Binomial Trees: Pricing Options Step by Step

FRM Part I

1 The story: price it from the end

- An option's value depends on an unknown future price
- Chop time into steps; each step: up or down
- At the far end the payoff is easy to read
- Walk backward to today: that is the price

IN THE LESSON

- Imagine you are offered a ticket that pays you money only if a share price ends above a set level in a year.
- What is the ticket worth today?
- Nobody knows where the share will end up, so it feels impossible.
- The binomial tree makes it manageable with one simple trick.
- Chop the year into small steps, and let the share do only two things in each step: move up, or move down.
- Now the future is a map of branches.
- At the far end of the map the ticket's value is obvious, and then we walk backward, one step at a time, until we reach today.

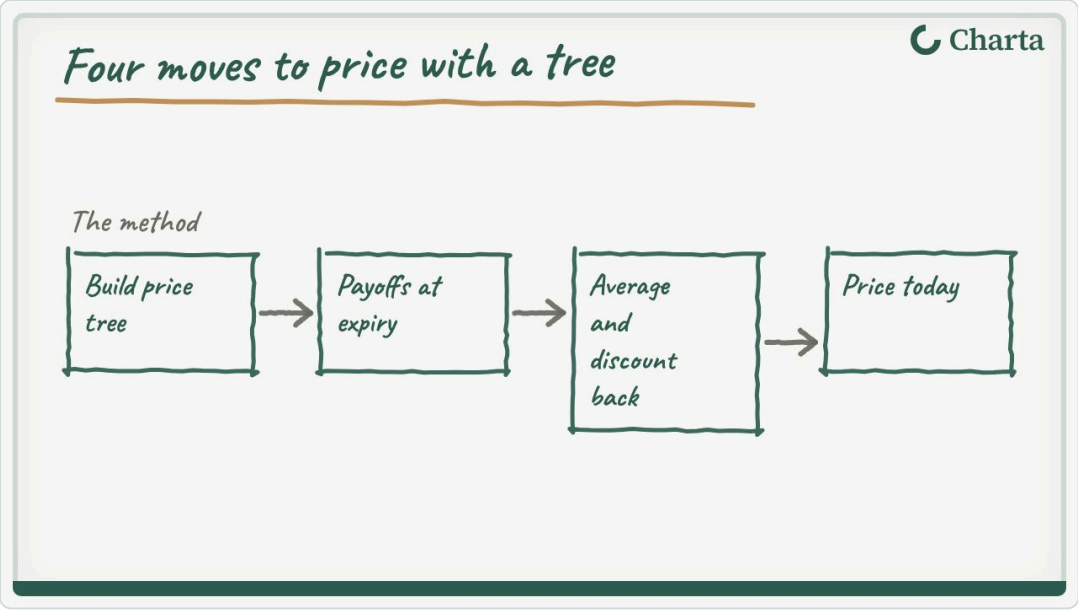
2 Calls, puts, European and American

- Call: right to buy at the strike. Put: right to sell
- Call payoff at expiry = max(S − K, 0)
- Put payoff at expiry = max(K − S, 0)
- European: only at expiry. American: any time

IN THE LESSON

- Four words matter before we start.
- A call option gives its holder the right to buy a share at a fixed price, called the strike.
- A put gives the right to sell at the strike.
- At expiry, a call is worth the share price minus the strike, or zero if that is negative, because nobody uses a right that costs them money.
- A put is worth the strike minus the share price, or zero.
- A European option can be used only at expiry.
- An American option can be used at any time up to expiry, which is extra freedom, and extra freedom can only add value.

3 Four moves to price with a tree



IN THE LESSON

- Here is the whole method in four moves, and we will repeat it in every example.
- First, build the tree of share prices, so that each step multiplies the price by an up factor or a down factor.
- Second, at expiry, write each end node's payoff.
- Third, step backward: each earlier node is worth a weighted average of the two nodes it leads to, shrunk by one period of discounting.
- Fourth, repeat until you reach the first node, which is the option's price today.
- For an American option, one extra rule applies at every node: compare waiting with using the option now, and keep the larger.

4 The risk-neutral probability

p = (e^(r × Δt) − d) ÷ (u − d)
d < e^(r × Δt) < u, so 0 < p < 1

Up probability, p	Weight on the up branch
Risk-free rate, r	Continuously compounded rate
Step length, dt	Length of one step in years
Moves u, d	Share times u or d

IN THE LESSON

- Here is the key formula, and it hides a clever idea.
- We need weights for the up and down branches.
- These are not the real chances of the share rising or falling.
- They are special weights, called risk-neutral probabilities, built so that the share itself grows at exactly the risk-free rate on average.
- The up weight is p, equal to e to the power of r times delta t, minus d, all divided by u minus d.
- Here r is the continuously compounded risk-free rate and delta t is the step length in years.
- If the share could ever beat or lag the safe rate on both branches, a free lunch would exist, so p must sit strictly between zero and one.

5 Example 1: a one-step call and put

Share 100, up to 110 or down to 90, r 5%, strike 95, one step of one year

Probability p	(e^0.05 − 0.9) ÷ 0.2	**0.7564**
Payoffs: up, down	110 − 95 and 0	**15 and 0**
Discount factor	e^−0.05	**0.9512**
Call value	0.9512 × 0.7564 × 15	**10.79**
Put value	0.9512 × 0.2436 × 5	**1.16**

IN THE LESSON

- Let us price both options on one tree.
- The share is one hundred, and in one year it becomes one hundred and ten or ninety, so u is one point one and d is nought point nine.
- The risk-free rate is five per cent and the strike is ninety-five.
- First, p is e to the power of nought point nought five, minus nought point nine, over nought point two, which is about nought point seven five six four.
- The call pays fifteen up and zero down.
- Discount by nought point nine five one two, and the call is worth about ten point seven nine.
- The put pays zero up and five down, so it is worth about one point one six.

6 Working back one step

V = e^(−r × Δt) × [p × V_up + (1 − p) × V_down]

Option value, V	Value of the node
Discount factor	One step of discounting
Probability, p	Weights p and one minus p
Next-step values	Values of the two nodes ahead

IN THE LESSON

- Now the rule for stepping backward, which we use at every node.
- The value of a node is the discount factor, e to the power of minus r times delta t, multiplied by the weighted average of the two values ahead.
- The weight p goes on the up value and one minus p on the down value.
- Notice the two ingredients.
- The weighted average replaces our ignorance about the future, and the discount factor pays for time.
- Because we repeat the rule at every step, a tree with two steps means two rounds of discounting.
- Forgetting a round of discounting is one of the commonest slips.

7 Example 2: a two-step European put

Share 100, u 1.1, d 0.9, r 6%, put strike 105, two steps of half a year

Payoffs at 121, 99, 81	put pays 0, 6, 24	**0, 6, 24**
Probability p	(e^0.03 − 0.9) ÷ 0.2	**0.6523**
Up node (110)	0.9704 × 0.3477 × 6	**2.02**
Down node (90)	0.9704×(0.6523×6+0.3477×24)	**11.90**
Put today	0.9704×(0.6523×2.02+0.3477×11.9)	**5.30**

IN THE LESSON

- Now two steps.
- The share starts at one hundred and each step is half a year, with u of one point one and d of nought point nine.
- The rate is six per cent and the put has strike one hundred and five.
- After two steps the share is one hundred and twenty-one, ninety-nine or eighty-one, and the put pays zero, six or twenty-four.
- Here p is about nought point six five two three, and one step of discounting is about nought point nine seven zero four.
- The up node is worth about two point nought two, the down node about eleven point nine, and stepping back once more gives a put today of about five point three.

8 American options: check every node

- Continuation: discounted average of the next nodes
- Exercise: the payoff if used right now
- Keep the larger, at every node, even the first
- Carry the larger value backward

IN THE LESSON

- An American option adds one question at every node: should I use the option right now?
- To answer it, compute two numbers.
- The continuation value is what the tree says the option is worth if you keep it, which is the discounted average we already know.
- The exercise value is the payoff you would get from using it immediately.
- Keep the larger of the two, and carry that larger number backward.
- This applies at every node, including the very first one, though at the first node it rarely matters.
- Early use tends to pay for puts, when the share has fallen a long way and the cash you receive can earn interest.

9 Example 3: the same put, American

Same tree and put. At each node: continue, or use it now?

Down node, continue	0.9704×(0.6523×6+0.3477×24)	**11.90**
Down node, use now	105 − 90	**15.00**
Up node: continue wins	continue 2.02, use now 0	**2.02**
Today, continue	0.9704×(0.6523×2.02+0.3477×15)	**6.34**
American versus European	6.343 against 5.296	**+1.05**

IN THE LESSON

- Take the same tree, but let the holder use the put early.
- At the up node of one hundred and ten, continuing is worth two point nought two and using it now pays nothing, so we keep two point nought two.
- At the down node of ninety, continuing is worth only eleven point nine, but using the put now pays one hundred and five minus ninety, which is fifteen.
- Fifteen is larger, so early use wins and the node is worth fifteen.
- Stepping back, the discounted average is about six point three four, which beats the immediate payoff of five.
- The American put is worth about six point three four, against five point three for the European.

10 American against European

Same underlying, strike and expiry; the American is never below the European

Option	American versus European
Put	Can be worth more
Call, no dividends	Equal (rates not negative)
Call, with dividends	Can be worth more
Any American	Never worth less

IN THE LESSON

- Let us collect what the examples taught us about American options.
- The American value can never be below the European value, because it contains every right the European has and more.
- For a put, the extra is often real, as we just saw: six point three four against five point three.
- For a call on a share that pays no dividends, however, there is nothing to gain from using early.
- Waiting keeps your money working at the interest rate and keeps the option's insurance alive, so the American call equals the European call, provided interest rates are not negative.
- Dividends and other cash flows can change that, which we return to later.

11 Slips to avoid in a tree

- Do not set p to one half; use the formula
- Discount once for every step back
- American: test early use at every node
- p is a pricing weight, not a forecast

IN THE LESSON

- Before moving on, three slips are worth naming because they cost marks and money.
- The first is using one half for p.
- It is tempting, because there are two branches, but p comes from the no-arbitrage formula and is usually not one half.
- The second is skipping a discount, treating a two-step tree as if one discount covered everything.
- The third is forgetting the early use test in an American problem, or doing it only at the end and not at every node.
- Also remember that p is not a forecast.
- It is a pricing weight that makes the tree consistent with the risk-free rate.

12 Volatility sets the size of the moves

- Volatility = typical yearly wobble of the share
- More volatility: bigger u, smaller d, wider tree
- Wider tree means a more valuable option
- Cox, Ross and Rubinstein: sigma fixes u and d

IN THE LESSON

- So far we handed you u and d.
- Where do they come from?
- They come from volatility, the typical size of the share's yearly wobble.
- The model builds volatility straight into the tree.
- A more volatile share moves further in each step, so its up factor is bigger and its down factor is smaller, and the tree fans out wider.
- A calm share has branches that stay close together.
- Because the price of an option depends on how far the share might travel, wide branches mean a more valuable option.
- In the standard set-up, called Cox, Ross and Rubinstein, one single number, volatility, fixes both u and d.

13 Up and down factors from volatility

u = e^(σ × √Δt) d = 1 ÷ u
Implied volatility: σ = ln(u) ÷ √Δt

Volatility, sigma	Annual volatility
Step length, dt	Step length in years
Up factor, u	Up multiplier
Down factor, d	Down multiplier

IN THE LESSON

- Here are the factors.
- The up factor u is e to the power of sigma times the square root of dt, where sigma is annual volatility and dt is the step length in years.
- The down factor d is simply one divided by u.
- The square root is there because variance, not volatility, grows in proportion to time: a share that wobbles by twenty per cent over a year wobbles by only half that over a quarter, not a quarter of that.
- A famous mistake is to scale by dt instead of its square root.
- And if a question hands you u and dt, you can run the formula backward to find the volatility.

14 Example 4: building the tree from volatility

Volatility 20%, step 0.25 year, r 4%

Up factor u	e^(0.2 × √0.25) = e^0.1	**1.1052**
Down factor d	1 ÷ 1.1052	**0.9048**
Growth over a step	e^(0.04 × 0.25)	**1.0101**
Probability p	(1.0101 − d) ÷ (u − d)	**0.5252**

IN THE LESSON

- Let us build a tree from volatility.
- Take twenty per cent volatility, a step of one quarter of a year, and a risk-free rate of four per cent.
- The square root of nought point two five is nought point five, so sigma times the square root is nought point one.
- The up factor is e to the power of nought point one, which is about one point one zero five two.
- The down factor is one divided by that, about nought point nine zero four eight.
- The growth over a step is about one point nought one nought one, so p is about nought point five two five two.
- Notice that p is close to a half here because the step is short, but it is still not exactly a half, so always use the formula.

15 Why the tree recombines, and what is missing

- d = 1 ÷ u: up then down returns to the start
- Paths meet, so the tree recombines and stays small
- Real-world expected return plays no part in the price
- Sigma measures wobble, not drift

IN THE LESSON

- Two features of this construction are worth understanding.
- First, because d equals one over u, an up move followed by a down move multiplies the price by u times one over u, which is one.
- The price returns to where it started, so the up-down and down-up paths meet in the same node.
- The tree recombines, which keeps it small.
- Second, notice what is missing: the share's real-world expected return.
- It appears nowhere in p or in the price.
- The option can be copied using the share and the risk-free asset, so how fast the share is expected to grow does not matter.
- Confusing sigma with drift is a common error.

16 Example 5: reading volatility from a tree

Monthly steps (Δt = 1/12), up factor u = 1.0905

Natural log of u	ln 1.0905	**0.0866**
Square root of Δt	√(1/12)	**0.2887**
Implied volatility	0.0866 ÷ 0.2887	**30%**
Down factor	1 ÷ 1.0905	**0.9170**

IN THE LESSON

- Sometimes the tree is given and you are asked what volatility it assumes.
- Suppose the steps are one month long, so dt is one twelfth of a year, and the up factor is one point nought nine nought five.
- Take the natural logarithm of u, which is about nought point nought eight six six.
- The square root of one twelfth is about nought point two eight eight seven.
- Dividing the first by the second gives nought point three, so the tree assumes thirty per cent annual volatility.
- As a check, the down factor must be one divided by u, about nought point nine one seven.

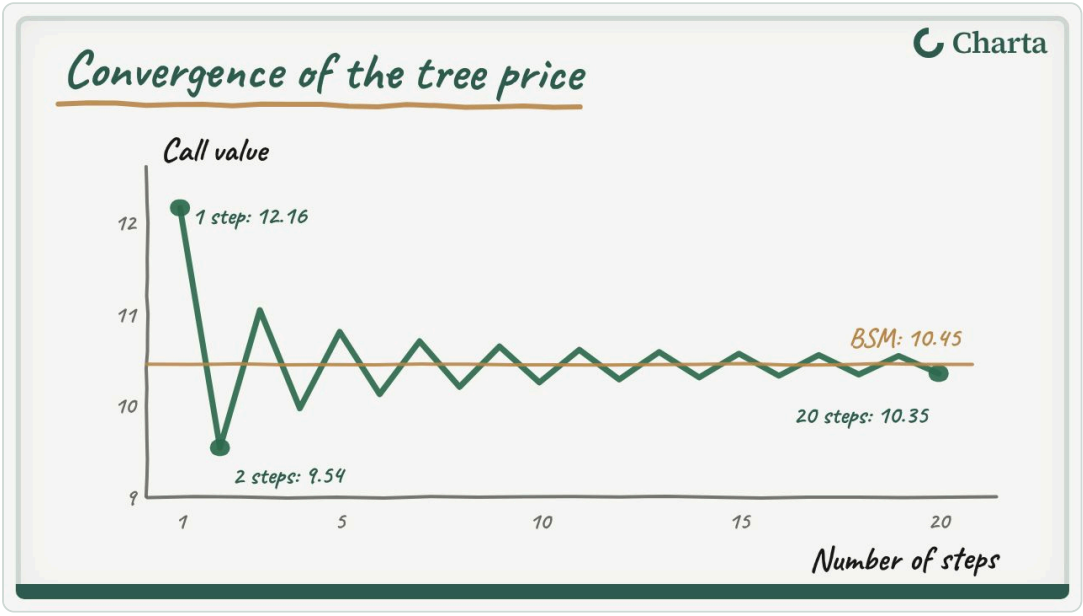
17 More steps, smaller steps

- Fixed life T, n steps: each step lasts T ÷ n
- u and d move closer to 1 as n rises
- Final prices approach a lognormal distribution
- European tree value approaches the B S M value

IN THE LESSON

- A tree with one or two steps is a very rough picture of a share.
- A real share moves almost continuously.
- So we split the option's life into more steps.
- For a fixed life of T years and n steps, each step lasts T divided by n, which shrinks as n grows.
- The factors u and d also move closer to one, since each little step moves the price only a little.
- With more steps the final prices spread across more and more values, and their shape approaches the smooth bell shape of a lognormal distribution.
- Because of that, the tree price of a European option approaches the famous Black, Scholes and Merton value.

18 Convergence of the tree price



IN THE LESSON

- Here is a real run.
- A call on a share at one hundred, strike one hundred, five per cent rate, twenty per cent volatility, one year.
- The flat line is the Black, Scholes and Merton value, about ten point four five.
- The wiggly line is the tree price as the number of steps grows from one to twenty.
- With one step the tree says about twelve point one six, with two steps only nine point five four.
- It jumps above and below the limit, but each swing is smaller than the last.
- This zigzag is not a failure: the error shrinks, even though the sequence is not smoothly one directional.

19 How the tree price settles

European call, S 100, K 100, r 5%, volatility 20%, one year

1 step, gap 1.71	**12.16**
2 steps, gap 0.91	9.54
10 steps, gap 0.20	10.25
100 steps, gap 0.02	10.43
500 steps, gap 0.00	10.45
Black-Scholes-Merton	10.45

IN THE LESSON

- The same run in numbers.
- One step gives twelve point one six, two steps give nine point five four, ten steps give ten point two five.
- At one hundred steps the tree gives about ten point four three, and at five hundred steps about ten point four five.
- The target is ten point four five.
- Notice how the answer is still on the wrong side of the target at times, yet the gap keeps shrinking: roughly one point seven, then nought point nine, then nought point two, then nought point nought two.
- A coarse tree is therefore a rough guide, not an answer, and a small number of steps should never be quoted as the price.

20 Example 6: how big does the tree get?

Nodes in a recombining tree with n steps: (n + 1) (n + 2) ÷ 2

n = 2	3 × 4 ÷ 2	**6**
n = 10	11 × 12 ÷ 2	**66**
n = 100	101 × 102 ÷ 2	**5,151**
Nodes at expiry, n = 100	n + 1	**101**

IN THE LESSON

- Why not use a huge number of steps straight away?
- Because the work grows quickly.
- A recombining tree with n steps has n plus one nodes at expiry, and altogether n plus one times n plus two, divided by two nodes.
- With two steps that is three times four over two, six nodes.
- With ten steps, eleven times twelve over two, which is sixty-six.
- With one hundred steps it is one hundred and one times one hundred and two over two, which is five thousand one hundred and fifty-one nodes.
- So the work grows roughly with n squared, which is why we choose enough steps to be accurate, but not more.

21 Using convergence wisely

- Zigzag is not divergence: the error shrinks
- A coarse tree value is not the answer
- American: add steps until the value stabilises
- Use put-call parity as a check

IN THE LESSON

- A few habits follow.
- Do not mistake the zigzag for divergence: a tree value that goes up, down, up, down is still closing in on the limit.
- Do not treat a coarse tree as the final answer.
- For a European option there is a closed form, so you can compare the tree with the Black, Scholes and Merton value.
- For an American option no such closed form exists, so you run the tree with more and more steps until the value stops changing in the digits you care about.
- Relationships that hold exactly in the limit, like put-call parity, make good sanity checks on your result.

22 Delta: the slope of the option

- Delta = change in option value per unit change in share
- Call delta: between 0 and 1
- Put delta: between −1 and 0
- A tree gives delta from two values and two prices

IN THE LESSON

- Next, delta.
- Suppose the share price rises by one pound.
- How much does the option price change?
- That rate of change is the delta: the slope of the option value plotted against the share price.
- A call gains when the share rises, so its delta is between zero and one.
- A put gains when the share falls, so its delta is between minus one and zero.
- Delta matters because it tells a dealer how many shares move in step with one option.
- And it also shows up directly inside a binomial tree, because each node of the tree has a pair of option values and a pair of share prices.

23 Delta in a binomial tree

Δ = (V_up − V_down) ÷ (S_up − S_down)
Put delta = call delta − 1 (no dividends)

Delta	Option change per share change
Option values	Option value, up minus down
Stock prices	Share price, up minus down

IN THE LESSON

- In a tree, delta at a node is the option value in the up state minus the value in the down state, divided by the share price in the up state minus the share price in the down state.
- Think of it as rise over run: the difference in option value over the difference in share price.
- Two common mistakes: dividing by the wrong spread, for example by the option values rather than the share prices, and getting the sign wrong.
- For a European call and put on a share with no dividends, the same strike and expiry, put delta equals call delta minus one.

24 Example 7: delta and the hedge

Share 100 to 110 or 90, strike 95, one step

Share price spread	110 − 90	**20**
Call delta	(15 − 0) ÷ 20	**0.75**
Put delta	(0 − 5) ÷ 20	**−0.25**
Check: call delta − 1	0.75 − 1	**−0.25**
Short 1,000 calls: buy	0.75 × 1,000	**750 shares**

IN THE LESSON

- Back to example one: share at one hundred, up to one hundred and ten, down to ninety, strike ninety-five.
- The share price spread is twenty.
- The call pays fifteen up and zero down, so its delta is fifteen divided by twenty, nought point seven five.
- The put pays zero up and five down, so its delta is zero minus five, over twenty, which is minus nought point two five.
- Check: call delta minus one is also minus nought point two five.
- Now the hedge.
- A dealer who sold one thousand calls buys seven hundred and fifty shares, so a small rise in the share is offset.
- This is called being delta neutral.

25 Example 8: delta changes from node to node

American put from example 3, delta at each node

Up node (110)	(0 − 6) ÷ (121 − 99)	**−0.273**
Down node (90)	(6 − 24) ÷ (99 − 81)	**−1.000**
Start (100)	(2.02 − 15) ÷ (110 − 90)	**−0.649**

IN THE LESSON

- In a tree with several steps, every node has its own delta.
- Use the American put from example three.
- At the up node, the share is a hundred and ten and the next shares are one hundred and twenty-one and ninety-nine.
- The put pays zero and six, so delta is minus six over twenty-two, about minus nought point two seven.
- At the down node, delta is six minus twenty-four, over ninety-nine minus eighty-one, which is exactly minus one.
- And at the start, delta uses the node values two point nought two and fifteen, over twenty: about minus nought point six five.
- Delta moves as the share moves, so the hedge must be rebalanced.

26 Hedging with delta, and its limits

- Short call, delta 0.6: buy 0.6 shares per option
- Long put, delta −0.4: buy 0.4 shares per option
- Watch the sign, and divide by the share spread
- Delta moves, so the hedge is rebalanced

IN THE LESSON

- Let us read delta as a hedging instruction.
- A dealer who is short a call with delta nought point six buys nought point six of a share per option, to be delta neutral.
- A holder of a put with delta minus nought point four also buys, nought point four of a share per option, because the put loses value as the share rises and the shares gain by the same amount.
- Sign is the main danger: a long put has negative delta, so you buy shares to offset it, you do not sell them.
- And because delta changes as the share moves and as time passes, the hedge is only good for a moment, and has to be rebalanced.

27 One tree, many underlyings

- Same tree, same backward steps
- Discounting is always at the risk-free rate r
- Only the growth factor a changes
- Need d < a < u so that 0 < p < 1

IN THE LESSON

- So far the share paid no dividends.
- But options are written on indices, currencies and futures too.
- The beauty of the tree is that almost nothing changes.
- The shape of the tree, the way we step backward and the discounting all stay the same.
- Discounting is always at the risk-free interest rate r.
- Only one thing differs: the growth factor, called a, which tells us how fast the underlying grows on average in the risk-neutral world.
- The up probability is built from a in place of the plain interest growth, and a must still sit between d and u, so that p stays between zero and one.

28 The general probability and growth factor

p = (a − d) ÷ (u − d)
a = e^((r − y) × Δt), y = q (stock or index) or rf (currency)
Futures: a = 1

Probability, p	Up probability
Growth factor, a	Growth factor of the underlying
Yield q or foreign rf	Dividend yield or foreign rate
Step length, dt	Step length in years

IN THE LESSON

- The general probability is p equals a minus d, over u minus d.
- For a share paying no dividends, a is e to the power of r times delta t, and we are back to the first formula.
- Now suppose the stock or index pays a continuous dividend yield q.
- Holders collect the yield, so the price itself grows more slowly, and the growth factor becomes e to the power of r minus q, times delta t.
- A currency behaves the same way, because the foreign interest rate acts like a yield: the growth factor uses domestic rate minus foreign rate.
- A futures price grows at no rate at all in this world, so a is one.

29 Which growth factor for which underlying

Only a changes; discounting stays at the risk-free rate r

Underlying	Growth factor a
Stock, no dividends	exp(r × Δt)
Stock or index, yield q	exp((r − q) × Δt)
Currency, foreign rate rf	exp((r − rf) × Δt)
Futures price	1

IN THE LESSON

- This table is worth memorising.
- A share with no dividends has a equal to e to the power of r delta t.
- A stock or index with dividend yield q has e to the power of r minus q, times delta t.
- A currency option has e to the power of domestic rate minus foreign rate, times delta t.
- A futures contract costs nothing to enter, so in the risk-neutral world it has no expected growth, and a is simply one.
- Two frequent errors: discounting at r minus q or at the foreign rate, when the discounting is always at the domestic risk-free rate, and using the plain share formula for a currency or a future.

30 Example 9: a call with a dividend yield

Call strike 95, share 100 to 110 or 90, r 5%, dividend yield 3%

Growth factor a	e^(0.05 − 0.03)	**1.0202**
Probability p	(1.0202 − 0.9) ÷ 0.2	**0.6010**
Call value	0.9512 × 0.6010 × 15	**8.58**
Without the dividend	example 1	**10.79**

IN THE LESSON

- Return to the call from example one, strike ninety-five, but now the share pays a continuous dividend yield of three per cent.
- The growth factor is e to the power of nought point nought five minus nought point nought three, that is e to the power of nought point nought two, which is about one point nought two nought two.
- The probability becomes one point nought two nought two minus nought point nine, over nought point two, about nought point six zero one.
- The up payoff is still fifteen.
- Discounting stays at five per cent, nought point nine five one two.
- The call is worth about eight point five eight, lower than ten point seven nine without the dividend, which makes sense because the dividends go to the share owner.

31 Example 10: a call on a futures price

Futures price 100, u 1.1, d 1 ÷ 1.1, r 5%, call strike 100, one year

Growth factor a	futures: no growth	**1**
Probability p	(1 − 0.9091) ÷ (1.1 − 0.9091)	**0.4762**
Payoffs: up, down	110 − 100 and 0	**10 and 0**
Call value	0.9512 × 0.4762 × 10	**4.53**

IN THE LESSON

- Finally, an option on a futures contract.
- The futures price is one hundred.
- Take u of one point one and d equal to one over u, about nought point nine zero nine one, with a rate of five per cent and one step of a year.
- A futures contract has no expected growth, so a is one.
- The probability is one minus nought point nine zero nine one, over one point one minus nought point nine zero nine one, about nought point four seven six two.
- A call with strike one hundred pays ten if the futures price goes up to one hundred and ten, and zero otherwise.
- Discounting at five per cent gives about four point five three.

32 Recap: the main ideas

- Work back: payoffs, then discounted averages with p
- American: larger of continue and use now, every node
- u = e^(σ√dt), d = 1 ÷ u; more steps, closer to B S M
- Delta = value spread ÷ share spread; changes by node
- Only a changes: e^((r − q)Δt), foreign rate, or 1

IN THE LESSON

- Let us pull it together.
- A binomial tree prices an option by working backward from expiry: payoffs at the end, then discounted risk-neutral averages, with p from the no-arbitrage formula and never simply one half.
- American options compare waiting with using at every node, and the American put can be worth more than the European.
- Volatility enters through u equal to e to the power of sigma root dt, with d equal to one over u.
- More steps bring the European value to Black, Scholes and Merton, with oscillation.
- Delta is rise over run.
- And for dividends, currencies and futures, only the growth factor a changes.

33 Try three quick questions

1. Up 1.2, down 0.9, r = 0, one step. Probability p?
   Answer: p = (1 − 0.9) ÷ (1.2 − 0.9) = 0.333. Not one half.
2. Call worth 8 up and 0 down; share 120 up, 80 down. Delta?
   Answer: Delta = (8 − 0) ÷ (120 − 80) = 0.2.
3. Currency: r 4%, foreign 1%, step 1 year. Growth factor a?
   Answer: a = e^(0.04 − 0.01) = e^0.03 = 1.0305.

IN THE LESSON

- Pause the video and think about each question before you listen.
- Question one.
- A one step tree has an up factor of one point two, a down factor of nought point nine and a rate of zero.
- What is the up probability?
- One minus nought point nine, over one point two minus nought point nine, which is nought point three three three, not one half.
- Question two.
- A call is worth eight up and zero down, and the share is one hundred and twenty up and eighty down.
- What is delta?
- Eight minus zero, over one hundred and twenty minus eighty, which is nought point two.
- Question three.
- For a currency option, the domestic rate is four per cent, the foreign rate is one per cent and the step is one year.
- What is the growth factor a?
- It is e to the power of nought point oh four minus nought point oh one, so e to the nought point oh three, which is one point oh three oh five.