CHARTA LEARNING · LESSON NOTES

Introduction to Derivatives

FRM Part I

1 A story: the farmer and the baker

- A farmer fears a low wheat price; a baker fears a high one
- They fix today a price for delivery in the autumn
- The promise gets its value from wheat, the underlying
- Both swap an uncertain price for a certain one

IN THE LESSON

- Picture a wheat farmer who plants in spring and harvests in autumn.
- Between those two dates the price of wheat can swing either way, and a bad swing could ruin her year.
- Across town a baker needs wheat in the autumn and fears the opposite swing.
- They agree today that she will deliver wheat in autumn at a price fixed now.
- Neither pays anything today.
- That simple promise is a derivative.
- Its value depends on the price of wheat, which is the underlying.
- If wheat ends up dear, the baker wins and the farmer regrets it.
- If wheat ends up cheap, the reverse.
- Both have swapped an uncertain price for a certain one.

2 What a derivative is and its main features

- Value derived from an underlying asset, rate, index or event
- Settled in the future; little or no outlay now (leverage)
- Zero-sum between the two parties, before costs
- Uses: hedge, speculate, arbitrage, change exposure cheaply

IN THE LESSON

- A derivative is a contract whose value is derived from the price or level of an underlying asset, rate, index or event.
- Three features define it.
- First, it settles in the future, not today.
- Second, it needs little or no money at the start compared with the exposure it creates.
- That is leverage.
- Third, before costs, it is zero sum: whatever one side gains, the other side loses.
- Derivatives serve four main purposes.
- People hedge to reduce a risk they already have.
- They speculate, betting on a price move.
- They arbitrage, locking in a riskless profit from inconsistent prices.
- And they change an exposure cheaply, without trading the underlying itself.

3 Example 1: leverage cuts both ways

Illustration: 100 units at $100; margin 10% of the notional

Notional exposure	100 units × $100	**$10,000**
Margin posted	10% of notional	**$1,000**
Price up 1%	100 units × $1	**+$100 (+10%)**
Price down 5%	100 units × −$5	**−$500 (−50%)**

IN THE LESSON

- Let's see leverage in numbers, using made up figures.
- Suppose you control one hundred units of an asset priced at one hundred dollars through a contract, so the notional exposure is ten thousand dollars.
- You post a margin of one thousand dollars, ten per cent of the exposure.
- Now the price rises by one per cent, one dollar a unit.
- You gain one hundred dollars, which is ten per cent of your margin.
- A tiny move in the price became a large move in your money.
- But the same leverage works backwards.
- If the price falls by five per cent, five dollars a unit, you lose five hundred dollars, which is fifty per cent of your margin.
- Leverage magnifies gains and losses equally.
- It is not extra expected return.

4 Linear and non-linear derivatives

Two families of derivative

	Linear | Non-linear
Examples	Forward, future, swap | Option
Payoff shape	Straight line | Kinked at strike
Value depends on	Spot, rates | Spot, rates, volatility
Exposure to price move	One for one | Changes with price

IN THE LESSON

- Derivatives come in two broad families.
- Linear derivatives are forwards, futures and swaps.
- Their payoff moves one for one with the underlying: if the price rises by a dollar, the position gains a dollar, and if it falls, it loses a dollar.
- Plot the payoff and you get a straight line.
- Because of that, they can be valued from the spot price and financing rates alone.
- Non-linear derivatives are options.
- Their payoff is kinked: flat on one side of a price called the strike, and rising on the other.
- To value an option you also need an estimate of volatility, how much the underlying is likely to move.
- Remember linear means a straight line, non-linear means a kink.

5 Exchange-traded derivatives

- Standard size, maturity and delivery terms
- Clearing house is the counterparty to both sides
- Initial margin, then daily marking to market
- Losers pay variation margin daily; prices visible
- Easy to close out with an opposite trade

IN THE LESSON

- Now to where derivatives trade.
- Exchange-traded contracts are standardized in size, maturity and delivery terms, and they trade on an organized exchange.
- A clearing house sits in the middle and becomes the counterparty to both sides, so the buyer and the seller each face the clearing house and not each other.
- To control credit risk, it asks for initial margin at the start, and it marks every position to market each day.
- The side that lost that day pays variation margin, and the side that gained receives it.
- Because contracts are standard, prices are visible to everyone, and a position is easy to close out by simply taking the opposite trade.

6 Example 2: daily margin on a futures position

Illustration: long 100 units at $100; initial margin $1,000

Day 1, settles at $98	100 × (98 − 100)	**−$200 paid**
Day 2, settles at $101	100 × (101 − 98)	**+$300 received**
Net variation margin	−200 + 300	**+$100**
Check against the start	100 × (101 − 100)	**+$100**

IN THE LESSON

- Let's follow the daily settlement on a futures position, with made up prices.
- You are long one hundred units, bought at one hundred dollars, and you posted an initial margin of one thousand dollars.
- At the end of day one, the settlement price is ninety eight.
- You lost two dollars a unit, so you pay two hundred dollars of variation margin.
- On day two the price closes at one hundred and one, up three dollars from the day before, so you receive three hundred dollars.
- Net, you have received one hundred dollars.
- Check that against the start: the price is one dollar above where you bought, on one hundred units, so one hundred dollars.
- The daily payments add up to the total gain, just paid in small pieces.

7 Over-the-counter derivatives

- Negotiated privately between a dealer and a client
- Tailored exactly to the exposure being covered
- Counterparty credit risk, less transparency, less liquidity
- After 2008: more standardized OTC trades cleared
- Non-cleared trades face margin requirements

IN THE LESSON

- Over the counter, or OTC, means a trade negotiated directly between two parties, usually a dealer and a client, with no exchange involved.
- The great advantage is customization.
- The size, the date, the underlying and the settlement can be tailored exactly to the exposure the client wants to cover.
- The price is paid in other ways.
- There is counterparty credit risk, because each side depends on the other to pay.
- There is less transparency, since the terms are private.
- And the contract is usually less liquid, because to leave it you must negotiate with the dealer or find someone to take it over.
- Since the two thousand eight crisis, standardized OTC trades are increasingly cleared through central counterparties, and trades that are not cleared face margin requirements.

8 Exchange against over the counter

The trade-off: customization versus protection and liquidity

	Exchange | OTC
Contract terms	Standard | Tailored
Credit risk	Clearing house | Counterparty
Transparency	High | Low
Closing out	Easy | Harder

IN THE LESSON

- Here is the comparison side by side.
- On terms, an exchange offers standard contracts, while the OTC market lets you tailor every detail.
- On credit risk, the clearing house and its margin system protect exchange traders, whereas OTC users carry the risk of the other party defaulting, which is reduced only by collateral, netting or central clearing.
- On transparency, exchange prices are public, while OTC prices are private.
- On liquidity, exchange positions are easy to close, while OTC positions are harder.
- The trade-off to remember is customization on one side, against credit protection and liquidity on the other.
- A hedger with an unusual exposure often accepts the credit risk to get the exact fit.

9 Forward, futures and option compared

Obligation or right?

	Forward | Future | Option
Venue	OTC | Exchange | Either
Obligation	Both | Both | Writer only
Paid at the start	Nothing | Margin | Premium
Settlement	At maturity | Daily | On exercise

IN THE LESSON

- Now let's separate the three basic contracts.
- A forward is a private agreement to buy or sell an asset at a fixed price on a future date.
- Both sides are obliged to go through with it.
- Nothing is paid at the start, and it settles only at maturity, which leaves credit risk.
- A futures contract carries the same obligation, but it is standardized, exchange-traded and marked to market daily, with margin and a clearing house limiting credit risk.
- An option is different in one decisive way.
- The buyer holds a right, not an obligation, and pays a premium up front for it.
- The seller of an option, called the writer, must perform if the buyer chooses to exercise.

10 Calls, puts and exercise styles

- Call = right to buy at the strike; put = right to sell
- Holder pays the premium and holds the right
- Writer receives the premium and must perform if exercised
- European: at expiry only. American: any time to expiry

IN THE LESSON

- An option comes in two flavors.
- A call gives its holder the right to buy the underlying at the strike price.
- A put gives the right to sell at the strike price.
- The strike is the fixed price written into the contract.
- The buyer, or holder, pays the premium at the start and has the right but not the duty.
- The writer receives the premium and must perform if the holder exercises: sell at the strike for a call, buy at the strike for a put.
- There are also two exercise styles.
- A European option can be exercised only at expiry, while an American option can be exercised at any time up to expiry.
- The names say nothing about geography.

11 Payoffs at maturity

Long forward = S − K (S final price, K strike)
Long call = max(S − K, 0)
Long put = max(K − S, 0)

Long forward payoff	Short = K − S; both linear
Long call payoff	Never negative: kinked at K
Long put payoff	Never negative: kinked at K
Writer's side	Writer profit = premium − payoff
Profit after premium	Buyer profit = payoff − premium

IN THE LESSON

- The payoff is what a position is worth at maturity, before any cost of entry.
- Let K be the delivery price or strike, and S the price of the underlying at maturity, the final price.
- A long forward pays S minus K, and a short forward pays K minus S.
- Both can be positive or negative, and both are straight lines.
- A long call pays the larger of the final price minus K, or zero.
- A long put pays the larger of K minus the final price, or zero.
- The writer of either option pays the negative of that amount.
- Profit is different from payoff: for the buyer it is the payoff minus the premium, and for the writer it is the premium minus the payoff.

12 Example 3: forward payoffs, long and short

Illustration: delivery price K = 100, contract size 1,000 units

Long, final price 120	1,000 × (120 − 100)	**+$20,000**
Long, final price 80	1,000 × (80 − 100)	**−$20,000**
Short, final price 120	1,000 × (100 − 120)	**−$20,000**
Short, final price 80	1,000 × (100 − 80)	**+$20,000**
Long + short, any price	+20,000 − 20,000	**$0**

IN THE LESSON

- Let's compute some forward payoffs.
- These are made up numbers.
- A forward has a delivery price of one hundred, on a contract size of one thousand units.
- If the final price is one hundred and twenty, the long position gains twenty a unit, so twenty thousand dollars.
- If the final price is eighty, the long position loses twenty a unit, minus twenty thousand dollars.
- The short forward is the mirror image.
- At one hundred and twenty the short loses twenty thousand, because it must sell at one hundred something worth one hundred and twenty.
- Now add the long and the short at any final price: twenty thousand gained and twenty thousand lost, so the total is zero.
- That is the zero sum feature.
- Notice that the multiplication by contract size comes last.

13 Example 4: call and put, payoff and profit

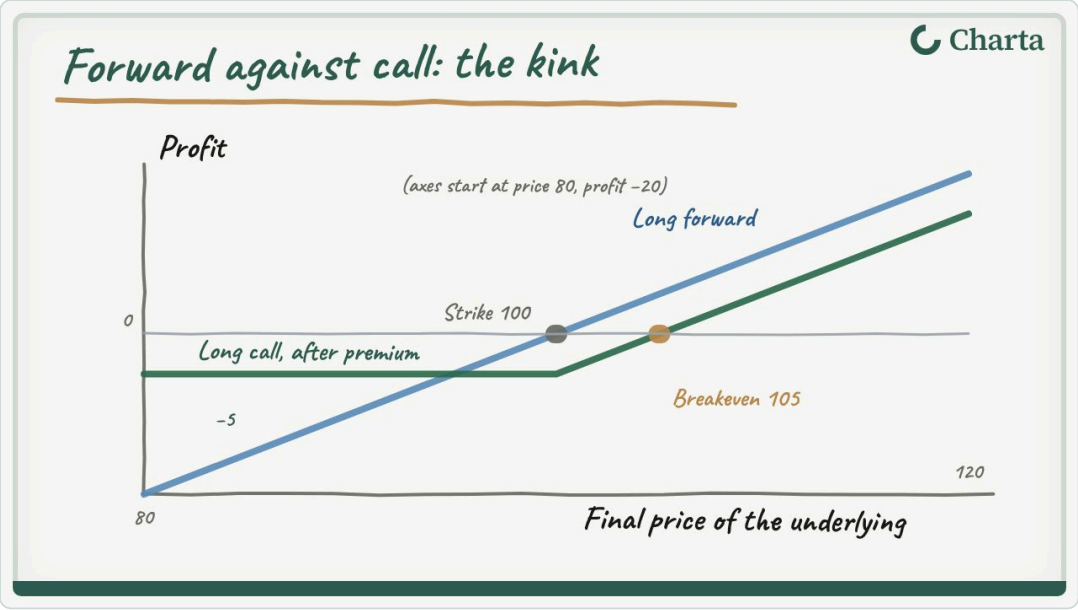
Illustration: strike 100; call premium $5; put premium $4

Call payoff, price 120	max(120 − 100, 0)	**20**
Call profit	20 − 5 premium	**15**
Put payoff, price 120	max(100 − 120, 0)	**0**
Put profit	0 − 4 premium	**−4**
Put profit, price 80	max(100 − 80, 0) − 4	**16**

IN THE LESSON

- Now options, with the same strike of one hundred.
- The call costs five dollars a unit and the put costs four.
- Say the final price is one hundred and twenty.
- The call pays the larger of twenty and zero, which is twenty.
- Subtract the five dollar premium and the profit is fifteen.
- The put at that price pays the larger of minus twenty and zero, which is zero, so the profit is minus four: the whole premium is lost.
- Now say the final price is eighty.
- The put pays twenty, and its profit is twenty minus four, which is sixteen.
- On a contract of one thousand units, each of these profits is simply multiplied by one thousand.
- A buyer can never lose more than the premium.

14 Forward against call: the kink



IN THE LESSON

- Here are two payoffs drawn on the same picture.
- Along the bottom is the final price of the underlying, from eighty to one hundred and twenty.
- Up the side is profit, and the grey line marks zero.
- The blue line is a long forward with a delivery price of one hundred.
- It is a single straight line through the point where the price equals one hundred, rising and falling dollar for dollar.
- The green line is a long call with the same strike, after paying the five dollar premium.
- It is flat at minus five all the way up to the strike, then bends upward.
- It crosses zero only at one hundred and five, the strike plus the premium.
- The kink is the signature of an option.

15 Example 5: the writer's side

Illustration: written call, strike 100, premium received $5

Writer payoff, price 120	−max(120 − 100, 0)	**−20**
Writer profit	−20 + 5 premium	**−15**
Writer payoff, price 90	−max(90 − 100, 0)	**0**
Writer profit, price 90	0 + 5 premium	**+5**
Loss on 1,000 units at 120	−15 × 1,000	**−$15,000**

IN THE LESSON

- The writer of an option sees the buyer's picture upside down.
- Take the writer of the call from the last example, who received five dollars a unit.
- If the final price is one hundred and twenty, the holder exercises, and the writer's payoff is minus twenty.
- Add the five dollar premium received, and the writer's profit is minus fifteen.
- If the final price is ninety, the call expires worthless, the payoff is zero, and the writer keeps the premium: a profit of five.
- That is the most the writer can ever make.
- Multiply the loss at one hundred and twenty by a contract of one thousand units and it is fifteen thousand dollars, and a higher price would lose even more.
- A common mistake is forgetting to flip the sign for the writer.

16 Hedgers, speculators and arbitrageurs

- Hedger: has a price risk, takes an offsetting position
- Speculator: no exposure, bets on a price move, adds liquidity
- Arbitrageur: locks in a riskless profit, restores price links
- The category depends on purpose, not on the instrument

IN THE LESSON

- Derivative users fall into three broad groups.
- Hedgers already face a price risk, such as a farmer who owns wheat, or an importer who owes foreign currency.
- They take an offsetting derivative position to reduce that risk, and they accept that they give up some potential gain.
- Speculators have no underlying exposure.
- They take positions to bet on price moves, which adds risk in search of profit, and they supply liquidity to the market.
- Arbitrageurs lock in a riskless profit by trading the same or equivalent assets at inconsistent prices, and their trading pushes prices back into line.
- The category depends on purpose, not on the instrument.
- The same forward may be a hedge for one party and speculation for the other.

17 Classify the trader

Judge the purpose, not the product

Case	Category
Farmer sells forward on her harvest	Hedger
Fund buys oil futures, no oil business	Speculator
Buys cheap, sells dear, locks in gain	Arbitrageur
Owes $1m, buys $5m forward	Hedge $1m, speculate $4m

IN THE LESSON

- Let's practice classifying, because the test is always purpose.
- A wheat farmer sells a forward on her expected harvest.
- She has a price risk and is reducing it, so she is a hedger.
- A fund manager with no oil business buys oil futures because she expects prices to rise.
- No underlying exposure, a bet on direction: a speculator.
- A trader sees the same asset cheaper in one market than in another and buys one while selling the other, locking in a gain.
- That is an arbitrageur.
- Last, an importer who owes about a million dollars buys forward currency worth five million.
- The first million is a hedge, and the extra four million is speculation.
- Position size matters, not just the instrument.

18 Hedging with a forward or with an option

- Sell a forward: price fixed, nothing paid, upside given up
- Buy a put: floor at the strike, upside kept, premium paid
- Put revenue = max(S, K) − premium; forward: price fixed
- Forward can show a loss; the put costs the premium
- Options suit contingent exposures

IN THE LESSON

- Take a hedger who will sell an asset in the future.
- One choice is to sell a forward.
- That fixes the price.
- Gains and losses on the exposure are offset, and nothing is paid up front.
- But the hedger gives up any favorable move, and the forward itself can show a loss.
- The other choice is to buy a put.
- This sets a floor, like an insurance policy.
- The downside is protected at the strike and the upside is kept, but the premium must be paid.
- In effect the revenue per unit is the larger of the final price and the strike, minus the premium.
- Options also suit contingent exposures, because a forward can leave an unwanted position if the underlying deal falls through.

19 Example 6: the farmer's two hedges

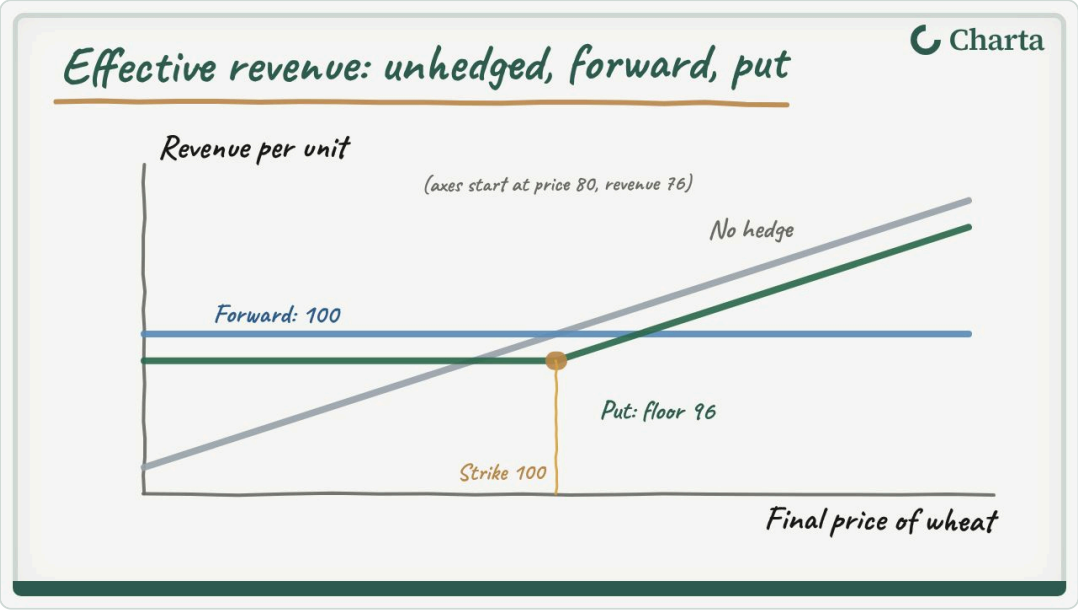
Illustration: forward price 100; put strike 100, premium 4; interest ignored

Forward, final price 80	revenue is F	**100**
Forward, final price 120	revenue is F	**100**
Put, final price 80	max(80, 100) − 4	**96**
Put, final price 120	max(120, 100) − 4	**116**

IN THE LESSON

- Back to the farmer, with made up figures.
- She will sell one unit of wheat in three months.
- Today's forward price is one hundred, and a put with a strike of one hundred costs four.
- Interest is ignored.
- Suppose she sells the forward.
- If the final price turns out to be eighty, her revenue is one hundred.
- If it turns out to be one hundred and twenty, her revenue is still one hundred, because the forward fixes the price.
- Now suppose she buys the put instead.
- At eighty she exercises, getting the larger of eighty and one hundred, which is one hundred, less four: ninety six.
- At one hundred and twenty she lets the put lapse, selling at the market for one hundred and twenty, less four: one hundred and sixteen.

20 Effective revenue: unhedged, forward, put



IN THE LESSON

- Here is the farmer's revenue per unit for every final price.
- The grey line is no hedge at all: revenue simply equals the market price, rising from eighty to one hundred and twenty.
- The blue line is the forward: flat at one hundred, whatever happens.
- It removes the risk and also the chance of gain.
- The green line is the put.
- At low prices it is flat at ninety six, one hundred less the premium of four.
- That is the floor.
- Above the strike it rises with the price, always four below the grey line.
- So the put costs the farmer four dollars in the good case, and pays off in the bad one.
- Which is better depends on the view and on whether she is willing to pay for insurance.

21 Example 7: the importer's two hedges

Illustration: forward price 100; call strike 100, premium 5

Long forward, price 120	cost is F	**100**
Long forward, price 80	cost is F	**100**
Call, final price 120	min(120, 100) + 5	**105**
Call, final price 80	min(80, 100) + 5	**85**

IN THE LESSON

- Now the mirror image.
- An importer will need to buy one unit of the same asset in three months and fears a price rise.
- A long forward at one hundred fixes the cost.
- Whether the final price is one hundred and twenty or eighty, the cost is one hundred.
- Notice that at eighty the forward looks regrettable, since the importer could have bought for eighty.
- Alternatively, the importer buys a call with a strike of one hundred, for a premium of five.
- At a final price of one hundred and twenty, the call is exercised, and the cost is the smaller of the market price and the strike, so one hundred, plus five: one hundred and five.
- At eighty, the call lapses, the importer buys at eighty, and pays five more: eighty five.
- The call caps the cost.

22 Example 8: speculating with futures or calls

Illustration: capital $1,000; futures margin 10%; call strike 100 costs $8

Futures, price to 120	100 × (120 − 100)	**+$2,000**
Calls, price to 120	125 × 20 − 1,000	**+$1,500**
Futures, price to 80	100 × (80 − 100)	**−$2,000**
Calls, price to 80	0 − 1,000	**−$1,000**

IN THE LESSON

- Now two speculators with the same view and the same one thousand dollars of capital, and made up prices.
- The asset trades at one hundred.
- The first posts that money as ten per cent margin on a futures position, so she controls one hundred units.
- The second buys calls with a strike of one hundred at eight dollars each, so she can afford one hundred and twenty five calls.
- Suppose the price rises to one hundred and twenty.
- The futures gain is one hundred times twenty, two thousand dollars.
- The calls pay twenty each, so twenty five hundred, less the thousand paid: fifteen hundred.
- Now suppose the price falls to eighty.
- The futures lose two thousand, more than the margin posted.
- The calls lose just the thousand premium.

23 Futures against calls on equal capital

Bullish view, profit on $1,000: futures | calls

Final price	Futures | Calls
80	−2,000 | −1,000
90	−1,000 | −1,000
100	0 | −1,000
108	+800 | 0
120	+2,000 | +1,500

IN THE LESSON

- Here is the full profile on the same thousand dollars.
- At a final price of eighty, the futures lose two thousand, but the calls lose only one thousand.
- At ninety, each loses one thousand.
- At one hundred, the futures break even while the calls lose the entire premium, because a call needs the price to move beyond the strike plus the premium before it profits.
- The calls break even at one hundred and eight, where the futures are already up eight hundred.
- At one hundred and twenty, the futures make two thousand and the calls fifteen hundred.
- So for moderate moves the futures earn more when the view is right, because the calls must first recover the premium.
- Futures can lose more than the margin posted, while a purchased option can never lose more than its premium.
- With a bearish view the comparison flips, selling futures against buying puts, and written options and short futures can lose very large amounts.

24 Example 9: cash and carry arbitrage

Illustration: spot 100; rate 5% a year; T = 1; no income; quoted forward 108

No-arbitrage forward	F = S(1 + r)^T = 100 × 1.05	**105**
Today: borrow, buy asset	100 − 100; sell forward	**net 0**
At maturity, deliver	receive the forward price	**+108**
Repay the loan	100 × 1.05	**−105**
Riskless profit	108 − 105	**3**

IN THE LESSON

- Now arbitrage.
- For an asset with no income or carrying cost, the no-arbitrage forward price equals the spot price times one plus the interest rate, raised to the power of time in years.
- With a spot price of one hundred, a rate of five per cent and one year, the fair forward price is one hundred and five.
- Suppose the market quotes one hundred and eight, which is too high.
- Do cash and carry.
- Today borrow one hundred, buy the asset and sell the forward at one hundred and eight.
- Net cash today is zero.
- At maturity, deliver the asset and receive one hundred and eight.
- Repay the loan with interest, one hundred and five.
- The riskless profit is three, exactly the quoted price minus the fair price.

25 Example 10: reverse cash and carry, with income

Illustration: spot 100; 5% rate; T = 1; income worth 3 at maturity; quoted 99

Fair forward	100 × 1.05 − 3 (costs: add)	**102**
Quoted forward is below	so reverse cash and carry	**99**
Loan repaid to you	100 × 1.05	**+105**
Pay forward and the income	99 + 3	**−102**
Riskless profit	105 − 102	**3**

IN THE LESSON

- Now the opposite direction, and with income.
- Suppose the asset pays income whose future value at maturity is three dollars.
- The fair forward price is the financed spot price, one hundred and five, minus that future value of three, so one hundred and two.
- Suppose the market quotes ninety nine, which is too low.
- Do reverse cash and carry.
- Today sell the asset short and lend the proceeds of one hundred, and buy the forward at ninety nine.
- At maturity the loan returns one hundred and five.
- Pay ninety nine for the asset under the forward and use it to close the short, and also pay the three of income that the lender of the asset is owed.
- That totals one hundred and two, so the profit is three.
- Storage or other carrying costs work the other way: add their future value.
- The usual mistakes are ignoring the financing cost, forgetting the income, and trading in the wrong direction.

26 Risks that come with derivatives

- Counterparty credit risk: the other side defaults
- Largest OTC; reduced by clearing, netting and collateral
- Netting: offset what you owe and are owed
- Wrong-way: exposure rises as the other side weakens

IN THE LESSON

- Derivatives transfer risk, but they create risks of their own.
- Start with counterparty credit risk, the chance that the other side defaults when the contract has a positive value to you.
- It is largest in OTC trades, and it is reduced by clearing, netting and collateral.
- Netting means offsetting what you owe and are owed under one agreement.
- A nasty variation is wrong-way risk.
- This occurs when your exposure rises just as the counterparty's credit quality worsens, so the contract is worth the most exactly when the other side is least able to pay.
- Think of buying protection against an event from a firm that would itself be badly hurt by that event.

27 More risks to recognize

Name the risk

Risk	Source
Leverage	Small margin, large notional
Basis risk	Hedge not one for one
Liquidity and funding	Margin calls; hard to close
Model risk	Flawed valuation assumptions
Operational risk	Errors, unauthorized trading

IN THE LESSON

- Several other risks deserve a name.
- Leverage magnifies market risk, because a small margin controls a large notional amount.
- Basis risk arises when the hedge does not move one for one with the exposure, for example when it uses a different asset, maturity or grade.
- Liquidity risk includes funding risk, meaning the cash needed for margin calls, and also the difficulty of closing a position.
- Model risk comes from valuing complex products with flawed assumptions.
- And operational risk comes from errors and unauthorized trading.
- When you meet a case in the course, ask which of these is at work, and whether the instrument, the market or the people are the source.

28 Example 11: a sound hedge, a cash crisis

Illustration: 10,000 units; short futures against inventory; price +$10

Futures loss	10,000 × (−10)	**−$100,000**
Inventory gain, on paper	10,000 × (+10)	**+$100,000**
Net economic result	−100,000 + 100,000	**$0**
Cash due now as margin	the futures loss	**$100,000**

IN THE LESSON

- Here is why a sound hedge can still cause a cash crisis.
- A company holds inventory of ten thousand units, bought when the price was one hundred, and hedges by selling futures on the same ten thousand units.
- Then the price rises by ten dollars.
- On the economics, the hedge works.
- The futures lose ten dollars a unit, minus one hundred thousand.
- The inventory gains ten dollars a unit, plus one hundred thousand, though only on paper, since the company has not sold it.
- Net, the economic result is zero.
- But the futures loss is settled in cash today, as variation margin.
- The company must find one hundred thousand dollars now, while the matching gain arrives only when the inventory is sold.
- Funding, not market risk, is the danger.

29 Four mistakes to avoid

- Put is max(K − S, 0); the writer pays the negative
- Profit = payoff − premium; multiply by size last
- Arbitrage: finance, subtract income, right direction
- Leverage is not extra return; hedges keep other risks

IN THE LESSON

- Mistake one: payoff formula errors.
- Use the call formula for a call and the put formula for a put, and remember that the writer pays the negative of the buyer's payoff.
- Mistake two: forgetting the premium when the question asks for profit rather than payoff, or forgetting to multiply by the contract size at the end.
- Mistake three: in arbitrage, ignoring the financing cost, forgetting income such as dividends, or doing the strategy in the wrong direction.
- Transaction costs can also remove an apparent opportunity.
- Mistake four: believing leverage means a higher expected return, or that a hedge removes all risk.
- Leverage magnifies gains and losses equally, and hedges still carry basis, credit and funding risk.

30 Recap: the main ideas

- Linear: forward, future, swap. Non-linear: options
- Exchange: standard, cleared. OTC: tailored, credit risk
- Forward S − K; call max(S − K, 0); put max(K − S, 0)
- Hedger, speculator, arbitrageur: purpose decides
- Forward fixes the price; put sets a floor; check the risks

IN THE LESSON

- Let's pull it together.
- A derivative takes its value from an underlying, settles later, needs little money at the start, and is zero sum.
- Forwards, futures and swaps are linear, options are kinked.
- Exchange-traded contracts are standard and cleared, with daily margin, while OTC contracts are tailored but carry credit risk.
- A forward and a future are obligations, an option is a right bought for a premium.
- Payoffs are the final price minus K for a forward, and kinked for options, and profit deducts the premium.
- Hedgers cut risk, speculators add it and arbitrageurs remove price gaps.
- A forward fixes the price, a put sets a floor.
- And derivatives carry credit, basis, liquidity, model and operational risks.

31 Try three quick questions

1. Long forward, K = 50, 200 units, final price 58. Payoff?
   Answer: (58 − 50) × 200 = $1,600.
2. Put, strike 60, premium 3, final price 52. Profit?
   Answer: Payoff 60 − 52 = 8; profit 8 − 3 = 5.
3. Spot 80, rate 10%, T = 1, no income. Forward quoted 90?
   Answer: Fair = 88. Cash and carry earns 90 − 88 = 2.

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- A long forward has a delivery price of fifty on two hundred units, and the final price is fifty eight.
- What is the payoff?
- Eight dollars a unit times two hundred is sixteen hundred dollars.
- Question two.
- You buy a put with a strike of sixty for a premium of three, and the final price is fifty two.
- What is the profit?
- The payoff is eight, so the profit is five.
- Question three.
- The spot price is eighty, the rate is ten per cent for one year, there is no income, and the forward is quoted at ninety.
- What is the arbitrage profit?
- The fair forward is eighty eight, so the profit is two, by cash and carry.