CHARTA LEARNING · LESSON NOTES

Properties of Options

FRM Part I

1 A story: the price of a ticket

- Call: a right to buy at an agreed price
- Put: a right to sell at an agreed price
- You can walk away: the most you lose is the premium
- What moves the price? Six factors

IN THE LESSON

- Imagine a ticket that lets you buy a house next year at today's agreed price, whatever the house is worth then.
- If house prices jump, the ticket is valuable.
- If they fall, you simply tear it up, and your loss is only what you paid for the ticket.
- That is a call option.
- A put option is the mirror image: a ticket that lets you sell at the agreed price, so it gains when prices fall.
- Now ask what makes such a ticket worth more or less.
- The current price, the agreed price, how long the ticket lasts, how wildly prices swing, interest rates, and any income the house pays out.
- These are exactly the six factors we study.

2 The six factors

- 1. Current stock price, S
- 2. Strike price, K
- 3. Time to expiration, T
- 4. Volatility of the stock
- 5. Risk-free rate, and 6. Dividends during the life

IN THE LESSON

- Here are the six factors that influence the price of a stock option.
- One, the current stock price, which we call S.
- Two, the strike price, K, the agreed price at which the option holder can buy or sell.
- Three, the time to expiration.
- Four, the volatility of the stock, meaning how much its price swings.
- Five, the risk free interest rate.
- Six, the dividends expected during the life of the option.
- Notice that five of these describe the stock and the contract, while the interest rate describes the market around them.
- In the next slides we take each factor, ask whether it raises or lowers a call and a put, and then explain why in plain words.

3 Sign table: price, strike, volatility

Raise one factor, hold the others fixed

Factor goes UP	Call | Put
Stock price S	Up | Down
Strike price K	Down | Up
Volatility	Up | Up

IN THE LESSON

- Let's fill in the first half of the sign table, always raising one factor while holding the others fixed.
- A higher stock price raises a call, because the right to buy is worth more when the thing is worth more, and lowers a put.
- A higher strike does the reverse.
- It lowers a call, because you must pay more to buy, and raises a put, because you receive more when you sell.
- Higher volatility raises both.
- The reason is the heart of option pricing: the holder keeps the upside of big moves but is protected from the downside, since the option can only be worth zero at worst.
- Wild swings therefore add value to either kind of option.

4 Sign table: rate, dividends, time

Raise one factor, hold the others fixed

Factor goes UP	Call | Put
Risk-free rate	Up | Down
Expected dividends	Down | Up
Time to expiration	Usually up | up*
*European exception	Deep ITM put, big-div call

IN THE LESSON

- Now the second half.
- A higher risk free interest rate raises a call and lowers a put.
- The strike is paid or received later, and a higher rate means that future payment is worth less in today's money.
- For a call buyer that means the strike costs less today, which helps.
- For a put, the strike you will receive is worth less today, which hurts.
- Higher expected dividends lower a call and raise a put, because the stock price drops on the ex dividend date, the day the buyer no longer gets the payout.
- Finally, a longer maturity usually raises the value of both.
- But a European option can be an exception, especially a deep in the money put, as we see next.

5 Why time can hurt a European option

- American: longer maturity is never worth less
- European: cannot exercise early, so no such guarantee
- Deep ITM European put: capped payoff arrives later
- Large dividend can also hurt a longer European call
- So time is usually, not always, positive

IN THE LESSON

- Why is time usually good?
- More time means more chance of a big favorable move, and an American option can always be exercised, so a longer American option is never worth less than a shorter one.
- A European option has no such freedom, because it can only be exercised at the very end.
- Take a deep in the money European put.
- The most it can ever pay is the strike, and that payment arrives only at expiry.
- A longer wait pushes that capped payment further away, so it is worth less in today's money.
- Something similar can happen to a European call when a large dividend falls before the longer option expires but after the shorter one ends.
- So the sign for time is usually positive, not always.

6 Example 1: why volatility adds value

Illustration: S = 100, K = 100, two equally likely outcomes

Calm: ends 95 or 105	call (0 + 5) ÷ 2	**$2.50**
Wild: ends 80 or 120	call (0 + 20) ÷ 2	**$10.00**
Calm: put	(5 + 0) ÷ 2	**$2.50**
Wild: put	(20 + 0) ÷ 2	**$10.00**

IN THE LESSON

- Let's see volatility at work with made up numbers.
- A stock trades at one hundred dollars and a call has a strike of one hundred.
- In a calm world, the stock ends at either ninety five or one hundred and five with equal chance.
- The call pays nothing in the first case and five dollars in the second, so the average payoff is two dollars fifty.
- Now a wild world: the stock ends at either eighty or one hundred and twenty.
- The call pays zero or twenty, and the average payoff is ten dollars.
- The puts tell the same story, two fifty in the calm world and ten in the wild one.
- More swing, more value, for both.
- We ignore discounting here to keep the idea clean.

7 Notation and the no-arbitrage idea

- S stock price, K strike, T maturity, r continuous rate
- D = present value of dividends before expiry
- Discounted strike = K × exp(−rT)
- Outside the bounds: buy cheap, sell dear, riskless gain
- So prices must stay inside the bounds

IN THE LESSON

- Next, the bounds on option prices.
- We use a little notation.
- S is the stock price, K the strike, r the continuously compounded risk free rate, T the maturity in years, and D the present value of the dividends paid before expiry.
- To discount the strike, we multiply it by e to the power of minus r T, written as K times exp of minus r T.
- Bounds come from one principle: no arbitrage.
- An arbitrage is a riskless profit with no net investment.
- If an option traded outside its bounds, you could buy the cheap side and sell the expensive side and lock in a gain, and traders would do so until the price came back.
- So the bounds hold in any sensible market.

8 Upper bounds on option prices

Call: c ≤ S (American C ≤ S)
European put: p ≤ K·e^{−rT}
American put: P ≤ K

S, stock price	Call can never exceed the stock
K·e^{−rT}, strike PV	European put: strike discounted
K, American cap	American put: just the strike K

IN THE LESSON

- Start with the highest value an option can have.
- A call can never be worth more than the stock.
- The call gives you the right to buy the stock, so paying more for the right than the stock itself costs would be foolish; you would just buy the stock.
- A put can never be worth more than the strike, because the best that can ever happen is that the stock falls to zero and you sell it at the strike.
- For an American put that cap is the strike itself, since you can exercise on day one.
- For a European put the payoff only arrives at expiry, so the cap is the strike discounted back to today.
- Forgetting to discount the European cap is a very common slip.

9 Example 2: an upper bound breaks

Illustration: S = 40, K = 50, T = 1 year, r = 5%

Cap, European put	50 × e^−0.05	**$47.56**
Cap, American put	the strike	**$50.00**
Quote is too high	$48.50 > $47.56	**Sell it**
Invest $48.50 for a year	48.50 × e^0.05	**$50.99**

IN THE LESSON

- Let's use the upper bound to spot an arbitrage.
- A stock trades at forty dollars.
- A one year European put has a strike of fifty dollars, and the risk free rate is five percent.
- The most this put can be worth is the strike discounted: fifty times e to the minus point zero five, which is forty seven dollars fifty six.
- The American cap would be the full fifty.
- Now suppose someone quotes the European put at forty eight dollars fifty.
- That is above forty seven fifty six, so it is too expensive.
- Sell it and invest the forty eight fifty at five percent.
- After a year it grows to about fifty dollars ninety nine.
- The worst you ever pay out is fifty, so you keep at least ninety nine cents, risk free.

10 Lower bounds for European options

c ≥ max(S − D − K·e^{−rT} , 0)
p ≥ max(K·e^{−rT} + D − S , 0)

Call price	European call, lowest fair value
Put price	European put, lowest fair value
Dividends, present value	Dividends before expiry, in PV
Strike, discounted	Strike, discounted at the rate

IN THE LESSON

- Now the lowest value an option can have.
- For a European call, the bound is the stock price, minus the present value of any dividends, minus the discounted strike, or zero if that number is negative.
- For a European put, it is the discounted strike plus the present value of dividends, minus the stock price, again floored at zero.
- Without dividends, simply set the dividend term to zero.
- These bounds come from comparing two portfolios.
- For the call: hold the call and the discounted strike in cash, against holding the stock.
- At expiry the first is worth at least as much as the stock, so it must cost at least as much today.
- The put works the same way, with the roles swapped.

11 Example 3: lower bound of a call

Illustration: S = 100, K = 90, T = 1 year, r = 5%

Discounted strike	90 × e^−0.05	**$85.61**
Lower bound, no dividends	100 − 85.61	**$14.39**
Intrinsic value (contrast)	100 − 90	**$10.00**
With dividends PV 4	100 − 4 − 85.61	**$10.39**

IN THE LESSON

- Let's compute a lower bound for a call.
- The stock is one hundred dollars, the strike ninety, the maturity one year and the rate five percent.
- First discount the strike: ninety times e to the minus point zero five, which is eighty five dollars sixty one.
- The stock minus the discounted strike is fourteen dollars thirty nine.
- That is the lower bound for the call with no dividends.
- Compare it with the intrinsic value, what you would get exercising today: the stock minus the strike, ten dollars.
- The bound of fourteen thirty nine is higher, so it is the tighter, better one.
- Now suppose four dollars of dividends are expected, in present value.
- The bound falls to ten dollars thirty nine, and the call must still be worth at least that.

12 Example 4: lower bound of a put

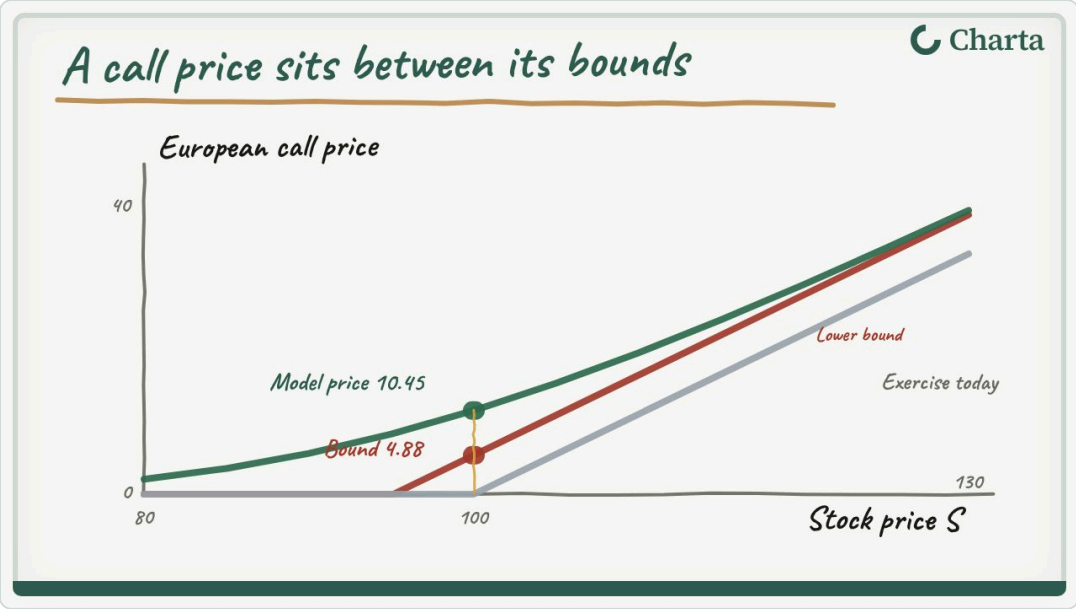
Illustration: S = 80, K = 90, T = 1 year, r = 5%

European, no dividends	85.61 − 80	**$5.61**
European, dividends PV 3	85.61 + 3 − 80	**$8.61**
American intrinsic	90 − 80	**$10.00**
Tightest for an American put	largest of the bounds	**$10.00**

IN THE LESSON

- Now a put.
- The stock is eighty dollars, the strike ninety, one year, five percent.
- The discounted strike is still eighty five dollars sixty one.
- The European put lower bound is the discounted strike minus the stock: five dollars sixty one.
- Add expected dividends with present value three dollars, and the bound rises to eight dollars sixty one, since dividends push the stock price down, which helps a put.
- Now compare with an American put, which can be exercised immediately for the strike minus the stock, ten dollars.
- So an American put must be worth at least ten, higher than either European bound.
- The lesson is to always ask for the tightest bound, and not to apply a European bound where the American one is stronger.

13 A call price sits between its bounds



IN THE LESSON

- Here is a picture of a call with a strike of one hundred, one year to go, and a five percent rate, for stock prices from eighty to one hundred and thirty.
- The red line is the lower bound: zero until the stock reaches ninety five twelve, which is the discounted strike, then rising one for one with the stock.
- The grey line below it is what you would get exercising today, zero up to one hundred, then rising one for one.
- The smooth curve is a sample call price, from a pricing model with twenty percent volatility.
- It sits above both lines.
- At a stock price of one hundred, the bound is four eighty eight and the curve shows about ten forty five.
- The gap above the bound is the value of the insurance.

14 Example 5: cashing in a broken lower bound

Illustration: call quoted at $12; bound is $14.39; S = 100, K = 90

Buy call, short, invest	−12 + 100 = 88 to invest	**$88**
Investment after one year	88 × e^0.05	**$92.51**
Worst case: pay 90 for share	92.51 − 90	**$2.51**
Worth today	2.51 × e^−0.05 = 14.39 − 12	**$2.39**

IN THE LESSON

- Suppose the call from the last lower bound example is quoted at only twelve dollars, below its lower bound of fourteen thirty nine.
- Here is how to harvest the gap.
- Buy the call for twelve, sell the stock short for one hundred, and invest the difference, eighty eight dollars, at five percent.
- After a year the investment is worth about ninety two dollars fifty one.
- At expiry, if the stock is above ninety, you exercise the call, pay ninety to get a share, and hand it back to close the short.
- You are left with about two fifty one.
- If the stock is below ninety, you buy it cheaply on the market and keep even more.
- Either way at least two fifty one is yours, worth two thirty nine today, which is the gap between fourteen thirty nine and twelve.

15 Put-call parity: two portfolios, one payoff

- A = European call + cash worth the discounted strike
- B = European put + one share of the stock
- Same strike K, same maturity T, same stock
- Stock above K: both end up holding one share
- Stock below K: both end up holding K in cash

IN THE LESSON

- Now the most useful relationship in this module, put call parity.
- Build two portfolios with European options on the same stock, with the same strike and the same maturity.
- Portfolio A holds one call plus an amount of cash equal to the discounted strike.
- Portfolio B holds one put plus one share of the stock.
- At expiry, if the stock ends above the strike, A exercises the call, pays the strike out of its grown cash and holds a share, while B holds a share and the put is worthless.
- If the stock ends below the strike, A holds just the strike in cash, while B uses the put to sell the share for the strike.
- Either way, both end with the same value, so they must cost the same today.

16 The put-call parity equations

No dividends: c + K·e^{−rT} = p + S
With dividends: c + D + K·e^{−rT} = p + S

Call price	European call, strike K
Put price	European put, same K and T
K·e^{−rT}, strike PV	Strike discounted over time T
Dividends, present value	Dividends before expiry, PV
Stock price	Stock price today

IN THE LESSON

- Writing it out, the call price plus the discounted strike equals the put price plus the stock price.
- That is for a stock paying no dividends.
- If the stock pays dividends, with D the present value of the dividends before expiry, portfolio A also needs to hold that amount, because B owns the stock and receives the dividends while A does not.
- So the equation becomes call plus D plus discounted strike equals put plus stock.
- Rearranged, a call minus a put equals the stock minus the discounted strike, and with dividends, the stock minus D minus the discounted strike.
- Remember this applies only to European options with the same strike and maturity, and both the strike and the dividends must be discounted.

17 Example 6: find the put from the call

Illustration: S = 100, K = 100, T = 1 year, r = 5%, call = $10

Discounted strike	100 × e^−0.05	**$95.12**
Left side: call + strike PV	10 + 95.12	**$105.12**
Put = left side − stock	105.12 − 100	**$5.12**
Check: put + stock	5.12 + 100	**$105.12**

IN THE LESSON

- Let's use parity to price a put.
- The stock is one hundred dollars, the strike one hundred, one year, five percent, no dividends.
- A European call with that strike trades at ten dollars.
- First discount the strike: one hundred times e to the minus point zero five is ninety five dollars twelve.
- Put call parity says the call plus the discounted strike equals the put plus the stock.
- So the put equals the call, plus the discounted strike, minus the stock: ten plus ninety five twelve minus one hundred, which is five dollars twelve.
- Check by adding the other side: five twelve plus one hundred is one hundred and five twelve, the same as ten plus ninety five twelve.
- The put should therefore trade at five dollars twelve.

18 Example 7: parity with a dividend

Illustration: S = 100, K = 100, T = 1, r = 5%, dividends PV = 3, call = $9

Left side: c + D + strike PV	9 + 3 + 95.12	**$107.12**
Put = left side − stock	107.12 − 100	**$7.12**
Forgetting D gives	9 + 95.12 − 100	**$4.12**
Wrong by the dividend	7.12 − 4.12	**$3.00**

IN THE LESSON

- Now add a dividend.
- Same stock at one hundred, strike one hundred, one year, five percent.
- The stock is expected to pay dividends with a present value of three dollars.
- A European call trades at nine dollars.
- The parity equation is call plus D plus discounted strike equals put plus stock.
- Left side: nine plus three plus ninety five twelve, which is one hundred and seven twelve.
- Subtract the stock price of one hundred and the put is seven dollars twelve.
- Compare with the no dividend case, where the put was five twelve for a call of ten.
- The dividend lowers the call and lifts the put, just as the sign table said.
- A common slip is to forget the dividend term, which would give a put of only four twelve.

19 Example 8: trading a parity violation

Illustration: call $10 and put $4, S = 100, K = 100, r = 5%, T = 1

Left side: call + strike PV	10 + 95.12	**$105.12**
Right side: put + stock	4 + 100	**$104.00**
Gap, riskless profit today	105.12 − 104.00	**$1.12**
At expiry: all cancels	stock+put−call = 100 = loan	**$0 owed**

IN THE LESSON

- Back to the no dividend case with the call at ten dollars, where parity says the put should be five twelve.
- Suppose the market put trades at only four dollars.
- Check the sides.
- Call plus discounted strike is one hundred and five twelve.
- Put plus stock is one hundred and four.
- The left side is more expensive, so sell the call and buy the put and the stock.
- Fund it by borrowing the discounted strike, ninety five twelve.
- Cash today: plus ten from the call, minus four for the put, minus one hundred for the stock, plus ninety five twelve borrowed.
- That is a gain of one dollar twelve right now.
- At expiry the stock, the put and the call you sold together are worth exactly the strike, one hundred, and you owe one hundred on the loan.
- Everything cancels, so the dollar twelve is riskless.

20 Parity through the forward price

c − p = (F − K) · e^{−rT}
F = (S − D) · e^{rT}

Forward price	Forward price for the same date
K, strike	Strike of both, not discounted
c − p	European call minus put
Stock price	Stock price today
Dividends, present value	Dividends, PV, if any

IN THE LESSON

- There is a second way to write the same relationship, through the forward price F.
- The forward price is the stock price grown at the risk free rate, after removing any dividends.
- Call minus put equals the forward price minus the strike, all discounted.
- Why?
- A long call and a short put struck at K behave like a forward contract with delivery price K.
- They gain when the stock ends high and lose when it ends low, exactly as a forward does.
- So their price difference is the discounted value of the forward's gain.
- One consequence is neat.
- If the strike equals the forward price, the right side is zero, so a call and a put struck at the forward price have exactly the same value.

21 Example 9: parity via the forward

Illustration: S = 100, r = 5%, T = 1, no dividends, K = 100

Forward price	100 × e^0.05	**$105.13**
Forward minus strike	105.13 − 100	**$5.13**
Call minus put	5.13 × e^−0.05	**$4.88**
Check: S − discounted K	100 − 95.12	**$4.88**

IN THE LESSON

- Let's check that the forward form agrees.
- Stock one hundred dollars, no dividends, five percent, one year, strike one hundred.
- The forward price is one hundred times e to the point zero five, one hundred and five dollars thirteen.
- The forward minus the strike is five thirteen.
- Discount it by multiplying by e to the minus point zero five and we get four eighty eight.
- So the call minus the put is four eighty eight.
- The earlier form gives the stock minus the discounted strike, one hundred minus ninety five twelve, also four eighty eight.
- They match.
- Now set the strike equal to the forward, one hundred and five thirteen.
- Then the forward minus the strike is zero, so the call and put are worth the same.

22 American options obey only inequalities

- Exact parity: European options only
- American, no dividends: $S - K \le C - P \le S - K \cdot e^{-rT}$
- Early exercise rights turn the equality into a range
- With dividends the lower end moves down by D
- American question: use the range, not the equal sign

IN THE LESSON

- A warning: parity as an exact equation is for European options.
- American options can be exercised early, which breaks the exact link, so we only get inequalities.
- For a stock paying no dividends, the American call minus the American put is at least the stock minus the strike, and at most the stock minus the discounted strike.
- If dividends are paid, the lower end shifts down by their present value.
- The upper end follows from the European relationship, while the lower end reflects the early exercise right.
- The practical rule: if the question says American, do not use the equal sign.
- Use the range, and remember that discounted values appear on the high side and undiscounted on the low side.

23 Example 10: testing an American range

Illustration: S = 100, K = 100, T = 1, r = 5%, no dividends

Lower end: S − K	100 − 100	**$0**
Upper end: S − discounted K	100 − 95.12	**$4.88**
C = 11, P = 7: C − P	11 − 7 = 4 in range	**No arbitrage**
C = 12, P = 6: C − P	12 − 6 = 6 above 4.88	**Arbitrage**

IN THE LESSON

- Let's apply the range.
- Stock one hundred dollars, strike one hundred, one year, five percent, no dividends.
- The lower end is the stock minus the strike, zero.
- The upper end is the stock minus the discounted strike: one hundred minus ninety five twelve, four eighty eight.
- So the American call minus the American put must lie between zero and four eighty eight.
- Suppose we see a call at eleven and a put at seven, a difference of four.
- That is inside the range, so there is no arbitrage here.
- Now suppose a call at twelve and a put at six, a difference of six.
- That is above four eighty eight, so the call is too dear relative to the put.
- Sell the call, buy the put and the stock, borrow the discounted strike, and lock in a gain.
- With no dividends, the call you sold will not be exercised early.

24 Early exercise of an American call

- No dividends: never exercise an American call early
- 1. Exercising gives up the insurance against a fall
- 2. Paying K now loses the interest on K
- Call is worth at least S − K·e^{−rT} > S − K
- Selling beats exercising; American = European call

IN THE LESSON

- Now the last topic: when should an American option be exercised early?
- Take a call on a stock that pays no dividends.
- The answer is never.
- There are two reasons.
- First, exercising throws away the insurance.
- Before expiry, if the stock collapses you can still walk away, but once you exercise you own the stock and its downside.
- Second, exercising means paying the strike now instead of later, so you lose the interest you could have earned on that money.
- The lower bound makes it exact: the call is worth at least the stock minus the discounted strike, which exceeds the stock minus the strike.
- So if you want out, selling the option always beats exercising it.
- That also means an American call without dividends is worth the same as a European one.

25 Example 11: sell, do not exercise

Illustration: S = 120, K = 100, T = 1, r = 5%, no dividends

Exercise now: S − K	120 − 100	**$20.00**
Sell: at least S − K·e^−rT	120 − 95.12	**$24.88**
Interest on the strike	100 × (e^0.05 − 1)	**$5.13**
Better to sell by	24.88 − 20.00	**$4.88**

IN THE LESSON

- Let's put numbers on it.
- The stock is at one hundred and twenty dollars, the strike one hundred, one year left, five percent, and no dividends.
- If you exercise now, you receive twenty dollars: the stock minus the strike.
- But the call's lower bound is the stock minus the discounted strike, one hundred and twenty minus ninety five twelve, which is twenty four eighty eight.
- So the call is worth at least twenty four eighty eight in the market, nearly five dollars more than twenty.
- The gap has a clear source.
- By waiting, you keep your hundred dollars invested for a year, and at five percent it earns about five dollars and thirteen cents.
- That interest arrives in a year, so today it is worth four eighty eight, exactly the gap.
- So sell the option and take at least twenty four eighty eight, rather than exercise for twenty.

26 When dividends make early exercise sensible

- Early exercise only just before ex-dividend
- Test: dividend > interest saved on K
- Dividend below that interest: keep the call
- Away from ex-dividend dates: still do not exercise

IN THE LESSON

- Dividends change the picture for calls.
- When a stock pays a dividend, its price drops on the ex dividend date, and the call holder who has not exercised does not get that payout.
- So early exercise can be optimal, but only just before an ex dividend date.
- The test in the course is that the dividend must be larger than the interest saved by not paying the strike over the remaining life.
- If the dividend is smaller than that interest, waiting is better.
- Even when the test is passed, early exercise is only worth considering, not guaranteed.
- And exercising at any other time, away from the ex dividend date, is still never sensible, since the dividend is the only thing you gain by exercising early.

27 Example 12: the dividend test

Illustration: K = 100, half a year left, r = 5%

Interest saved on K	100 × (1 − e^−0.025)	**$2.47**
Dividend $4.00	4.00 > 2.47	**Consider it**
Dividend $2.00	2.00 < 2.47	**Keep the call**

IN THE LESSON

- A worked test.
- A call has a strike of one hundred dollars and half a year of life left.
- The risk free rate is five percent.
- The interest saved by delaying payment of the strike is one hundred times one minus e to the minus point zero two five, which is about two dollars forty seven.
- Case one: a dividend of four dollars is about to go ex.
- Four is larger than two forty seven, so early exercise just before the ex date may be sensible.
- Case two: the dividend is two dollars.
- That is below two forty seven, so keep the call.
- The gain from the dividend does not make up for the interest and insurance you would give up.
- One more point: the test is a screen and a full model decides for sure.

28 Early exercise of an American put

- Put gain is capped at K: little left to wait for
- Exercise: receive K now and earn interest on it
- High interest rates favor early exercise
- Low volatility favors early exercise
- Do not apply call logic to puts

IN THE LESSON

- Puts are different, and the classic trap is applying call logic to them.
- An American put can be worth exercising early when it is deep in the money.
- The reason is that the most a put can ever pay is the strike, and the gain is capped.
- If the stock has fallen almost to zero, there is almost nothing left to gain from waiting.
- Meanwhile, exercising now gives you the strike in cash, and you can invest it and earn interest.
- Once the interest on the cash received outweighs the remaining insurance value, exercise wins.
- So high interest rates and low volatility favor early exercise of a put: high rates make cash today more valuable, and low volatility makes the remaining insurance less valuable.

29 Example 13: a put worth exercising

Illustration: American put, K = 100, S = 2, T = 1 year, r = 5%

Exercise now: K − S	100 − 2	**$98**
Invest for a year	98 × e^0.05	**$103.02**
Best possible by waiting	stock falls to zero	**$100**
Better to exercise by	103.02 − 100	**$3.02**

IN THE LESSON

- Let's see a put in this position.
- The strike is one hundred dollars, the stock has crashed to two dollars, and there is one year left at five percent.
- Exercise now and you receive the strike minus the stock: ninety eight dollars.
- Invest that for a year at five percent, and it grows to about one hundred and three dollars two cents.
- Now think about waiting.
- The most a put can ever pay is the strike, one hundred dollars, and only if the stock falls to exactly zero.
- So waiting cannot produce more than one hundred, while exercising now can produce one hundred and three.
- Exercising is clearly better, because you give up almost no insurance and gain the interest on the cash.

30 Four mistakes to avoid

- Discount the strike: use $K \cdot e^{-rT}$, not K
- American is tighter: $P \geq K - S$; parity is inequalities
- Subtract D for calls and add D for puts
- Calls differ from puts: deep ITM puts go early

IN THE LESSON

- Mistake one: forgetting to discount the strike.
- The bounds and the parity relationship all use the strike discounted, and the bound for a European put uses it too.
- Mistake two: applying the European result where the American one is tighter.
- An American put is at least its intrinsic value, and American options obey inequalities rather than parity.
- Mistake three: leaving out the present value of dividends in parity and in the bounds.
- They shift the call down and the put up.
- Mistake four: applying call logic to puts.
- An American call without dividends is never exercised early, but a deep in the money American put often is.
- Also remember that time to expiry is not always positive for a European option.

31 Recap: the main ideas

- Six factors; learn the sign table
- Bounds: call $\leq S$; European put $\leq K \cdot e^{-rT}$
- Lower: $c \geq S - D - K \cdot e^{-rT}$; $p \geq K \cdot e^{-rT} + D - S$
- Parity: $c + D + K \cdot e^{-rT} = p + S$ (European only)
- Early exercise: no-dividend calls never; deep puts yes

IN THE LESSON

- Let's pull it together.
- Six factors drive a stock option price: the stock price, the strike, the time to expiration, the volatility, the risk free rate and the dividends.
- Memorize the sign table.
- A higher stock price raises calls and lowers puts, a higher strike does the reverse, volatility raises both, a higher rate raises calls and lowers puts, and dividends lower calls and raise puts.
- The bounds follow from no arbitrage: a call is below the stock, a European put below the discounted strike, and the lower bounds use the discounted strike and dividends.
- Put call parity is exact for European options and a range for American ones.
- An American call without dividends is never exercised early, but a deep in the money put may be.

32 Try three quick questions

1. S 50, K 45, r 4%, T 1, no dividends. European call floor?
   Answer: 50 − 45·e^−0.04 = 50 − 43.24 = 6.76.
2. c = 4, S = 40, K = 40, r 5%, T 1, no dividends. Put?
   Answer: p = 4 + 38.05 − 40 = 2.05.
3. Exercise an American call early on a no-dividend stock?
   Answer: No: sell it; you keep insurance and interest.

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- A stock is at fifty, the strike is forty five, the rate is four percent for one year, and there are no dividends.
- What is the lower bound for a European call?
- Use the stock minus the discounted strike: about six seventy six.
- Question two.
- A European call is at four dollars and the stock at forty, strike forty, one year at five percent, no dividends.
- What should the put be?
- Four plus thirty eight oh five minus forty gives two oh five.
- Question three.
- Should you exercise an American call early on a stock that pays no dividends?
- No.
- Sell it, because the option is worth more than exercising and you keep the insurance and the interest on the strike.
- The answers are on the board.