Swaps

FRM Part I

1 A story: a loan you wish were different

- A floating-rate loan makes the interest bill uncertain
- Changing the loan itself is slow and costly
- A separate deal can offset the floating payments
- That deal is a swap: it reshapes cash flows

IN THE LESSON

- Imagine a small company that borrowed money from its bank at a floating rate.
- Every six months the rate is reset, so the interest bill moves up and down.
- The owner now loses sleep, because a jump in rates could wreck the budget.
- What the owner wants is a fixed bill.
- Renegotiating the loan is slow and costly.
- So instead the company makes a separate deal with someone else, a deal that pays it the floating amount when rates are high and takes a fixed amount from it.
- The loan is untouched, yet the owner's real cost is now steady.
- That separate deal is a swap, and the rest of this lesson unpacks it.

2 The plain vanilla interest rate swap

- Exchange of interest on a notional over a stated life
- One side pays fixed, receives floating; other reverses
- The notional is never exchanged: it only sizes payments
- Only the net amount moves on each date
- Floating rate set at period start, paid at its end

IN THE LESSON

- A plain vanilla interest rate swap is an agreement to exchange interest payments on a stated amount over a stated life.
- One party pays a fixed rate and receives a floating rate.
- The other party does the opposite.
- The stated amount is called the notional principal, and here is the surprise: the notional is never exchanged.
- It only sizes the payments.
- Better still, on each payment date only the difference moves.
- If you owe me fifty and I owe you forty, one party simply pays ten.
- The floating rate is set at the start of each period and paid at its end, so the very first payment is already known on day one.

3 The net payment on a swap date

Net payment = N × (K − R) × 0.5
Positive: fixed payer pays. Negative: fixed payer receives

Notional	Notional: sizes the payment
Fixed rate	Fixed rate agreed in the swap
Floating rate	Floating rate set at period start
Accrual fraction	Accrual: 0.5 for six months
Day counts	Fixed 30/360, floating act/360

IN THE LESSON

- For a swap that pays every six months, the net payment is the notional, times the fixed rate minus the floating rate, times one half.
- That half is the accrual fraction, the share of a year the period covers.
- If the answer is positive, the fixed payer pays that amount.
- If it is negative, the fixed payer receives it.
- Day counts matter too.
- Fixed legs often use thirty over three sixty, meaning twelve equal months, while floating legs often use actual days over three sixty.
- Three slips to avoid: dropping the accrual fraction, reversing who pays, and using the floating rate seen on the payment date instead of the rate set at the start of the period.

4 Example 1: net payments for the fixed payer

Illustration: notional $100m, fixed rate 4%, paid every six months

Period 1: floating 3.0%	100m × (4% − 3.0%) × 0.5	**pay $0.50m**
Period 2: floating 4.5%	100m × (4% − 4.5%) × 0.5	**get $0.25m**
Period 3: floating 5.0%	100m × (4% − 5.0%) × 0.5	**get $0.50m**
Net over three periods	−0.50 + 0.25 + 0.50	**get $0.25m**

IN THE LESSON

- Here is an illustration.
- The notional is one hundred million dollars, the fixed rate is four per cent, and payments are every six months.
- In period one, the floating rate was set at three per cent.
- Four minus three is one per cent, times one hundred million, times one half, which is half a million dollars.
- The fixed payer pays.
- In period two the floating rate was set at four and a half per cent.
- Four minus four and a half is negative half a per cent, so the fixed payer receives a quarter of a million.
- In period three, floating is five per cent, so the fixed payer receives half a million.
- Over the three periods the fixed payer nets a quarter million received.

5 Example 2: a borrower turns floating into fixed

Illustration: loan at floating + 1.5%. Swap: pay 4% fixed, receive floating

Loan interest, floating 4.5%	100m × (4.5% + 1.5%) × 0.5	**pay $3.00m**
Swap net payment	100m × (4% − 4.5%) × 0.5	**get $0.25m**
Net cash paid	3.00 − 0.25	**$2.75m**
All-in rate	4% + 1.5%	**5.5%**
Check	2.75 ÷ 100 × 2	**5.5%**

IN THE LESSON

- A swap can change the character of an existing loan.
- Take a borrower paying the floating rate plus one and a half per cent on one hundred million dollars.
- The borrower enters a swap, paying four per cent fixed and receiving the same floating rate.
- Suppose floating is four and a half per cent this period.
- Loan interest is four and a half plus one and a half, six per cent, times one hundred million, times one half: three million dollars paid.
- On the swap, four minus four and a half, times the notional, times one half, is a quarter million received.
- The net cash paid is two million seven hundred and fifty thousand.
- The all in rate is four per cent plus the one and a half spread, which is five and a half per cent, and the check agrees.

6 Example 3: an investor turns fixed into floating

Illustration: $100m bond with 5% coupon. Swap: pay 4% fixed, receive floating

Bond coupon received	100m × 5% × 0.5	**get $2.50m**
Swap net payment	100m × (4% − 3%) × 0.5	**pay $0.50m**
Net cash received	2.50 − 0.50	**$2.00m**
Asset return	3% + (5% − 4%)	**4.0%**
Check	2.00 ÷ 100 × 2	**4.0%**

IN THE LESSON

- Now the mirror image.
- An investor holds a one hundred million dollar bond with a coupon of five per cent.
- The investor enters a swap that pays four per cent fixed and receives floating.
- Suppose floating is three per cent this period.
- The bond pays five per cent times one hundred million times one half, which is two and a half million received.
- On the swap, four minus three, times the notional, times one half, is half a million paid.
- Net cash received is two million.
- The asset return is the floating rate plus the gap between coupon and swap rate: three per cent plus five minus four, which is four per cent.
- The check, two over one hundred times two, agrees.
- The fixed bond now behaves like a floating rate asset.

7 The swap dealer in the middle

- Dealer quotes a bid and an offer fixed rate
- Pays the lower rate, receives the higher
- Matched book: two offsetting swaps, dealer keeps gap
- No match yet: warehouse the swap and hedge it
- Two contracts: if one side defaults, the other stays

IN THE LESSON

- Most companies do not find another company with exactly opposite needs.
- So a financial intermediary, usually a bank acting as swap dealer, steps into the middle.
- The dealer quotes two fixed rates against the floating rate: a bid and an offer.
- It pays the lower rate and receives the higher, and the gap, often a few basis points, is its pay.
- In a matched book, the dealer enters two offsetting swaps and keeps the spread.
- When no match exists yet, the dealer warehouses the swap, meaning it holds the position and hedges it with bonds, futures or other swaps until a counterparty appears.

8 Example 4: the dealer's matched book

Illustration: $100m. Dealer quotes bid 3.97% and offer 4.00%

Receives from company A	fixed offer rate	**4.00%**
Pays to company B	fixed bid rate	**3.97%**
Floating legs	received from B, paid to A	**cancel**
Spread per year	4.00% − 3.97%	**0.03%**
Dealer income per year	100m × 0.03%	**$30,000**

IN THE LESSON

- Take a dealer quoting a bid of three point nine seven per cent and an offer of four per cent, on one hundred million dollars.
- Company A wants to pay fixed, so it pays the dealer four per cent and receives floating.
- Company B wants to receive fixed, so the dealer pays it three point nine seven per cent and takes floating from it.
- The dealer receives four, and pays three point nine seven.
- The floating legs cancel exactly, since the dealer receives floating from one party and pays it to the other.
- What remains is the spread, three basis points, or point zero three per cent.
- On one hundred million dollars, that is thirty thousand dollars a year.
- But remember, the dealer holds two separate contracts, so a matched book is not risk free.

9 The confirmation and the master agreement

- Confirmation: the exact terms of one swap trade
- Parties, notional, dates, rates, day counts
- Issued under a master agreement (for example ISDA)
- Master agreement: default, close-out, netting
- If they conflict, the confirmation prevails

IN THE LESSON

- Once a trade is agreed, someone must write down exactly what was agreed.
- That document is the confirmation.
- It records the parties, the notional, the trade date and effective date, the fixed rate, the floating reference rate, the payment dates, the day count conventions and the maturity.
- A confirmation is normally issued under a master agreement, such as the one published by the International Swaps and Derivatives Association.
- The master agreement sets the general terms for every trade between the two parties, including events of default, close out and netting.
- Each confirmation becomes part of that single agreement.
- Where the two conflict, the confirmation wins for that transaction.

10 What the confirmation does and does not do

Three documents, three jobs, and what none covers

Document	Its job
Confirmation	Records one trade's terms
Master agreement	Default, netting
Credit support annex	Sets the collateral
Not covered by any of them	Market risk remains

IN THE LESSON

- So what is a confirmation for?
- Its practical job is to catch any disagreement about terms before the first payment, so that the two sides do not discover a difference when money is due.
- That lowers legal and operational risk.
- But be clear about its limits.
- It does not remove market risk, because the swap still gains or loses as rates move.
- And it does not provide collateral.
- Collateral comes from a separate document, called the credit support annex.
- Questions usually ask either what a confirmation does, or how it relates to the master agreement.
- The answer is simple: it records one trade, inside the master agreement's framework for every trade between the two parties.

11 Comparative advantage: the borrowing rates

Illustration: A wants floating, B wants fixed

Borrowing rates	Fixed | Floating
Firm A (stronger)	4.0% | R + 0.2%
Firm B (weaker)	5.2% | R + 1.0%
B pays more by	1.2% | 0.8%
Total gain a year	1.2% − 0.8% = 0.4%

IN THE LESSON

- Now an argument for why swaps exist.
- Suppose firm A is stronger than firm B.
- A can borrow fixed at four per cent, or floating at the floating rate plus point two per cent.
- Firm B must pay five point two per cent fixed, or the floating rate plus one per cent.
- B pays more in both markets, but the extra it pays differs.
- In the fixed market B pays one point two per cent more than A.
- In the floating market it pays only point eight per cent more.
- A's advantage is larger in the fixed market.
- So the two firms can share a total gain equal to the difference of the spreads: one point two minus point eight, which is point four per cent a year.

12 Example 5: sharing the gain

A borrows fixed 4.0%, B borrows R + 1.0%. A pays B floating R, B pays A fixed 4.0%

Total gain a year	1.2% − 0.8%	**0.4%**
A: direct floating	vs swapped cost	**saves**
R + 0.2%	R	**0.2%**
B: direct fixed 5.2%	vs swapped 1.0% + 4.0%	**saves** **0.2%**
Each gain on $100m	100m × 0.2%	**$200,000**

IN THE LESSON

- Here is how the gain is captured.
- Firm A borrows fixed at four per cent, where its advantage is greatest.
- Firm B borrows floating at the floating rate plus one per cent.
- Then they swap, with A paying B the floating rate and B paying A four per cent fixed.
- After the swap, A's cost is the floating rate, instead of the floating rate plus point two if it had borrowed floating itself.
- That saves point two.
- B pays the floating rate plus one, receives the floating rate and pays four, a net five per cent instead of five point two.
- Another point two saved.
- The total is point four.
- On one hundred million dollars, each firm gains two hundred thousand dollars a year, before any dealer fee.

13 An objection to the argument

- Gain = fixed spread − floating spread = 0.4% a year
- Shared as agreed, less any dealer fee
- No gain if each firm wants its own advantage market
- Objection: floating spread resets; fixed is locked in
- So part of the gain pays for lenders' repricing risk

IN THE LESSON

- The argument is neat, but there are two cautions.
- First, check who wants what before calculating any gain.
- If each firm already wants to borrow in the market where its relative advantage lies, there is no gain to share, because no swap is needed.
- Second, and more important, there is an objection to the gain itself.
- The floating rate spread is not fixed for the life of the deal.
- It is reset as credit quality changes, while the fixed rate spread is locked in for the whole term.
- So part of the apparent gain is simply compensation for the risk that lenders reprice floating borrowing for the weaker firm.

14 Bootstrapping the discount factors

$c \times (DF_1 + \ldots + DF_n) + DF_n = 1$
$DF_n = (1 - c \times \text{sum of earlier DF}) \div (1 + c)$

Par swap rate	Par swap rate: swap worth zero
Discount factor	Factor for the last date
Earlier factors	Earlier factors, already found
Zero rate	Zero rate: DF = e^(−z × T)

IN THE LESSON

- To value a swap we need discount rates, and these come from a zero curve built by bootstrapping.
- Short maturities come from money market rates or overnight index swap rates.
- Longer ones are solved one maturity at a time from par swap rates.
- A par swap rate is the fixed rate that makes the swap worth zero, so a bond paying that coupon is worth exactly par.
- For annual payments, the coupon times all the discount factors, plus the last factor for the principal, equals one.
- Solve for the last factor using the earlier ones: one minus the coupon times the earlier factors, all divided by one plus the coupon.
- The zero rate then follows from the factor.

15 Example 6: bootstrapping three years

Illustration: annual par swap rates 3.0%, 3.5%, 4.0% for 1, 2, 3 years

1-year factor	1 ÷ 1.03	**0.9709**
2-year factor	(1 − 0.035×0.9709) ÷ 1.035	**0.9334**
3-year factor	(1 − 0.04×1.9042) ÷ 1.04	**0.8883**
3-year zero rate	−ln(0.8883) ÷ 3	**3.95%**

IN THE LESSON

- Illustration: annual par swap rates of three, three and a half, and four per cent for one, two and three years.
- For one year, the factor is one divided by one point zero three, which is point nine seven zero nine.
- For two years, one minus point zero three five times point nine seven zero nine, all divided by one point zero three five, gives point nine three three four.
- For three years, the earlier factors add to one point nine zero four two.
- One minus point zero four times that, divided by one point zero four, gives point eight eight eight three.
- Last, the continuously compounded zero rate for three years is minus the natural logarithm of point eight eight eight three, over three, about three point nine five per cent.

16 Par rates are not zero rates

From the bootstrapped curve (continuous compounding)

Maturity	Par rate | zero rate
1 year	3.00% | 2.96%
2 years	3.50% | 3.45%
3 years	4.00% | 3.95%
Collateralized swaps	Use the OIS curve

IN THE LESSON

- Compare the two sets of rates we just found.
- The par rates are three, three and a half and four per cent.
- The zero rates, continuously compounded, are two point nine six, three point four five and three point nine five per cent.
- They are close but never equal.
- Even at one year they differ, because the zero rates here are continuously compounded; compounded annually, the one year zero would equal the par rate.
- The usual mistake is to treat a par swap rate as the zero rate of the same maturity.
- Each zero rate is a single discount rate for one date, whereas a par rate summarizes a whole stream of payments.
- Collateralized swaps are generally discounted on an overnight index swap curve instead.

17 A swap as two bonds

Fixed payer: V = B_float − B_fixed
Fixed receiver: V = B_fixed − B_float

Swap value	Swap value to the fixed payer
Floating bond	Par at reset, else next pay + N
Fixed bond	PV of coupons and principal

IN THE LESSON

- The first way to value a swap is as two bonds held at once.
- A fixed payer is long a floating rate bond and short a fixed rate bond, with the same notional.
- So the swap's value to the fixed payer is the floating bond's value minus the fixed bond's value.
- The fixed receiver holds the reverse.
- The fixed bond is worth the present value of its coupons and principal at the current zero rates.
- The floating bond is worth par just after a reset date, because its next coupon is set at the market rate.
- Between resets, its next payment is already known, so it is worth the notional plus that payment, discounted only from the next payment date.

18 Example 7: valuing a swap with bonds

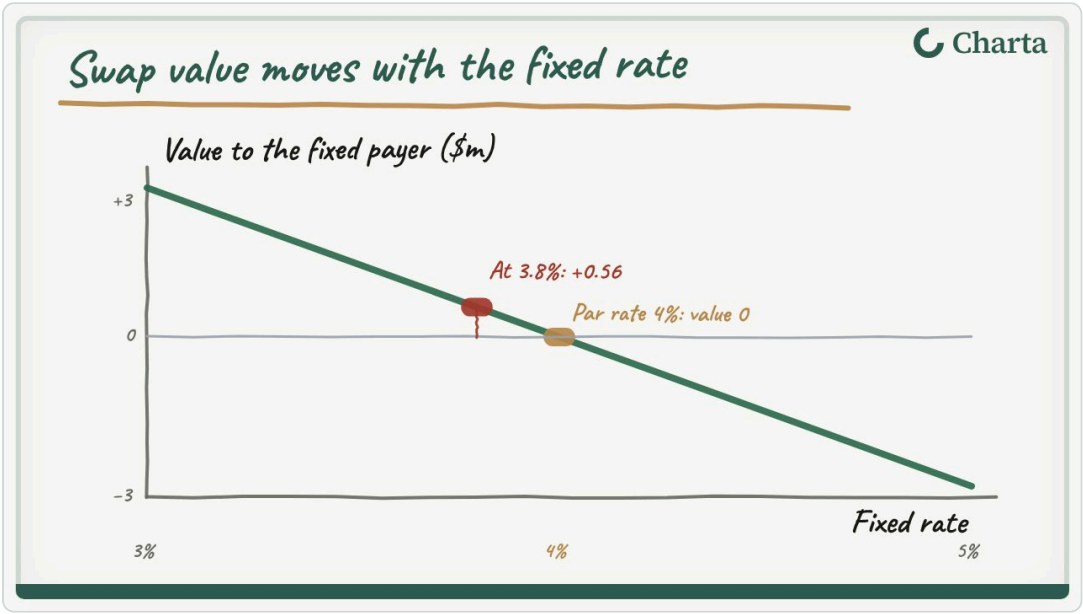
Illustration: 3-year annual swap, $100m, fixed 3.8%. Just after a reset

Annuity of factors	0.9709 + 0.9334 + 0.8883	**2.7925**
Fixed bond value	3.8 × 2.7925 + 100 × 0.8883	**$99.44m**
Floating bond value	par at a reset date	**$100.00m**
Fixed payer's value	100.00 − 99.44	**+$0.56m**
Fixed receiver's value	99.44 − 100.00	**−$0.56m**

IN THE LESSON

- Use the factors from before.
- A three year annual swap has a notional of one hundred million dollars and a fixed rate of three point eight per cent, just after a reset.
- The factors add up to two point seven nine two five.
- The fixed bond is worth three point eight times two point seven nine two five, plus one hundred times point eight eight eight three, which comes to ninety nine point four four million.
- The floating bond is worth par, one hundred million.
- So the fixed payer's swap is worth plus point five six million.
- The fixed receiver holds the reverse, minus point five six.
- The market swap rate is four per cent, so a payer of only three point eight has a gain.

19 Swap value moves with the fixed rate



IN THE LESSON

- Here is the swap's value to the fixed payer for different fixed rates, using the same curve.
- Along the bottom is the fixed rate, from three to five per cent.
- Up the side is value in millions of dollars.
- The line slopes downward, because a higher fixed rate means a worse deal for the payer.
- It crosses zero exactly at four per cent, the par swap rate, where the swap is worth nothing at the start.
- Our swap at three point eight per cent sits above zero, at plus point five six million.
- At three per cent the payer would be worth nearly two point eight million, and at five per cent the payer would owe the same amount.

20 A swap as a strip of FRAs

$FRA_i = N \times (F_i - K) \times \tau \times DF_i$
Swap value = sum of the FRA values (fixed payer)

Notional	Notional
Forward rate	Forward rate for period i
Fixed rate	Fixed swap rate
Accrual fraction	Accrual fraction
Discount factor	Discount factor to payment date

IN THE LESSON

- The second way to value a swap is as a strip of forward rate agreements, one for each payment date.
- Each one exchanges the swap's fixed rate for that period's floating rate.
- We value each by assuming the floating rate turns out equal to today's forward rate.
- For the fixed payer, the value is the notional, times the forward rate minus the fixed rate, times the accrual fraction, times the discount factor to the payment date.
- The first floating rate was already set at the last reset, so it replaces the first forward rate.
- Add the values and you have the swap.
- Flip the sign for the fixed receiver.
- The mistakes are using zero rates instead of forwards, and forgetting to discount.

21 Example 8: the same swap as FRAs

Same swap, fixed 3.8%. Forward = earlier DF ÷ this DF − 1

Period 1: rate already set	(3.00% − 3.8%) × 0.9709	**−$0.78m**
Period 2: 0.9709÷0.9334−1	(4.02% − 3.8%) × 0.9334	**+$0.21m**
Period 3: 0.9334÷0.8883−1	(5.07% − 3.8%) × 0.8883	**+$1.13m**
Sum of the FRAs	−0.78 + 0.21 + 1.13	**+$0.56m**
At fixed rate 4.0%: sum	−0.97 + 0.02 + 0.95	**$0.00m**

IN THE LESSON

- Same swap, valued as FRAs.
- The forward rate for a period is the earlier factor divided by this factor, minus one.
- Period one is already set at three per cent.
- Three minus three point eight, times point nine seven zero nine, times one hundred, is minus point seven eight million.
- Period two's forward is point nine seven zero nine over point nine three three four, minus one, which is four point zero two per cent.
- That minus three point eight, times point nine three three four, gives plus point two one.
- Period three's forward is point nine three three four over point eight eight eight three, minus one, which is five point zero seven per cent, and the value is plus one point one three.
- The sum is plus point five six million, exactly what the bonds gave.
- Finally, at a fixed rate of four per cent, the par rate, the pieces are minus point nine seven, plus point zero two, plus point nine five.
- Individually not zero, but the sum is.

22 How a currency swap works

- Principals exchanged at the start, at the spot rate
- Interest paid on each principal, in its own currency
- Principals exchanged back at maturity, at the same rate
- Turns an asset or liability into another currency
- Net each currency; unmatched spread stays put

IN THE LESSON

- A currency swap exchanges principal and interest in two currencies.
- At the start, the principal amounts are exchanged at the spot rate.
- During the life, each side pays interest on the principal it holds, in that principal's own currency.
- At maturity, the principals are exchanged back, at the same initial rate, not the rate then in the market.
- This is what makes the swap a tool for changing currency.
- A firm with a dollar bond can swap to receive dollar interest and pay euro interest.
- The dollars received service the bond, and the firm pays euro interest and repays euro principal.
- Net each currency separately, and any unmatched spread stays in its original currency.

23 Example 9: a dollar bond becomes euro debt

Illustration: spot $1.10 per €. $55m bond at 6%. Swap: receive 6% in $, pay 4% in €

Start: principals swapped	give $55m, get €50m	**1.10 per €**
Each year: dollar leg	receive 6% × $55m	**+$3.30m**
Each year: euro leg	pay 4% × €50m	**−€2.00m**
Maturity: principals back	get $55m, give €50m	**same 1.10**
Result	dollar bond is now euro debt	**4% in euros**

IN THE LESSON

- Illustration.
- Spot is one dollar and ten cents per euro.
- A firm has a fifty five million dollar bond paying six per cent.
- It enters a swap.
- At the start it gives the counterparty fifty five million dollars and receives fifty million euros, since fifty five divided by one point one is fifty.
- Each year it receives six per cent on fifty five million, which is three point three million dollars, exactly enough to pay the bond coupons.
- And it pays four per cent on fifty million euros, which is two million euros.
- At maturity it receives fifty five million dollars to repay the bond, and pays fifty million euros, at the same one point one rate.
- The result is euro debt costing four per cent.

24 A currency swap as two bonds

$V = B_D - S_0 \times B_F$ (receive domestic, pay foreign)
At the start: $B_D = S_0 \times B_F$, so $V = 0$

Domestic bond	Domestic bond, domestic rates
Spot rate	Domestic units per foreign unit
Foreign bond	Foreign bond, foreign rates
Swap value	Value to the domestic receiver

IN THE LESSON

- A fixed for fixed currency swap equals a bond in each currency, held in opposite positions.
- The domestic bond is valued at domestic zero rates, in domestic currency.
- The foreign bond is valued at foreign zero rates, in foreign currency.
- Spot is quoted as domestic units per foreign unit.
- For the party receiving domestic currency and paying foreign, the value is the domestic bond minus spot times the foreign bond.
- For the other party, the sign flips.
- Each bond includes its coupons and its principal, because principals are exchanged at the end.
- At the start, the two sides are equal, so the value is zero.
- When the foreign currency strengthens, a party paying it loses.

25 Example 10: valuing the currency swap

Illustration: 2 years left. Rates: $ 5%, € 3% (continuous). Spot 1.10

Dollar bond B_D	3.3·e^{-.05} + 58.3·e^{-.10}	**$55.891m**
Euro bond B_F	2.0·e^{-.03} + 52·e^{-.06}	**€50.913m**
Euro bond in dollars	1.10 × 50.913	**$56.004m**
Value, receive dollars	55.891 − 56.004	**−$0.113m**

IN THE LESSON

- Use the same swap, with two years left.
- Zero rates, continuously compounded, are five per cent in dollars and three per cent in euros, with spot at one point one zero.
- The dollar bond pays three point three million in year one, and fifty eight point three, which is coupon plus principal, in year two.
- Its value is fifty five point eight nine one million dollars.
- The euro bond pays two million, then fifty two.
- Its value is fifty point nine one three million euros.
- Convert that at spot: fifty six point zero zero four million dollars.
- The swap value to the party receiving dollars is fifty five point eight nine one minus fifty six point zero zero four, which is minus point one one three million.

26 A currency swap as forward contracts

$F_t = S_0 \times e^{(r_D - r_F) \times t}$
$\text{Value} = (CF_D - F_t \times CF_F) \times e^{-r_D \times t}$

Forward exchange rate	Forward rate for time t
Domestic cash flow	Domestic cash flow on the date
Foreign cash flow	Foreign cash flow on the date
Domestic discount	Discount at the domestic rate

IN THE LESSON

- The third approach treats the currency swap as a portfolio of forward contracts, one for each exchange of cash, including the final principals.
- Quote rates as domestic units per foreign unit, with continuous compounding.
- The forward exchange rate for time t is the spot rate times e raised to the domestic rate minus the foreign rate, times t.
- For the party receiving domestic cash and paying foreign, each exchange is worth the domestic cash flow minus the forward rate times the foreign cash flow, all discounted at the domestic rate.
- Each foreign payment is converted at the forward rate for its own date, because that rate can be locked in today.
- Never use spot for every date.

27 Example 11: forwards give the same value

Same swap. Spot 1.10, rates $ 5% and € 3%. Receive dollars, pay euros

Year 1 forward	1.10 × e^{0.02}	**1.1222**
Year 1 exchange value	(3.3 − 1.1222×2) e^{-0.05}	**+$1.004m**
Year 2 forward	1.10 × e^{0.04}	**1.1449**
Year 2 exchange value	(58.3 − 1.1449×52) e^{-0.10}	**−$1.117m**
Swap value	1.004 − 1.117	**−$0.113m**

IN THE LESSON

- Same swap again.
- The year one forward rate is one point one zero times e raised to point zero two, which is one point one two two two.
- The year one exchange is three point three, minus one point one two two two times two, all discounted at five per cent: plus one point zero zero four million dollars.
- The year two forward rate is one point one zero times e raised to point zero four, or one point one four four nine.
- The year two exchange is fifty eight point three, minus one point one four four nine times fifty two, discounted for two years: minus one point one one seven.
- Add them: minus point one one three million, identical to the two bond answer.
- Two methods, one value.

28 Other swaps follow the same idea

Same exchange-of-cash-flows idea, different underlying

Swap	What is exchanged
Amortizing / accreting	Notional falls / rises
Forward-start / basis	Later start / two floating
Credit default swap	Premium for loss payment
Volatility / variance	Realized vs strike
Commodity swap	Fixed vs average spot price

IN THE LESSON

- Other swaps follow the same idea of exchanging cash flows.
- Exotic interest rate swaps include amortizing swaps, whose notional falls over time, and accreting swaps, whose notional rises.
- Forward start swaps begin at a later date.
- Basis swaps exchange two floating rates.
- A credit default swap has the protection buyer paying a periodic premium, while the seller pays the loss if the reference entity has a credit event.
- A volatility swap exchanges realized volatility for a fixed strike, and a variance swap does the same with variance.
- A commodity swap exchanges a fixed price per unit for a floating price, usually an average of spot prices, on an agreed quantity.
- Always match the product to the exposure it hedges.

29 Example 12: three payoffs

Illustrations: one payoff for each product

CDS: $10m, premium 1.2%	10m × 1.2% a year	**$120,000**
CDS loss, recovery 40%	10m × (1 − 0.40)	**$6.0m**
Vol swap: strike 20, got 24	$50,000 × (24 − 20)	**$200,000**
Oil swap: fixed 80, avg 90	10,000 barrels × (90 − 80)	**$100,000**

IN THE LESSON

- Illustrations of three payoffs.
- A credit default swap on ten million dollars with a premium of one point two per cent costs the buyer one hundred and twenty thousand dollars a year.
- If the reference entity defaults with a recovery rate of forty per cent, the seller pays ten million times one minus point four, which is six million dollars.
- Next, a volatility swap pays fifty thousand dollars per volatility point.
- With a strike of twenty and realized volatility of twenty four, the receiver of realized volatility gets fifty thousand times four, or two hundred thousand.
- Finally an oil consumer pays a fixed eighty a barrel and receives the average spot, ninety, on ten thousand barrels.
- That is ten thousand times ten, a hundred thousand dollars, which fixes its cost.

30 How credit risk arises in a swap

- Only the party owed money (positive value) is exposed
- Rate swaps: net payments. Currency swaps: principal too
- Netting: exposure on the sum of values per counterparty
- Exposure = max(sum of values − collateral, 0)
- Expected loss = PD × exposure × LGD

IN THE LESSON

- Now the risk side.
- Credit risk in a swap arises only for the party to which the swap has positive value, because that party is owed money.
- If the counterparty defaults, the loss is roughly the positive value at that moment, less any recovery.
- A party with negative value has no credit exposure, although it still owes its liability.
- Because rates and exchange rates move, exposure changes sign, so both parties can face exposure at different times.
- Interest rate swaps carry a small exposure, as only net payments are at risk.
- Currency swaps carry more, since principal is exchanged at maturity.
- With a netting agreement, exposure is computed on the sum of values with the same counterparty, less collateral.

31 Example 13: netting and collateral

Illustration: two swaps with one counterparty, worth +$8m and −$5m

No netting	max(8, 0) + max(−5, 0)	**$8m**
With netting	max(8 − 5, 0)	**$3m**
Netting and $2m collateral	max(8 − 5 − 2, 0)	**$1m**
Expected loss, no collateral	2% × $3m × 60%	**$36,000**
Expected loss, with collateral	2% × $1m × 60%	**$12,000**

IN THE LESSON

- Illustration.
- A bank has two swaps with the same counterparty.
- One is worth plus eight million dollars to the bank, the other minus five.
- Without netting, the bank counts only the positive one, so its exposure is eight million.
- With netting, the values are added: eight minus five is three million.
- Now suppose the counterparty has posted two million in collateral.
- Exposure is the larger of eight minus five minus two, and zero, which is one million.
- Finally the expected loss.
- If the probability of default is two per cent, exposure is the three million before collateral, and loss given default is sixty per cent, the expected loss is two per cent times three million times sixty per cent, thirty six thousand dollars.
- With the two million of collateral, exposure is one million, and the expected loss falls to twelve thousand dollars.

32 Four mistakes to avoid

- Dropping the 0.5 or reversing who pays: net = K − R
- Par ≠ zero rate; floating bond = par just after reset
- Final principal swaps at the agreed rate, not spot
- Matched dealer has credit risk; apply netting, collateral

IN THE LESSON

- Mistake one: dropping the accrual fraction or reversing who pays.
- Use the notional times fixed minus floating times one half, and read the sign carefully.
- Mistake two: treating a par swap rate as a zero rate.
- Bootstrap the discount factors first, then discount with them, and remember the floating bond is only worth par just after a reset.
- Mistake three, in currency swaps: converting the final principal at the spot rate then, when the contract fixes the rate used at the start.
- Mistake four: treating the dealer as risk free because the book is matched.
- It holds two separate contracts, and netting and collateral should always be applied when measuring exposure.

33 Recap: the main ideas

- Swap: fixed for floating on a notional; only net paid
- Swaps reshape loans and bonds; dealers earn the spread
- Confirmation = one trade; master agreement = framework
- Value: two bonds or FRAs; currency swap: forwards
- Exposure = max(sum of values − collateral, 0)

IN THE LESSON

- Let's pull it together.
- A plain vanilla swap exchanges fixed for floating interest on a notional that never moves, and only the net amount is paid.
- A swap turns a floating loan into a fixed one, or a fixed bond into a floating one.
- Dealers sit in the middle, quote a bid and an offer, and keep the spread, while the confirmation records each trade under a master agreement.
- Value comes three ways: two bonds, a strip of FRAs, and, for currencies, forwards.
- Currency swaps exchange principal at the start and the end.
- Other swaps cover credit, volatility and commodities.
- Credit exposure is the positive value, reduced by netting and collateral.

34 Try three quick questions

1. Notional $20m, fixed 5%, floating 6%, semi-annual. Fixed payer?
   Answer: 20m × (5% − 6%) × 0.5 = −$0.1m: receives $100,000.
2. Loan at floating + 2%. Swap: pay 4.5% fixed, get floating. All-in?
   Answer: 4.5% + 2% = 6.5%.
3. Swaps +$6m and −$2m, collateral $1m. Exposure?
   Answer: max(6 − 2 − 1, 0) = $3m.

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- A swap has a notional of twenty million dollars, a fixed rate of five per cent, and pays every six months.
- The floating rate set for the period is six per cent.
- What does the fixed payer pay or receive?
- Question two.
- A borrower pays the floating rate plus two per cent, and enters a swap paying four and a half per cent fixed and receiving floating.
- What is its all in rate?
- Question three.
- Two swaps with one counterparty are worth plus six million and minus two million, and collateral of one million is held.
- What is the exposure with netting?
- The answers are on the board.