Using Futures for Hedging

FRM Part I

1 A farmer and an airline

- Farmer: harvest later, afraid the price will FALL
- Airline: fuel to buy later, afraid the price will RISE
- A futures contract fixes a price today for a deal later
- Hedging means taking the opposite side of your exposure

IN THE LESSON

- Picture two businesses on the same morning.
- A farmer has a field of wheat that will be ready in three months.
- Every cent the wheat price drops before harvest comes straight out of her income.
- Across town, an airline knows it must buy a huge amount of jet fuel next season.
- Every cent the fuel price rises before then comes straight out of its profit.
- Both are exposed to a price, but in opposite directions.
- Both can use futures contracts to lock in a price today for a deal that happens later.
- The farmer locks in a selling price, and the airline locks in a buying price.
- The rest of this lesson is about doing that well.

2 The short hedge: sell futures

- Short hedge = SELL futures
- Fits: you own it, or will sell it later
- Fear: the price FALLS
- Examples: farmer, oil producer, fund holding bonds

IN THE LESSON

- A short hedge means selling futures.
- It suits someone who owns an asset, or who will receive it and sell it later, and who fears the price will fall.
- The farmer before harvest is the classic case.
- An oil producer is another, because its revenue falls when oil falls.
- A fund that holds bonds is a third, because bond values drop when rates rise.
- If the price does fall, the loss on the physical asset is offset by a gain on the futures, because a short futures position gains when the futures price goes down.
- The hedger gives up the chance of a windfall in return for a price it can plan around.

3 The long hedge: buy futures

- Long hedge = BUY futures
- Fits: you must buy it later
- Fear: the price RISES
- Examples: airline, copper user, fund awaiting cash

IN THE LESSON

- A long hedge means buying futures.
- It suits someone who will have to buy an asset later and who fears the price will rise.
- The airline buying fuel is the classic case.
- A manufacturer that needs copper for next quarter's output is another.
- A fund that expects a cash inflow and plans to invest it in shares is a third, because it worries that shares will be dearer by the time the cash arrives.
- If the price does rise, the extra cost of buying is offset by a gain on the futures, because a long futures position gains when the futures price goes up.
- Again the hedger fixes a cost it can plan around.

4 Which hedge fits? A classification

Ask: which price move hurts you?

Situation	Hedge
Farmer, crop unsold	Short: sell futures
Airline needing fuel	Long: buy futures
Fund holding bonds	Short: sell futures
Fund awaiting cash for shares	Long: buy futures
Fixed-price sale, metal not owned	Long: buy futures

IN THE LESSON

- Now let's practice choosing.
- The test is simple: which way does a price move hurt you?
- It is not whether your company has sold or bought something.
- Take a farmer with an unsold crop.
- A fall hurts, so she sells futures.
- An airline needing fuel is hurt by a rise, so it buys futures.
- A bond fund that owns bonds is hurt by a fall in their price, so it sells.
- A fund waiting for cash to invest is hurt by a rise, so it buys.
- The last case is the trap.
- A firm agrees today to sell a metal at a fixed price, but does not yet own the metal.
- It has sold, yet a rise in its purchase cost hurts it, so it needs a long hedge.

5 Example 1: hedge only the net amount

Illustration: same asset, same quarter

Buys later	expected purchases	**60,000 units**
Sells later	expected sales	**45,000 units**
Net exposure	60,000 − 45,000	**15,000 buy**
Hedge to place	net buyer, so long hedge	**Buy futures**

IN THE LESSON

- Real firms often have exposures that offset, so hedge only the net amount.
- Here is an illustration.
- A trading firm expects to buy sixty thousand units of an asset over the next quarter.
- In the same period it expects to sell forty five thousand units of that same asset.
- The two flows partly cancel, so there is no reason to hedge all sixty thousand and all forty five thousand.
- The net amount is sixty thousand minus forty five thousand, which is fifteen thousand units.
- The firm is net a buyer, so a price rise would hurt it, and it needs a long hedge.
- It buys futures for fifteen thousand units.
- Hedging more than that would create a new risk instead of removing one.

6 The case for hedging

- Less uncertainty in cash flows: easier planning
- Lower chance of financial distress
- Protects investment spending
- Can cut expected taxes when the tax schedule is convex

IN THE LESSON

- Why would a firm hedge at all?
- The central argument is that hedging reduces uncertainty in cash flows.
- That has several benefits.
- Steady cash flows make planning and budgeting easier.
- They lower the chance of financial distress, because a bad price move is less likely to leave the firm unable to pay its debts.
- They protect investment spending, since the firm does not have to cancel good projects when prices fall.
- And hedging can cut expected taxes when the tax schedule is convex, meaning that profits taxed at higher rates in good years are not offset by equal savings in bad years.
- Smoother income can therefore mean a smaller expected tax bill.

7 The case against hedging

- Shareholders can diversify or hedge on their own
- Costs money and management attention
- Firm-specific risk earns no risk premium
- A hedge can leave you worse off than unhedged rivals

IN THE LESSON

- There is also a case against.
- First, shareholders can diversify or hedge on their own account, so they do not need the firm to do it for them.
- Second, hedging costs money and managerial attention.
- Third, firm specific risk earns no risk premium, so removing it does not raise the value investors put on the firm.
- Fourth, a hedge can leave the firm worse off than competitors who did not hedge.
- Suppose the price of the input falls.
- Unhedged rivals buy it cheaply, while the hedged firm is locked into a dearer price and loses on its futures.
- Good hedging is a business decision, not a free lunch.

8 Example 2: a hedge removes variability

Illustration: futures sold at $80, basis ignored

Spot falls to	futures gain 80 − 70	**$70 → +$10**
Net price received	70 + 10	**$80**
Spot rises to	futures result 80 − 90	**$90 → −$10**
Net price received	90 − 10	**$80**
Unhedged would have been	spot only	**$90**

IN THE LESSON

- A hedge removes variability, not expected profit.
- Here is an illustration with a producer who sells oil later and sells futures now at eighty dollars a barrel.
- We ignore basis for simplicity.
- Case one: the spot price falls to seventy.
- The futures gain eighty minus seventy, ten dollars, so the net price is seventy plus ten, eighty dollars.
- Case two: the spot price rises to ninety.
- The futures result is eighty minus ninety, minus ten dollars, a loss, so the net price is ninety minus ten, again eighty dollars.
- The hedge delivered eighty either way.
- In the second case the hedged outcome is lower than the unhedged ninety, but judge a hedge by its purpose, not by hindsight on the outcome.

9 Basis and the effective price

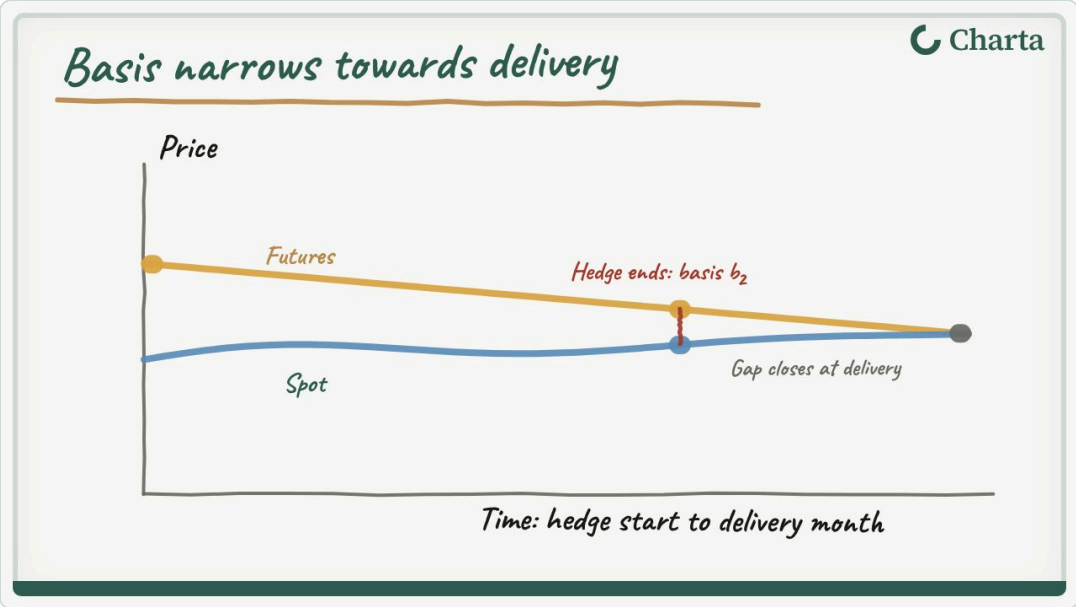
Basis $b = S - F$
Effective price $= S_2 + F_1 - F_2 = F_1 + b_2$

Spot price	S: spot price of the exposure
Futures price	F: futures price
Basis	b_2: basis when hedge ends
Effective price	Effective price: what you lock in

IN THE LESSON

- The basis is the spot price of the asset being hedged minus the futures price.
- In symbols, basis equals spot minus futures.
- This lesson uses that convention, but some texts define it as futures minus spot, so always read the question.
- If the hedge ends exactly in the delivery month, the basis is close to zero.
- But if the hedge ends before delivery, the basis is not zero and is not known in advance.
- That leads to the key result.
- The effective price a hedger gets equals the final spot price plus the starting futures price minus the final futures price.
- That simplifies to the starting futures price plus the final basis.
- Only the final basis is uncertain.

10 Basis narrows towards delivery



IN THE LESSON

- This picture shows why the basis matters.
- Along the bottom is time, from the start of the hedge to the delivery month.
- Up the side is price.
- The upper line is the futures price, and the lower line is the spot price.
- At the start there is a gap between them.
- As delivery approaches, futures and spot are pulled together, so the gap closes.
- Now suppose the hedger closes the position here, before delivery.
- The gap that remains is the final basis.
- It is not zero, and nobody knew its size at the start.
- That leftover gap, which can change the result of an otherwise perfect hedge, is called basis risk.

11 Where basis risk comes from

- Asset mismatch: grade, product or place differ
- Gap between hedge end date and delivery month
- The closing date is uncertain
- Rule: choose the delivery month just after the hedge ends

IN THE LESSON

- Basis risk comes from three places.
- The first is asset mismatch.
- The futures contract may be on a slightly different grade, product or location from the asset you are hedging, so the two prices drift apart.
- The second is a gap between the hedge end date and the delivery month.
- Because the basis is only reliably zero at delivery, a mismatch in time leaves a basis that moves.
- The third is uncertainty about when the hedge will end.
- If you do not know the closing date in advance, you cannot match it to a contract.
- A good rule of thumb: choose the delivery month just after the hedge ends, so the hedge ends close to delivery.

12 Example 3: a short hedge with basis

Illustration: short hedge, futures sold at $F_1 = \$100$

Starting basis $S_1 - F_1$	98 − 100	**−2**
Final basis $b_2 = S_2 - F_2$	93 − 96	**−3**
Futures gain on the short	100 − 96	**+4**
Effective price	93 + 4	**97**
Check: $F_1 + b_2$	100 + (−3)	**97**

IN THE LESSON

- Let's follow a short hedger.
- These are illustrative prices.
- She sells futures at one hundred, when spot is ninety eight, so the starting basis is ninety eight minus one hundred, minus two.
- When she lifts the hedge, spot is ninety three and futures are ninety six.
- The final basis is ninety three minus ninety six, minus three.
- The futures gain is one hundred minus ninety six, plus four.
- She sells the asset at ninety three and adds the four, so the effective price is ninety seven.
- Check the shortcut: the starting futures price of one hundred plus the final basis of minus three gives ninety seven.
- The basis weakened by one, so she did one worse than the ninety eight she would have had if it had not changed.

13 Strengthening and weakening basis

Basis $b = S - F$. Effective price $= F_1 + b_2$

Basis moves	Effect
Rises (strengthens)	Helps short, hurts long
Falls (weakens)	Helps long, hurts short
Unchanged	Price = F_1 + starting basis
Hedge ends before delivery	Final basis b_2 unknown

IN THE LESSON

- Now the direction of the basis change.
- A rising basis is called strengthening, and a falling basis is called weakening.
- A strengthening basis helps a short hedger and hurts a long hedger.
- A weakening basis does the opposite.
- You can see why from the formula: the effective price is the starting futures price plus the final basis.
- A short hedger is selling, so a higher final basis means a higher price received, which is good.
- A long hedger is buying, so a higher final basis means a higher cost paid, which is bad.
- If the basis ends exactly where it began, the effective price is the starting futures price plus the starting basis, and the hedge works as planned.

14 Profit or loss on a hedge

Short futures profit = $(F_1 - F_2) \times \text{size} \times N$
Long futures profit = $(F_2 - F_1) \times \text{size} \times N$

Futures profit	Futures profit or loss
Futures price	F_1 at start, F_2 at the end
Size and number	size: per contract; N: contracts
Effective price	Price = $F_1 + b_2$

IN THE LESSON

- A hedge's profit or loss has two parts: the change in value of the exposure, and the gain or loss on the futures.
- A short futures position gains when the futures price falls.
- Its profit is the price sold minus the price bought back, times the contract size, times the number of contracts.
- A long futures position is the reverse.
- It gains when the futures price rises.
- Both a short hedge and a long hedge reduce to the same expression per unit, the final spot price plus the starting futures price minus the final futures price.
- That equals the starting futures price plus the final basis.
- Always apply the quantity actually hedged, which may differ from the exposure.

15 Example 4: a short hedge in dollars

Illustration: sell 50 contracts of 1,000 bbl at $80

Sale value at the end	50,000 × $74	**$3,700,000**
Futures gain, 50 short	(80 − 75) × 1,000 × 50	**$250,000**
Total received	3,700,000 + 250,000	**$3,950,000**
Effective price per barrel	3,950,000 ÷ 50,000	**$79**
Check: $F_1 + b_2$	80 + (74 − 75)	**$79**

IN THE LESSON

- Here is a full short hedge in dollars, with made up figures.
- A producer will sell fifty thousand barrels.
- She sells fifty futures contracts of one thousand barrels each, at eighty dollars.
- When she closes, spot is seventy four and futures are seventy five.
- She sells the oil for fifty thousand times seventy four, which is three million seven hundred thousand dollars.
- The futures gain is eighty minus seventy five, times one thousand, times fifty, which is two hundred fifty thousand dollars.
- Her total is three million nine hundred fifty thousand.
- Divide by fifty thousand barrels and the effective price is seventy nine dollars.
- Check: eighty plus the final basis of seventy four minus seventy five gives seventy nine.

16 Example 5: a long hedge in dollars

Illustration: buy 4 contracts of 25,000 lb at $4.00

Cost of buying at the end	100,000 lb × $4.30	**$430,000**
Futures gain, 4 long	(4.35 − 4.00) × 25,000 × 4	**$35,000**
Net cost	430,000 − 35,000	**$395,000**
Effective cost per pound	395,000 ÷ 100,000	**$3.95**
Check: $F_1 + b_2$	4.00 + (4.30 − 4.35)	**$3.95**

IN THE LESSON

- Now a long hedge, again with made up figures.
- A manufacturer needs one hundred thousand pounds of metal.
- Each futures contract covers twenty five thousand pounds, so it buys four contracts at four dollars a pound.
- When it closes the hedge, spot is four thirty and futures are four thirty five.
- Buying the metal costs one hundred thousand times four thirty, which is four hundred thirty thousand dollars.
- The futures gain is four thirty five minus four, times twenty five thousand, times four, which is thirty five thousand dollars, and it reduces the cost.
- Net cost is three hundred ninety five thousand, or three ninety five a pound.
- Check: four dollars plus the final basis of four thirty minus four thirty five equals three ninety five.

17 Is the hedge working? Check the signs

- Futures result and exposure change: OPPOSITE signs
- Same signs = wrong direction or a poor match
- Profit uses the quantity actually hedged
- If h is not 1, hedged amount differs from exposure

IN THE LESSON

- Here is a quick test you can apply to any hedge.
- In a good hedge, the futures result and the change in the value of the exposure have opposite signs.
- If the producer's oil falls in value, the short futures should gain.
- If the manufacturer's metal gets dearer to buy, the long futures should gain.
- When both move the same way, something is wrong: perhaps the direction is wrong, or the contract is a poor match.
- Also remember that the futures result is measured on the quantity hedged.
- If a hedge ratio below or above one is used, that quantity is not the same as the exposure, so use the hedged amount when you compute the profit.

18 The cross hedge

- Cross hedge = futures on a DIFFERENT asset
- Used when no futures exist, or another is more liquid
- Example: jet fuel hedged with heating oil futures
- Prices move together, but not perfectly: basis risk

IN THE LESSON

- Sometimes no futures contract exists on the asset you are hedging.
- Or the available contracts are on a different asset that trades far more actively.
- Using a futures contract on a different asset from the one being hedged is called a cross hedge.
- An airline, for example, may hedge jet fuel with heating oil futures, because the two prices usually move together.
- A cross hedge brings basis risk, because the two prices do not move in lockstep.
- So the question becomes how much futures cover to use.
- A one for one hedge can leave the position more volatile than necessary.
- The answer is the minimum variance hedge ratio, which depends on how closely the two prices move together and how volatile each one is.

19 The minimum-variance hedge ratio

$\mathrm{Var}(h) = \sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F$
$h^* = \rho \times \sigma_S \div \sigma_F$ Effectiveness $= \rho^2$

Correlation, rho	ρ: correlation of price changes
Volatility, sigma	σ_{S}, σ_{F}: price volatility
Hedge ratio	h*: variance-minimizing ratio

IN THE LESSON

- Let's find the best hedge ratio.
- Suppose we hold a position in the spot asset and hedge with some amount of futures.
- The variance of the hedged position depends on the hedge ratio: it is the spot variance, plus the ratio squared times the futures variance, minus two times the ratio times the correlation times the two volatilities.
- Choosing the ratio that makes this smallest gives the answer: the hedge ratio equals the correlation times the spot volatility divided by the futures volatility.
- Here the volatilities are standard deviations of changes in the spot and futures prices.
- This ratio is also the slope from regressing spot changes on futures changes.
- And effectiveness is the proportion of variance removed, which is the correlation squared.

20 Example 6: a cross hedge

Illustration: $\rho = 0.8$, $\sigma_{S} = 0.030$, $\sigma_{F} = 0.040$

Hedge ratio h	$0.8 \times 0.030 \div 0.040$	**0.60**
Variance removed ρ^2	0.8^2	**64%**
Residual std deviation	$0.030 \times \sqrt{(1 - 0.64)}$	**0.018**
Cut in std deviation	$1 - 0.018 \div 0.030$	**40%**
Wrong: inverted ratio	$0.8 \times 0.040 \div 0.030$	**1.07**

IN THE LESSON

- Let's compute a cross hedge with illustrative numbers.
- The correlation between changes in jet fuel and heating oil is point eight.
- The volatility of jet fuel changes is point zero three zero dollars per gallon, and for heating oil it is point zero four zero.
- The hedge ratio is point eight times point zero three zero divided by point zero four zero, which is point six.
- The variance removed is the correlation squared, point eight squared, sixty four per cent.
- The residual standard deviation is point zero three zero times the square root of one minus point six four, which is point zero one eight.
- That is a forty per cent cut in risk.
- Do not invert the ratio.
- Point eight times point zero four divided by point zero three zero gives one point zero seven, which is too large here: it hedges more than the minimum variance amount.

21 How effective is a hedge?

Variance removed = ρ^2, not ρ

Correlation ρ	Variance removed ρ^2
0.5	25%
0.8	64%
0.9	81%
0.95	90.25%
1.0	100%

IN THE LESSON

- Effectiveness is the proportion of variance removed, and that is the correlation squared, the regression R squared.
- Look at the pattern.
- A correlation of point five removes only twenty five per cent of the variance.
- Point eight removes sixty four per cent.
- Point nine removes eighty one per cent, not ninety.
- Point nine five removes ninety point two five per cent.
- Only a correlation of one removes everything.
- So the common slip is to treat the correlation itself as the share of risk removed.
- It is not.
- The remaining standard deviation follows from the same figure: the spot volatility times the square root of one minus the correlation squared.
- Even a strong correlation leaves noticeable residual risk.

22 How many contracts? Tailing the hedge

N = h × QA ÷ QF
Tailed: N = h × VA ÷ VF

Number of contracts	N: contracts, to whole number
Exposure size	QA: size of exposure
Contract size	QF: size of one contract
Exposure value	VA: spot value of exposure
Contract value	VF: futures price × contract size

IN THE LESSON

- Now count the contracts.
- The number of futures contracts that minimizes variance equals the hedge ratio times the size of the position being hedged, divided by the size of one contract.
- The ratio is one if the hedge is exact.
- Otherwise it is the optimal ratio from the last section.
- The answer is rounded to a whole number.
- Tailing then adjusts for daily settlement, which pays futures gains and losses immediately rather than at the end.
- Instead of physical sizes, we use values.
- The tailed number equals the hedge ratio times the value of the exposure, divided by the value of one futures contract, which is the futures price times the contract size.
- Check units: gallons against barrels is a common slip.

23 Example 7: the number of contracts

Illustration: h = 0.60, long hedge for jet fuel

Exposure QA	jet fuel to buy	**2,000,000 gal**
One contract QF	heating oil	**42,000 gal**
Raw ratio QA ÷ QF	2,000,000 ÷ 42,000	**47.62**
Times h	0.60 × 47.62	**28.57**
Whole contracts to buy	round	**29**

IN THE LESSON

- Let's size the airline's hedge.
- These are illustrative numbers.
- The airline will buy two million gallons of jet fuel.
- The heating oil contract is forty two thousand gallons.
- Divide two million by forty two thousand and you get about forty seven point six two contracts for a one for one hedge.
- But the optimal ratio is point six, so multiply: point six times forty seven point six two gives twenty eight point five seven.
- Rounded to a whole number, that is twenty nine contracts.
- Because the airline needs to buy fuel, it buys the contracts, a long hedge.
- Notice that twenty nine is well below forty eight, the amount a naive one for one hedge would use.

24 Example 8: tailing the hedge

Illustration: spot $2.40, futures $2.60, h = 0.60

Exposure value VA	2,000,000 × $2.40	**$4,800,000**
Contract value VF	42,000 × $2.60	**$109,200**
Tailed N	0.60 × 4,800,000 ÷ 109,200	**26.37**
Whole contracts	round	**26**
Untailed, for comparison	from the last example	**29**

IN THE LESSON

- Now tail the same hedge.
- Spot jet fuel is two forty a gallon and the futures price is two sixty.
- These are made up.
- The value of the exposure is two million gallons times two forty, which is four million eight hundred thousand dollars.
- The value of one contract is forty two thousand gallons times two sixty, which is one hundred nine thousand two hundred dollars.
- The tailed number is point six times four million eight hundred thousand divided by one hundred nine thousand two hundred, about twenty six point three seven.
- Rounded, that is twenty six contracts, compared with twenty nine before tailing.
- The tailed number is slightly smaller here because the futures price is above spot.

25 Changing beta with index futures

N = (target β − current β) × P ÷ A
Full hedge: N = − current β × P ÷ A

Number of contracts	N > 0: buy. N < 0: sell
Exposure value	P: value of the portfolio
Contract value	A: index level × multiplier

IN THE LESSON

- Stock index futures also let a manager change the beta of an equity portfolio without trading the shares.
- To move beta from its current level to a target level, the number of contracts equals the target beta minus the current beta, times the portfolio value, divided by the value of the stocks underlying one contract.
- That value is the index level times the multiplier.
- A positive answer means buy futures to raise beta.
- A negative answer means sell futures to lower it.
- Setting the target to zero gives a full hedge, so the number of contracts is minus the current beta times the portfolio value divided by that contract value.
- The beta of a mixed portfolio is the value weighted average of its parts.

26 Example 9: moving a portfolio's beta

Illustration: $40m portfolio, contract A = $1m

Portfolio beta	(30×1.2 + 10×0.8) ÷ 40	**1.10**
Cut beta to 0.6	(0.6 − 1.1) × 40 ÷ 1	**−20: sell 20**
Full hedge, beta 0	−1.1 × 40 ÷ 1	**−44: sell 44**
Raise beta to 1.5	(1.5 − 1.1) × 40 ÷ 1	**+16: buy 16**

IN THE LESSON

- Let's work one case with illustrative numbers.
- A portfolio is worth forty million dollars.
- Thirty million has a beta of one point two, and ten million has a beta of point eight.
- The value weighted beta is thirty times one point two plus ten times point eight, divided by forty, which is one point one.
- One index futures contract has underlying value of one million dollars.
- To cut beta to point six, the number is point six minus one point one, times forty, divided by one: minus twenty, so sell twenty contracts.
- For a full hedge, minus one point one times forty is minus forty four, sell forty four.
- To raise beta to one point five, one point five minus one point one, times forty, is plus sixteen: buy sixteen.

27 Stack and roll: a long-term hedge

- Hedge the WHOLE exposure in the nearest contract
- Close it near expiry; open the next contract; repeat
- Strip hedge: each cash flow gets its own maturity
- Risks: rollover basis risk and funding risk

IN THE LESSON

- Futures usually have short lives, but sometimes the exposure lasts for years.
- Stack and roll builds a long term hedge out of short dated contracts.
- The whole exposure is hedged in the nearest contract.
- As that contract nears expiry, it is closed out and replaced by the next contract, and so on until the exposure ends.
- This is different from a strip hedge, which matches each future cash flow with its own maturity, so that each piece of the exposure gets a contract that ends when it does.
- Stack and roll is easier because the nearest contracts are the most liquid.
- The cost is two risks, which we will meet next: rollover basis risk, and funding risk.

28 Example 10: rollover basis risk

Illustration: short hedge, 100,000 bbl, 100 contracts

Roll 1: contango	new 82 vs expiring 80	**+$2 per bbl**
Roll 2: backwardation	new 84 vs expiring 85	**−$1 per bbl**
Net effect of two rolls	+2 − 1	**+$1 per bbl**
On 100,000 barrels	1 × 100,000	**+$100,000**

IN THE LESSON

- The first risk is rollover basis risk.
- At each roll, the price gap between the expiring contract and the new one is unknown in advance.
- Here is an illustration for a short hedger with a hundred thousand barrels, in a hundred contracts.
- At the first roll the expiring contract is at eighty and the new one is at eighty two.
- That is contango, and the short hedger gains two dollars a barrel.
- At the second roll the expiring contract is at eighty five and the new one at eighty four.
- That is backwardation, and she loses one dollar.
- Net, the rolls add one dollar a barrel, or one hundred thousand dollars.
- A long hedger gains and loses the other way round.

29 Example 11: funding risk

Illustration: long 100,000 bbl hedge, price falls $8

Futures loss on the long	8 × 100,000	**− $800,000**
Margin paid in cash now	settled daily	**$800,000**
Gain on fixed-price sales	not yet received	**$0 cash**
Cash gap today	800,000 − 0	**$800,000**

IN THE LESSON

- The second risk is funding risk.
- Futures losses are settled in cash through margin, while the gains on the hedged exposure are not yet realized.
- Here is an illustration.
- A firm has promised to sell fuel at fixed prices over several years and hedges with long futures on a hundred thousand barrels.
- The oil price falls by eight dollars a barrel.
- The loss on the futures is eight times a hundred thousand, eight hundred thousand dollars, and it must be paid in cash through margin now.
- The matching gain on the fixed price contracts is real but still to come over the years, so zero cash today.
- The cash gap is eight hundred thousand dollars.
- Metallgesellschaft in nineteen ninety three is the standard cautionary example.

30 Four mistakes to avoid

- Direction: ask which price move hurts, not what you did
- Basis: price = $F_1 + b_2$; check the definition S − F
- Ratio: ρσ_{S}÷σ_{F}, not inverted; effectiveness is ρ^2, not ρ
- Contracts: units match, tail by value, beta difference

IN THE LESSON

- Here are four mistakes to avoid.
- Mistake one: choosing the hedge by what the firm did rather than which price move hurts it.
- Ask which way the price hurts, and remember a fixed price sale of an asset not yet owned needs a long hedge.
- Mistake two: forgetting the basis.
- The effective price is the starting futures price plus the final basis, and check which way the basis is defined.
- Mistake three: getting the hedge ratio wrong.
- Use correlation times spot volatility over futures volatility, never inverted, and effectiveness is the correlation squared.
- Mistake four: slips in counting contracts.
- Match units, tail with values, and for beta use the difference between target and current beta, with its sign.

31 Recap: the main ideas

- Short = sell, fear a fall. Long = buy, fear a rise
- Cuts variability. Pro: steady cash. Con: cost, diversify
- Effective price = $F_1 + b_2$; only b_2 is uncertain
- h = ρσ_{S}÷σ_{F}; effectiveness ρ²; N = h × QA÷QF, tailed
- Beta: N = (target − current) × P÷A. Roll: basis, funding

IN THE LESSON

- Let's pull it together.
- A short hedge sells futures and suits someone who fears a fall.
- A long hedge buys futures and suits someone who fears a rise.
- The case for hedging is steadier cash flow, and the case against is cost, and shareholders' own diversification.
- A hedge removes variability, not expected profit.
- The effective price is the starting futures price plus the final basis, and the final basis is where the risk lies.
- A cross hedge uses the ratio of correlation times spot volatility divided by futures volatility, and effectiveness is the correlation squared.
- The number of contracts is the ratio times exposure over contract size, tailed with values.
- Index futures change beta, and stack and roll brings rollover and funding risk.

32 Try three quick questions

1. ρ 0.9, σ_{S} 0.05, σ_{F} 0.04. Hedge ratio and variance removed?
   Answer: h = 0.9×0.05÷0.04 = 1.125; ρ² = 81%.
2. Short hedge: F_1 50, S_2 47, F_2 48. Effective price?
   Answer: 47 + 50 − 48 = 49, or 50 + (47 − 48).
3. $20m, beta 1.0 to 1.5, A = $0.5m. Contracts?
   Answer: (1.5 − 1.0) × 20 ÷ 0.5 = +20: buy 20.

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- A cross hedge has a correlation of point nine, a spot volatility of point zero five and a futures volatility of point zero four.
- What are the hedge ratio and the share of variance removed?
- Question two.
- A short hedger sold futures at fifty.
- When the hedge ended, spot was forty seven and futures were forty eight.
- What was the effective price?
- Question three.
- A portfolio of twenty million dollars has a beta of one and the target beta is one point five.
- One contract has underlying value of half a million dollars.
- How many contracts, and buy or sell?
- The answers are on the board.