN THE LESSON

- Now the futures contract.
- The Treasury bond futures contract does not name one bond.
- The seller, the short party, may deliver any of several eligible bonds with different coupons and maturities.
- Those bonds trade at very different prices, so they cannot be swapped one for one.
- The conversion factor, or C F, puts them on a common footing.
- It is the price per one of face value at which the bond would trade if it yielded six per cent a year with semiannual compounding.
- A bond with a coupon above six per cent has a factor above one.
- A coupon below six per cent gives a factor below one.
- Exactly six per cent, with whole half years, gives one.
- The factor is fixed in advance and never moves with market yields.

14 Conversion factors for a 10-year bond

10 years, discounted at 6% semiannual, per 1 of face

Annual coupon	Conversion factor
4% (below 6%)	0.8512
6% (equal)	1.0000
8% (above 6%)	1.1488

IN THE LESSON

- Here are three ten year bonds with annual coupons of four, six and eight per cent.
- We price each at a six per cent yield, compounded semiannually, per one of face value.
- The six per cent bond is the benchmark: it trades at par, so its factor is exactly one.
- The eight per cent bond pays more than the benchmark, so it is worth more than par at six per cent, and its factor is one point one four eight eight.
- The four per cent bond pays less, so its factor is zero point eight five one two.
- Because the factor is fixed, it equalizes bonds only when yields really are six per cent.
- When yields differ, the bonds are not truly equal, and that creates the delivery choice we study soon.

15 Example 5: computing a conversion factor

Illustration: 8% bond, 10 years, discounted at 3% per half-year

Coupon and periods	8% ÷ 2 = 4; 10 × 2 = 20	**4; 20**
PV of coupons	4 × 14.8775	**59.51**
PV of face value	100 ÷ 1.03^20	**55.37**
Conversion factor	(59.51 + 55.37) ÷ 100	**1.1488**

IN THE LESSON

- Let's compute the factor for the eight per cent, ten year bond.
- The semiannual coupon is four per one hundred.
- There are ten times two, or twenty, periods.
- A common error is to discount annually, or to forget to double the periods.
- Discount each period at three per cent, half of six.
- The coupons are worth four times fourteen point eight seven seven five, about fifty nine point five one.
- The face value is worth one hundred divided by one point zero three to the twentieth, which is fifty five point three seven.
- Add them and you get one hundred and fourteen point eight eight.
- Divide by one hundred, and the conversion factor is one point one four eight eight.

16 Example 6: when three months are left over

Illustration: 8% bond, 10 years 3 months after rounding down

Value at first coupon date	114.8775 + 4	**118.88**
Discount three months	118.8775 ÷ 1.03^{0.5}	**117.13**
Less accrued, 3 months	half of 4	**−2.00**
Conversion factor	115.13 ÷ 100	**1.1513**

IN THE LESSON

- Maturities are first rounded down to the nearest three months.
- Suppose the eight per cent bond has ten years and four months left.
- Round down to ten years and three months.
- That is three months over a whole number of half years, so the first coupon is three months away.
- Start from the value of the twenty period bond, one hundred and fourteen point eight eight, and add the four coupon due at that first date, giving one hundred and eighteen point eight eight.
- Discount back three months, half a period at three per cent, to get one hundred and seventeen point one three.
- Then subtract the accrued interest for the three months, which is half the semiannual coupon, so two.
- That leaves one hundred and fifteen point one three, and a factor of one point one five one three.

17 The cost of delivering a bond

Invoice = F × CF + accrued
Cost = Bond price − Futures price × CF
Lowest cost = CTD; negative = gain

Delivery cost	Per 100 of face value
Quoted bond price	Quoted, accrued cancels
Futures price	Settlement price of the future
Conversion factor	Multiply, never divide

IN THE LESSON

- Which bond should the short party deliver?
- Start with the cash received.
- The invoice price equals the settlement futures price times the conversion factor, plus the accrued interest on the delivered bond.
- The cost of buying that bond is its quoted price plus the same accrued interest.
- The accrued interest appears on both sides and cancels.
- What remains is the delivery cost: the quoted bond price, minus the settlement futures price times the conversion factor, per one hundred of face value.
- The bond with the lowest cost is the cheapest to deliver, or C T D.
- A negative cost means that buying and delivering the bond produces a gain.
- The usual errors are multiplying the bond price by the factor, or dividing.

18 Example 7: finding the cheapest to deliver

Illustration: settlement futures price 110.00

Bond A: 118.50, CF 1.0650	118.50 − 117.15	**1.35**
Bond B: 99.20, CF 0.9000	99.20 − 99.00	**0.20**
Bond C: 131.40, CF 1.1900	131.40 − 130.90	**0.50**
Cheapest: bond B	0.20 per 100	**$200/100k**

IN THE LESSON

- Settlement futures price is one hundred and ten, an illustration.
- Bond A is quoted at one hundred and eighteen point five with a factor of one point zero six five.
- The futures price times the factor is one hundred and seventeen point one five, so the cost is one point three five.
- Bond B is quoted at ninety nine point two with a factor of point nine.
- One hundred and ten times point nine is ninety nine, so the cost is point two.
- Bond C is quoted at one hundred and thirty one point four with a factor of one point one nine.
- The product is one hundred and thirty point nine, so the cost is point five.
- Bond B is cheapest, at twenty cents per hundred, or two hundred dollars on one hundred thousand of face value.

19 Yield level and the cheapest bond

Rules of thumb, not guarantees

Yield level or curve	Cheapest bond tends to be
Yields above 6%	Low coupon, long maturity
Yields below 6%	High coupon, short maturity
Upward-sloping curve	Long maturity
Downward-sloping curve	Short maturity

IN THE LESSON

- The conversion factor system assumes a six per cent yield, so which bond is cheapest depends on where yields really are.
- When yields are above six per cent, low coupon, long maturity bonds, with high duration, tend to be cheapest.
- Their prices fall furthest relative to factors fixed at six.
- When yields are below six per cent, high coupon, short maturity bonds, with low duration, tend to be cheapest.
- The shape of the curve matters too.
- With an upward sloping curve the cheapest bond tends to have a long maturity, and with a downward sloping curve a short one.
- These are rules of thumb, not guarantees.
- The usual mistake is reversing them.

20 Example 8: the winner flips with yields

X: 4% coupon, 25 yrs, CF 0.7427. Y: 10%, 5 yrs, CF 1.1706

Yields 8%: X price ÷ CF	57.04 ÷ 0.7427	**76.79**
Yields 8%: Y price ÷ CF	108.11 ÷ 1.1706	**92.35**
Yields 4%: X price ÷ CF	100.00 ÷ 0.7427	**134.64**
Yields 4%: Y price ÷ CF	126.95 ÷ 1.1706	**108.45**

IN THE LESSON

- Compare two bonds.
- Bond X has a four per cent coupon and twenty five years left, a factor of point seven four two seven.
- Bond Y has a ten per cent coupon and five years left, a factor of one point one seven zero six.
- Market yields are flat, an illustration.
- The market prices the future near the lowest price per unit of factor, so the lower ratio identifies the cheaper bond.
- At eight per cent, X is priced at fifty seven point zero four, a ratio of seventy six point seven nine.
- Y is priced at one hundred and eight point one one, a ratio of ninety two point three five.
- X is cheapest.
- At four per cent, X is at one hundred, a ratio of one hundred and thirty four point six four, and Y is at one hundred and twenty six point nine five, a ratio of one hundred and eight point four five.
- Now Y wins.

21 When the cheapest bond changes

- The future tracks the cheapest bond to deliver
- Low coupon, long maturity: high duration
- High coupon, short maturity: low duration
- A change of cheapest bond changes the future's duration
- Rankings can flip as yields move

IN THE LESSON

- Look at what that flip does.
- The futures contract behaves like the cheapest bond, so when the cheapest bond changes, the contract changes too.
- A long maturity, low coupon bond has a high duration; a short maturity, high coupon bond has a low one.
- If the cheapest to deliver switches from X to Y, the futures contract's duration falls, and it reacts less to yield changes than before.
- This matters for pricing, because the futures price follows the cheapest bond, and it matters for hedging, because the hedge ratio uses that duration.
- Remember too that the ranking can change as yields move, so the rules of thumb are only a guide.

22 The theoretical price of the future

Cash F = (S − I) × e^{rT}
Quoted F = (Cash F − accrued at delivery) ÷ CF

Cash price today	Quoted price + accrued
Coupons paid before delivery	PV at the risk-free rate
Compounding forward	Grow to delivery date T
Quoted futures price	Divide by CF, never multiply

IN THE LESSON

- How should the futures price be set?
- Treat the contract as a forward on the cheapest bond, which pays coupons.
- Step one: take that bond's cash price today, S, its quoted price plus accrued interest.
- Step two: subtract I, the present value of coupons paid before delivery, each discounted at the risk free rate.
- Step three: compound what is left forward to the delivery date.
- That gives the cash futures price.
- Step four: subtract the accrued interest at delivery to get a quoted price, then divide by the conversion factor.
- Typical errors are starting from the quoted price, adding the coupons instead of subtracting, forgetting the accrued interest, and multiplying by the factor.
- If the market is above this value, sell futures and buy the bond.

23 Example 9: pricing the future

Illustration: 8% CTD, quote 112, CF 1.10, r = 5%, delivery in 0.5 year

Cash price today	112 + 1.33	**113.33**
PV of the coupon	4 × e^{−0.05 × 1/3}	**3.93**
Cash futures price	(113.33 − 3.93) × e^{0.025}	**112.17**
Quoted futures price	(112.17 − 1.33) ÷ 1.10	**100.76**

IN THE LESSON

- An illustration.
- The cheapest bond has an eight per cent coupon, a quote of one hundred and twelve, a factor of one point one, and accrued interest today of one point three three.
- Its cash price is one hundred and thirteen point three three.
- The next coupon of four is paid in four months, and delivery is in six months.
- The risk free rate is five per cent, continuously compounded.
- The present value of that coupon is three point nine three.
- Subtract it to leave one hundred and nine point four, and compound for half a year to get a cash futures price of one hundred and twelve point one seven.
- At delivery, accrued interest is again one point three three.
- The quoted bond price is one hundred and ten point eight four, and dividing by one point one gives a futures price of one hundred point seven six.

24 Duration hedging: how many contracts

N = (P × D_P) ÷ (F × D_F)

P: portfolio value	Market value of the bonds
D_P: portfolio duration	Modified duration of portfolio
F: one contract's value	Quote × contract face size
D_F: cheapest bond's duration	Duration of the CTD bond

IN THE LESSON

- Now the second main use of these contracts: hedging.
- A duration based hedge matches the price sensitivity of the portfolio to that of the futures.
- The number of contracts is the portfolio value times its duration, divided by the value of one futures contract times the duration of the cheapest bond at delivery.
- A quote of one hundred and ten on a contract with one hundred thousand face value means one hundred and ten thousand dollars per contract; do not use the face value.
- Decide the direction first.
- If the portfolio loses when yields rise, sell futures.
- If you will buy bonds later and fear yields will fall, buy futures.
- Inverting the duration ratio, and getting the sign wrong, are the common mistakes.

25 Example 10: hedging a bond portfolio

Illustration: P = 10,000,000, D_P = 6.5, F = 110,000, D_F = 8.0

Portfolio sensitivity	10,000,000 × 6.5	**65,000,000**
One contract's sensitivity	110,000 × 8.0	**880,000**
Contracts	65,000,000 ÷ 880,000	**73.86**
Trade	loses if yields rise	**Sell 74**

IN THE LESSON

- A manager holds ten million dollars of bonds with a duration of six point five.
- The futures price is one hundred and ten, so one contract is worth one hundred and ten thousand dollars, and the cheapest bond has a duration of eight.
- All are illustrations.
- The portfolio's sensitivity is ten million times six point five, which is sixty five million.
- One contract's sensitivity is one hundred and ten thousand times eight, which is eight hundred and eighty thousand.
- Divide sixty five million by eight hundred and eighty thousand to get seventy three point eight six.
- Round to seventy four.
- The portfolio loses when yields rise, so the manager sells seventy four contracts.

26 Example 11: moving to a target duration

N = (D_T − D_P) × P ÷ (F × D_F). Target D_T = 3.0

Change in duration	3.0 − 6.5	**−3.5**
Required sensitivity	−3.5 × 10,000,000	**−35,000,000**
Contracts	−35,000,000 ÷ 880,000	**−39.77**
Trade	negative means sell	**Sell 40**

IN THE LESSON

- The same idea can change the portfolio's duration rather than remove it.
- The formula becomes the target duration minus the current duration, times the portfolio value, divided by the futures value times its duration.
- Suppose the manager wants to cut duration from six point five to three.
- This is an illustration.
- The change in duration is minus three point five.
- Times ten million, that is minus thirty five million.
- The sensitivity of one contract is still eight hundred and eighty thousand.
- Divide, and the answer is minus thirty nine point seven seven.
- The negative sign means sell, so the manager sells forty contracts.
- That is fewer than the full hedge, because only part of the risk is removed.

27 Limits of a duration hedge

- Assumes small, parallel yield shifts
- Linear approximation: convexity gaps on big moves
- Cheapest bond changes: the hedge ratio goes stale
- Portfolio and Treasury yields differ: spread risk
- Durations drift: rebalance

IN THE LESSON

- A duration hedge works well only under restrictive assumptions.
- It assumes small, parallel shifts in yields.
- If short and long yields move by different amounts, portfolio and futures respond differently, and a residual loss remains.
- Duration is a linear approximation, so for large moves, differences in convexity leave a gap.
- The futures duration is that of the cheapest bond, so when the cheapest bond changes, the hedge ratio goes stale.
- The portfolio's yield may not move one for one with the futures yield: a corporate bond portfolio hedged with Treasury futures carries credit spread risk.
- And durations drift over time, so the hedge needs rebalancing.

28 Example 12: a hedge that leaks

Illustration: sell 74 contracts; portfolio yield +0.10%

Portfolio loss	10,000,000 × 6.5 × 0.001	**−$65,000**
Futures gain, +0.10%	74 × 110,000 × 8 × 0.001	**+$65,120**
Futures gain, +0.08%	74 × 110,000 × 8 × 0.0008	**+$52,096**
Net if futures +0.08%	−65,000 + 52,096	**−$12,904**

IN THE LESSON

- Return to the seventy four contracts.
- Suppose the portfolio's yield rises by ten basis points, point one per cent.
- The portfolio loses ten million times six point five times point zero zero one, which is sixty five thousand dollars.
- If the futures yield also rises by ten basis points, the futures gain seventy four times one hundred and ten thousand times eight times point zero zero one, or sixty five thousand one hundred and twenty dollars.
- A near perfect hedge.
- But suppose the futures yield rises only eight basis points.
- The futures gain just fifty two thousand and ninety six dollars, and the net result is a loss of twelve thousand nine hundred and four.
- The hedge protects against parallel moves in the relevant yield curve, and is weaker otherwise.

29 Four mistakes to avoid

- One day count everywhere: match it to the market
- Accrued subtracted or annual: add semiannual accrued
- Bond price × CF: use futures price × CF
- Wrong hedge: use quote not face; sell if yields hurt

IN THE LESSON

- Four mistakes cause most errors in this topic.
- First, using one day count for every instrument.
- Match the convention to the market: actual over actual for Treasury bonds, thirty over three hundred and sixty for corporates, actual over three hundred and sixty for bills.
- Second, subtracting accrued interest when you should add it, or using the annual coupon.
- Cash paid is clean plus accrued, using the semiannual coupon.
- Third, multiplying the bond price by the conversion factor.
- The delivery cost uses the futures price times the factor.
- Fourth, getting the hedge wrong: use the futures price rather than the face value, and sell when the portfolio loses as yields rise.

30 Recap

- Day counts: match convention to market
- Bill price = 100 − d × n ÷ 360; dirty = clean + accrued
- CF = value at a 6% yield; cost = price − F × CF
- Futures price: forward on the cheapest bond
- Hedge: N = P × D_P ÷ (F × D_F); parallel moves only

IN THE LESSON

- Let's pull the lesson together.
- Day counts fix how many days accrue and how long a year is: actual over actual for Treasury bonds, thirty over three hundred and sixty for corporates, and actual over three hundred and sixty for bills.
- A bill price is one hundred minus the discount rate times days over three hundred and sixty, and the bond equivalent yield is higher.
- A bond's cash price is clean plus accrued.
- The conversion factor is the price at a six per cent yield.
- The cheapest bond minimizes the bond price less the futures price times the factor, and it depends on the level and shape of yields.
- The future is priced as a forward on that bond, and duration based hedging uses the number of contracts formula, with its limits.

31 Quiz: three questions

1. 182-day bill, discount rate 3%: price per 100?
   Answer: 100 − 3 × 182 ÷ 360 = 98.48
2. Clean 100, coupon 3 a half-year, day 60 of 180: cash price?
   Answer: Accrued 1.00, so 101.00
3. P $5m, D 4; F 100,000, CTD D 5: contracts?
   Answer: 20m ÷ 500,000 = sell 40

IN THE LESSON

- Time to test yourself.
- Question one.
- A Treasury bill has one hundred and eighty two days to maturity and a discount rate of three per cent.
- What is its price per one hundred of face value?
- Question two.
- A bond is quoted clean at one hundred.
- Its semiannual coupon is three, and it is sixty days into a coupon period of one hundred and eighty days.
- What cash price does the buyer pay?
- Question three.
- A portfolio worth five million dollars has a duration of four.
- The futures price is one hundred, on a contract of one hundred thousand face value, and the cheapest bond has a duration of five.
- How many contracts should be sold?
- The answers are on the board.