CHARTA LEARNING · LESSON NOTES

The Arbitrage Pricing Theory and multifactor models

FRM Part I

1 A story: one storm, or many kinds of weather?

- A farm is hit by rain, fuel prices, borrowing costs, demand
- Each force hits different assets by a different amount
- Is one force enough to describe risk, or do we need several?
- Today: measure each push, and price each one

IN THE LESSON

- Imagine you run a small farm.
- Is there one thing that decides whether you have a good year?
- You might say the weather.
- But think harder.
- Rain matters, and so does the price of fuel, and so does the cost of borrowing, and so does how rich your customers feel.
- Each of these hits you by a different amount.
- A neighbour who grows other crops feels them differently.
- Investors face the same puzzle.
- A share does not go up and down because of one force.
- It is pushed around by several, and each push is bigger for some assets than others.
- The question for today is how to measure those pushes and how much reward each one deserves.

2 The starting point: the CAPM

- Expected return depends on one thing: beta to the market
- Beta = how strongly the asset moves with the market
- Assumes investors care only about mean and variance
- Assumes everyone holds the same market portfolio

IN THE LESSON

- First, the model we already know, the capital asset pricing model, or C A P M.
- It says an asset's expected return depends on one thing only: its beta to the market portfolio.
- Beta measures how strongly the asset moves with the market as a whole.
- The asset earns the risk free rate, which is what you get for waiting without risk, plus its beta times the market risk premium, the extra return investors expect from holding the market instead of safe cash.
- The model rests on strong assumptions.
- Investors care only about the average return and the variance of their portfolio, and they all hold the same market portfolio.
- Hold on to those two assumptions, because the next theory does not need them.

3 The CAPM formula

$E(R) = Rf + \beta \times [E(Rm) - Rf]$

Expected return	What the asset is expected to earn
Risk-free rate, Rf	Return for waiting, no risk
Factor beta	Sensitivity to the market
Factor risk premium	Market return minus Rf

IN THE LESSON

- Here is the C A P M as a line.
- Expected return equals the risk free rate, plus beta times the market risk premium.
- Take the pieces one at a time.
- The risk free rate, written R f, is the return with no risk at all.
- Beta tells us how much market risk the asset carries: a beta of one moves with the market, a beta of two moves twice as much.
- The market risk premium is the expected return on the market minus the risk free rate.
- Multiply beta by the premium, and you have the reward for the market risk taken.
- Add the risk free rate, and you have the full expected return.
- Notice there is only one beta here, and only one premium.

4 The APT: a different starting point

- 1. Returns are linear in several systematic factors
- Plus an idiosyncratic term, unique to each asset
- 2. Enough assets to diversify idiosyncratic risk away
- 3. No arbitrage persists
- It does not name the factors or need mean-variance

IN THE LESSON

- The arbitrage pricing theory, or A P T, starts from a different place.
- It does not ask what investors prefer.
- It asks what the market must look like if there are no free lunches.
- It makes three assumptions.
- First, returns follow a linear model: each return is built from several systematic factors, plus a term that belongs to that asset alone, the idiosyncratic term.
- Second, there are enough assets for the idiosyncratic risk to diversify away in a large portfolio.
- Third, no arbitrage persists, meaning no one can earn a sure profit with no money and no risk.
- Nothing here says that investors care only about mean and variance.
- And nothing says what the factors are.

5 The APT pricing relation

$E(R) = Rf + \beta_1 \times premium_1 + \beta_2 \times premium_2 + \ldots + \beta k \times premiumk$

Expected return	Expected return of the asset
Risk-free rate, Rf	Risk-free rate
Factor beta	Beta to each factor
Factor risk premium	Reward per unit of that factor

IN THE LESSON

- From those assumptions comes the pricing line.
- Expected return equals the risk free rate, plus the sum, over every factor, of the factor beta times that factor's risk premium.
- Look at the pieces.
- Each factor has its own beta, which says how strongly the asset responds to that factor.
- And each factor has its own risk premium, which is the reward the market pays for carrying one unit of that risk.
- Multiply each beta by its own premium, add them up, and add the risk free rate.
- If there is only one factor, and it is the market, you get the C A P M back.
- So the C A P M is the single factor special case of the A P T.

6 CAPM and APT side by side

Same idea, different foundations

Feature	CAPM | APT
Sources of risk	One | Several
Investor behaviour	Mean-variance | Not needed
Which factors?	The market | Not specified
Core argument	Equilibrium | No arbitrage
Holds for	All assets | Diversified portfolios

IN THE LESSON

- Let's set them next to each other.
- The C A P M has one source of risk, the market, while the A P T allows several.
- The C A P M needs investors who care only about mean and variance and hold the same market portfolio, while the A P T needs only a factor model, diversification and no arbitrage.
- The C A P M tells you the one factor, the market.
- The A P T does not tell you the factors at all, which is both its freedom and its weakness: you must choose them yourself.
- And one last difference.
- The A P T relation holds closely for well diversified portfolios, but only approximately for single assets, because the idiosyncratic leftovers do not vanish for one asset alone.
- Finally, the C A P M is an equilibrium argument, about what prices result when investors behave in a certain way, while the A P T rests on no arbitrage.

7 Example 1: why no arbitrage forces the line

One factor. Rf 3%, premium 5%. C expects 10%, D 8%; both beta 1

Fair return, beta 1	3% + 1 × 5%	**8%**
Portfolio C expects	above the line	**10%**
Long C, short D	10% − 8%, per $100m	**$2m**
Expected profit, factor risk	betas cancel, zero cost	**$2, none**

IN THE LESSON

- Here is the argument in numbers, an illustration.
- Suppose one factor, with a risk free rate of three per cent and a factor premium of five per cent.
- A well diversified portfolio with a beta of one should then earn eight per cent.
- Now suppose portfolio C, also with a beta of one, is expected to earn ten per cent.
- Buy one hundred dollars of C and sell short one hundred dollars of portfolio D, which has a beta of one and earns eight.
- The two betas cancel, so there is no factor risk, and diversification has removed the rest.
- The cost is nothing, yet you expect to earn two per cent on one hundred, which is two dollars, with no factor risk.
- Everyone would do it, pushing C's price up and its return down, until the gap closes.

8 Only systematic risk earns a premium

- Only systematic factor risk earns a premium
- Idiosyncratic risk can be diversified away, so it is not paid
- A strike at one company's factory earns no extra return
- Close for diversified portfolios, rough for single assets

IN THE LESSON

- One lesson from the A P T matters more than any formula, and it is a key point.
- Only systematic factor risk earns a premium.
- Idiosyncratic risk never does.
- Why?
- Because it can be diversified away, and a risk you can remove for free should not be paid for.
- If a company's return is hurt by a strike at its own factory, that is idiosyncratic, and the market will not give you extra return for bearing it.
- Hold many assets and it fades.
- What cannot be diversified is the exposure to the broad factors, such as the economy, inflation or interest rates, and that is what the market pays for.
- Remember also that the relation holds closely for well diversified portfolios, but only approximately for single assets.

9 What goes into a multifactor model

- $R = a + b_1F_1 + b_2F_2 + \ldots + bkFk + e$
- a = constant; each F = a factor, usually a surprise
- Typical: market, GDP growth, inflation, interest rates
- e = residual: idiosyncratic and diversifiable

IN THE LESSON

- Now let's open up the multifactor model.
- It writes an asset's return as a sum of parts.
- First a constant, which is the part not tied to any factor.
- Then, for each factor, a beta times the factor.
- And last a residual, the leftover that belongs to that asset alone.
- The factors are systematic drivers, the broad forces that move many assets together.
- Typical choices are the return on the market, growth in the economy, inflation and interest rates.
- They are usually entered as surprises, the amount by which the outcome differs from what was expected.
- The reason is simple.
- Expected changes are already in the price today, and only the news moves it.

10 Factor betas and the residual

- Beta = change in return per one-unit move in that factor
- Others held constant: it is a partial effect
- Betas are estimated by regression on past data
- Residual: uncorrelated with the factors, diversifiable

IN THE LESSON

- Each factor beta has a plain meaning.
- It is the expected change in the asset's return when that one factor moves by one unit, with all the other factors held constant.
- If an asset has a beta of point five to economic growth, and growth surprises by one per cent, we expect the return to rise by half a per cent from that cause.
- In practice betas are not known.
- They are estimated by regression, using past returns of the asset and past values of the factors.
- The residual is what the regression cannot explain.
- It is idiosyncratic, uncorrelated with the factors, and the part we hope to diversify.
- A good model explains most of the risk with a few factors, leaving a small residual.

11 Why hedging with these models is hard

- Betas are estimated with error and drift
- Factors can be correlated with each other
- Macro factors are not tradable: proxies leave basis risk
- A hedge instrument loads on several factors
- A hedge is only as good as its betas

IN THE LESSON

- The model is meant to explain risk, and so it should help us hedge it.
- In practice hedging is difficult for several reasons.
- First, the betas are estimated with error, and they drift over time, so a hedge sized on last year's beta may be wrong today.
- Second, the factors may be correlated with each other, which makes the separate betas hard to pin down.
- Third, macroeconomic factors such as growth or inflation cannot be bought or sold directly.
- We must use a proxy, and the gap between the proxy and the real factor leaves basis risk.
- Fourth, a real hedging instrument usually loads on several factors at once.
- The takeaway: a hedge is only as good as the beta estimates and the instruments behind it.

12 Example 2: a beta from correlation and volatility

Beta = correlation × asset volatility ÷ factor volatility

Correlation with market		**0.8**
Asset vol ÷ market vol	30% ÷ 20%	**1.5**
Beta	0.8 × 30 ÷ 20	**1.2**

IN THE LESSON

- Now to the estimating of returns, starting with the single factor case.
- First we need a beta.
- For one factor, beta equals the correlation between the asset and the factor, times the asset's volatility, divided by the factor's volatility.
- Take an illustration.
- A share has a correlation of point eight with the market.
- Its volatility is thirty per cent a year, while the market's volatility is twenty per cent.
- Multiply point eight by thirty, and you get twenty four.
- Divide by twenty, and beta is one point two.
- Notice what this says.
- A higher correlation raises beta, and so does a more volatile asset, while a more volatile market lowers it, because the market is the yardstick.

13 Example 3: expected return, single factor

Rf = 3%, market risk premium = 6%, beta = 1.2

Reward for market risk	1.2 × 6%	**7.2%**
Add the risk-free rate	3% + 7.2%	**10.2%**
Expected return	single factor	**10.2%**

IN THE LESSON

- Now use that beta in the single factor model.
- Expected return equals the risk free rate, plus beta times the market risk premium.
- Take a risk free rate of three per cent.
- The market risk premium is six per cent.
- Be careful here: that is the market return minus the risk free rate, and not the market return itself.
- Beta is one point two.
- Multiply one point two by six, and you get seven point two.
- Add the risk free rate of three, and the expected return is ten point two per cent.
- Two slips to avoid.
- Forgetting to add the risk free rate would give seven point two.
- And applying beta to the whole market return, say nine per cent, would give a wrong answer of thirteen point eight.

14 The multifactor expected return

$E(R) = Rf + \beta_1 \cdot premium_1 + \beta_2 \cdot premium_2 + \ldots + \beta k \cdot premiumk$

Risk-free rate, Rf	Start from the risk-free rate
Factor beta	One beta per factor
Factor risk premium	Matching premium for that factor
Expected return	Sum of all the terms

IN THE LESSON

- For several factors the idea is the same, repeated.
- Expected return equals the risk free rate, plus a term for each factor: that factor's beta times that factor's risk premium.
- What is a factor risk premium?
- It is the expected return of a portfolio that has a beta of one on that factor and zero on all the others, minus the risk free rate.
- In other words, the price the market pays for one clean unit of that single risk.
- Because every factor has its own premium, the beta must be matched with the right one.
- Using the growth beta with the inflation premium is one of the most common errors, and we will see it again in the mistakes slide.

15 Signs, and what a premium really is

- Premium = return of a pure one-unit portfolio, minus Rf
- Positive beta × positive premium: adds to the return
- Negative beta × positive premium: lowers the return
- With surprises: predicted = expected + Σ beta × surprise

IN THE LESSON

- A few points before the big example.
- A factor premium comes from a clean portfolio: one unit of exposure to that factor and none to the others, earning its return above the risk free rate.
- Now the signs.
- A positive beta on a positive premium adds to the expected return.
- But a negative beta on a positive premium lowers it, because the asset moves against that risk and so acts as a partial hedge, and investors accept less return for it.
- A negative beta on a negative premium would add.
- Finally, when the factors enter as surprises, the predicted return is the expected return, plus each beta times the surprise in its factor.
- We will do both.

16 Example 4: expected return, three factors

Rf 3%. Premia: market 5%, growth 2%, inflation 1%

Market: 1.2 × 5%		**+6.0%**
Growth: 0.5 × 2%		**+1.0%**
Inflation: −0.8 × 1%	negative beta, so a drag	**−0.8%**
Risk-free rate		**+3.0%**
Expected return	3 + 6 + 1 − 0.8	**9.2%**

IN THE LESSON

- Here is a three factor illustration.
- The risk free rate is three per cent.
- Factor one is the market, with a premium of five per cent.
- Factor two is growth in the economy, with a premium of two per cent.
- Factor three is inflation, with a premium of one per cent.
- Our asset has betas of one point two to the market, point five to growth, and minus point eight to inflation.
- The market term is one point two times five, which is six.
- The growth term is point five times two, which is one.
- The inflation term is minus point eight times one, which is minus point eight, a drag on the return.
- Add the risk free rate of three: three plus six plus one minus point eight gives nine point two per cent.

17 Example 5: adding the surprises

Expected return 9.2%. Surprises: market −2, growth +1, inflation +0.5

Market: 1.2 × (−2)		**−2.4%**
Growth and inflation	0.5×1 and −0.8×0.5	**+0.1%**
Total from surprises	−2.4 + 0.1	**−2.3%**
Predicted return	9.2 − 2.3 (before residual)	**6.9%**

IN THE LESSON

- Now suppose the year arrives, and the factors surprise us.
- The market comes in two per cent below expectation, a surprise of minus two.
- Growth is one per cent better than expected, a surprise of plus one.
- Inflation is half a per cent higher, a surprise of plus point five.
- Multiply each by the beta from before.
- The market, one point two times minus two, is minus two point four.
- Growth, point five times one, is plus point five.
- Inflation, minus point eight times point five, is minus point four.
- The surprises together move the return by minus two point three.
- The predicted return is the expected nine point two, less two point three, which is six point nine per cent, before the residual.

18 Hedging several factors at once

- Goal: net beta of zero to every factor
- Pure factor portfolios: position = −beta × portfolio value
- Impure instruments: solve k equations for k positions
- Need at least k instruments with independent exposures

IN THE LESSON

- Now suppose we hold a portfolio and want to remove its exposure to the factors.
- The aim is simple to state: build a combined position whose net beta to every factor is zero.
- If we have pure factor portfolios, each carrying exposure to just one factor, the job is easy.
- For each factor, hold a position equal to minus the portfolio's beta to that factor, times the portfolio's value.
- But real instruments are impure.
- Each loads on several factors.
- Then you must solve a set of simultaneous equations, one for each factor, so that the total exposure from the portfolio and every hedge position is zero.
- For k factors you need at least k instruments, and their exposures must be linearly independent, meaning none is just a blend of the others.

19 Example 6: hedging with pure factor portfolios

Portfolio $100m. Betas: market 1.2, growth 0.7. Pure hedges

Market exposure	1.2 × 100	**$120m**
Growth exposure	0.7 × 100	**$70m**
Hedge positions	−120 market, −70 growth	**sell $190m**
Net exposure left	120−120 and 70−70	**0 and 0**

IN THE LESSON

- Take a portfolio worth one hundred million dollars, an illustration.
- Its beta to the market factor is one point two, and its beta to a growth factor is point seven.
- Imagine we can trade a pure market portfolio and a pure growth portfolio.
- For the market, the position is minus one point two times one hundred, minus one hundred twenty million: sell that much.
- For growth, minus point seven times one hundred, minus seventy million.
- After these two sales, the market exposure is one hundred twenty less one hundred twenty, which is zero, and the growth exposure is seventy less seventy, also zero.
- The portfolio is now immune to surprises in both factors, whatever the residual does.

20 Example 7: impure hedges, two equations

Instrument 1 betas (1.0, 0.2); instrument 2 betas (0.4, 1.0). $m

Market: 120 + x1 + 0.4·x2 = 0		**solve both**
Growth: 70 + 0.2·x1 + x2 = 0		**together**
Positions	x1 = −100, x2 = −50	**sell 100, 50**
Check market, growth	120−100−20, 70−20−50	**0 and 0**

IN THE LESSON

- Now the realistic case.
- We have the same portfolio, with exposures of one hundred twenty to the market and seventy to growth, in millions.
- But our two instruments are impure.
- Instrument one has a market beta of one and a growth beta of point two.
- Instrument two has a market beta of point four and a growth beta of one.
- We want positions x one and x two that cancel both exposures.
- The market equation is one hundred twenty plus x one plus point four x two equals zero.
- The growth equation is seventy plus point two x one plus x two equals zero.
- The solution is x one equal to minus one hundred, and x two equal to minus fifty.
- Check: market, one twenty minus one hundred minus twenty, is zero.
- Growth, seventy minus twenty minus fifty, is zero.

21 Example 8: a single-factor hedge, and a trap

$m. Trap: instr. 1 betas (mkt 1.0, growth 0.2); instr. 2 (0.4, 1.0)

Notional, instrument beta 0.8	1.2 × 100 ÷ 0.8	**$150m**
Sell 120 of instrument 1	market now 0	**growth 46**
Then sell 46 of instrument 2	0.4 × 46 added to market	**market −18.4**
Lesson	solve all together	**no one-by-one**

IN THE LESSON

- With only one factor, the hedge is sized very simply.
- The notional equals the portfolio's beta, times its value, divided by the instrument's beta.
- Our portfolio has a market beta of one point two and is worth one hundred million.
- Suppose the instrument has a beta of point eight.
- The notional is one point two times one hundred, divided by point eight: one hundred fifty million, sold short.
- Now the trap, hedging factors one at a time, using the two instruments of Example seven.
- Sell one hundred twenty million of instrument one to fix the market.
- That also reduces growth exposure by twenty four, leaving forty six.
- But selling forty six of instrument two to fix growth now changes the market exposure again, by eighteen point four.
- The cross exposures were ignored.

22 After the hedge: what is left?

- Zero betas, no idiosyncratic risk: only Rf
- Extra return is alpha: arbitrage or error
- Mistake: hedging one factor at a time
- Mistake: wrong sign on a negative beta
- Mistake: betas taken as certain

IN THE LESSON

- What should a perfectly hedged portfolio earn?
- If it has zero beta to every factor, and its idiosyncratic risk is diversified, it has no systematic risk left.
- Under the A P T it should then earn just the risk free rate.
- If the hedged portfolio still earns more than the risk free rate, the extra is an alpha, and under the A P T that signals an arbitrage, or else a mistake in the betas.
- In practice, because of estimation error and basis risk, some risk always remains.
- The three classic mistakes in hedging are these.
- Hedging one factor at a time and ignoring cross exposures.
- Getting the sign wrong on a negative beta, which doubles the exposure instead of cancelling it.
- And treating estimated betas as if they were certain.

23 The Fama-French three-factor model

- Excess = a + bm(Rm − Rf) + bs·SMB + bh·HML + e
- SMB = small-cap return minus large-cap return
- HML = high book-to-market (value) minus low (growth)
- Positive HML loading = value behaviour; negative = growth

IN THE LESSON

- The best known multifactor model in practice comes from Fama and French.
- It explains an asset's excess return, meaning its return above the risk free rate, with three factors.
- The first is the market's excess return.
- The second is S M B, small minus big: the return on small company stocks minus the return on large company stocks.
- The third is H M L, high minus low: the return on stocks with a high book to market ratio, called value stocks, minus those with a low ratio, called growth stocks.
- The equation is excess return equals a constant, plus a market loading times the market excess return, plus a size loading times S M B, plus a value loading times H M L, plus a residual.
- A positive value loading means the asset behaves like a value stock, and a negative one like a growth stock.

24 Example 9: building SMB and HML

Six portfolios (2 sizes × 3 value groups). Illustrative returns, %

Small: value, neutral, growth	12, 10, 8	**avg 10**
Big: value, neutral, growth	9, 7, 5	**avg 7**
SMB	10 − 7	**3**
HML	(12+9)/2 − (8+5)/2	**4**

IN THE LESSON

- How are the two new factors made?
- Both are zero cost, long short portfolios.
- Stocks are sorted two ways: into small and big by size, and into high, neutral and low by book to market.
- That makes six portfolios.
- Here are illustrative returns, in per cent.
- Small value twelve, small neutral ten, small growth eight.
- Big value nine, big neutral seven, big growth five.
- S M B averages the three small portfolios and subtracts the average of the three big ones: ten minus seven is three.
- H M L averages the two high portfolios, small value and big value, giving ten point five, and subtracts the two low ones, giving six point five.
- The result is four.

25 Expected return from the three factors

E(R) = Rf + bm·(market premium) + bs·(SMB premium) + bh·(HML premium)

Risk-free rate, Rf	Add back the risk-free rate
Market loading	Beta to the market excess return
Size loading	Loading on SMB, size
Value loading	Loading on HML, value

IN THE LESSON

- The expected return follows the same recipe as before.
- It is the risk free rate, plus each loading times its premium.
- The market loading multiplies the market risk premium.
- The size loading multiplies the average return on S M B, the size premium.
- The value loading multiplies the average return on H M L, the value premium.
- Note the premiums for the two new factors are simply the average returns of those long short portfolios, since they cost nothing to hold.
- Once you have that expected return, you can judge a fund.
- Its alpha is its actual average return less the figure the model predicts.
- A positive alpha means the manager beat what the three exposures alone would explain.

26 Example 10: a fund's expected return and alpha

Rf 2%. Premia: market 6%, SMB 2%, HML 3%. Loadings 1.1, 0.4, −0.3

Market 1.1×6, size 0.4×2	6.6 and 0.8	**+7.4%**
Value: −0.3 × 3	growth lean	**−0.9%**
Expected: 2 + 7.4 − 0.9		**8.5%**
Alpha: actual 9.7% − 8.5%		**+1.2%**

IN THE LESSON

- Let's apply it, with illustrative numbers.
- The risk free rate is two per cent.
- The market premium is six per cent, the size premium is two, and the value premium is three.
- A fund has a market loading of one point one, a size loading of point four, and a value loading of minus point three.
- The negative sign means it leans towards growth.
- The market term is one point one times six, which is six point six.
- The size term is point four times two, point eight.
- The value term is minus point three times three, minus point nine.
- The expected return is two plus six point six plus point eight minus point nine, which is eight point five.
- If the fund actually averaged nine point seven, its alpha is one point two.

27 Four mistakes to avoid

- 1. Forgetting the risk-free rate
- 2. Beta × full market return, instead of the premium
- 3. Wrong factor's premium, or the wrong sign on a beta
- 4. Hedging one factor at a time, ignoring cross-exposure

IN THE LESSON

- Here are four mistakes to avoid.
- One: forgetting the risk free rate.
- The premiums are extras, so the expected return always starts from the risk free rate, and the same goes for the Fama and French alpha.
- Two: applying a beta to the full market return instead of the premium, the market return minus the risk free rate.
- Three: using a beta with the wrong factor's premium, or ignoring the sign of a negative beta, which turns a drag into a boost.
- Four: hedging factors one at a time, ignoring that each instrument loads on several.
- And behind them all, a reminder: only systematic factor risk earns a premium, never idiosyncratic risk.

28 Recap: the main ideas

- CAPM = one factor; APT = many factors, no arbitrage
- E(R) = Rf + Σ beta × its own factor premium
- Hedge: net beta of zero on every factor, solved together
- FF: market, SMB, HML; alpha = actual − expected

IN THE LESSON

- Let's pull it together.
- The C A P M uses one factor, the market, and rests on mean variance investors.
- The A P T assumes a linear factor model, diversification and no arbitrage, and does not name the factors.
- The C A P M is its single factor special case, and only systematic risk earns a premium.
- A multifactor model writes return as a constant, betas times factors, and a residual, but hedging with it is hard because betas are noisy and instruments are impure.
- Expected return is the risk free rate plus each beta times its own premium.
- To hedge, make every net beta zero by solving the equations together.
- And Fama and French add size and value to the market, with alpha as the gap from the prediction.

29 Try three quick questions

1. Rf 3%. Beta 1 on factor A (premium 5%), beta 0.5 on factor B (premium 2%). Expected return?
   Answer: 3 + 5 + 1 = 9%.
2. Portfolio $200m, factor beta 0.9. Hedge instrument beta 1.5. Amount to sell?
   Answer: 0.9 × 200 ÷ 1.5 = $120m.
3. Rf 2%, loadings 1, 0.5, −0.5; premia 6%, 2%, 3%. Expected return?
   Answer: 2 + 6 + 1 − 1.5 = 7.5%.

IN THE LESSON

- Now try three quick questions yourself, and work each one before you hear the answer.
- Question one.
- The risk free rate is three per cent.
- A stock has a beta of one to a factor with a premium of five per cent, and a beta of point five to a second factor with a premium of two per cent.
- Expected return?
- Three plus five plus one, which is nine per cent.
- Question two.
- A portfolio worth two hundred million has a beta of point nine to a factor.
- The hedge instrument has a beta of one point five.
- How much is sold?
- Point nine times two hundred over one point five, which is one hundred twenty million.
- Question three.
- In the three factor model, the risk free rate is two, with loadings of one, point five and minus point five, and premiums of six, two and three.
- Expected return?
- Two plus six plus one minus one point five, which is seven point five per cent.