CHARTA LEARNING · LESSON NOTES

Bond Yields and Return Calculations

FRM Part I

1 Realized return: what you actually earned

- Three sources: price change, coupons, reinvested coupons
- The base is the price paid, not the face value
- Known only afterwards, once reinvestment rates are seen
- Gross: the bond itself. Net: after financing costs

IN THE LESSON

- Start with the plainest question.
- You bought a bond, held it for a while, and then sold it.
- What did you earn?
- A bond pays you in three ways.
- First, the price may change between the day you buy and the day you sell.
- Second, you collect coupons, the regular interest payments.
- Third, each coupon can be put to work again, which is called reinvesting, and that earns a little more.
- Gross realized return counts all three, compared with the price you paid.
- It is only known afterwards, because the reinvestment rate depends on rates available when each coupon arrives.

2 Total and annualised realized return

Total = (end price + coupons + reinvestment − price paid) ÷ price paid
Annualised = (1 + total)^(1/years) − 1

Total return	All earned in the holding period
Price paid	The base: what you actually paid
Annualised return	Compounded, never averaged
Net of financing	Net: subtract borrowing cost

IN THE LESSON

- Here are the two formulas.
- Total realized return is the ending price, plus all coupons, plus the income from reinvesting them, minus the purchase price, all divided by the purchase price.
- To compare holdings of different lengths, we annualise by compounding: one plus the total, raised to the power of one over the number of years, minus one.
- Net realized return is built the same way, but it also subtracts the cost of holding the position, mainly the interest on money borrowed to buy the bond.
- A leveraged position therefore earns less net than gross.

3 Example 1: gross realized return

Face 1000, coupon 6% a year. Buy at 950, sell after 3 years at 980. Reinvest at 4%

Coupon 1, reinvested 2 years	60×1.04^2	**64.90**
Coupon 2, reinvested 1 year	60×1.04	**62.40**
Coupon 3, just received	60	**60.00**
Total realized return	(980 + 187.30 − 950) ÷ 950	**22.87%**

IN THE LESSON

- A bond with face value one thousand pays a six per cent coupon each year, sixty.
- You buy it at nine hundred fifty and sell it three years later at nine hundred eighty.
- You reinvest each coupon at four per cent.
- The first coupon is reinvested for two years and grows to sixty four point nine.
- The second grows to sixty two point four.
- The third is just received, so it stays sixty.
- Together that is one hundred eighty seven point three, of which seven point three is reinvestment income.
- Add the sale price, subtract the purchase price, divide by nine hundred fifty: twenty two point eight seven per cent.

4 Example 2: annualised and net

Same trade. 600 of the 950 was borrowed at 3% a year, interest paid in cash

Annualised gross	1.2287^(1/3) − 1	**7.11%**
Financing cost	600 × 3% × 3 years	**54**
Net total return	(217.30 − 54) ÷ 950	**17.19%**
Annualised net	1.1719^(1/3) − 1	**5.43%**

IN THE LESSON

- Now annualise it.
- One plus point two two eight seven is one point two two eight seven.
- Raise that to the power of one third, subtract one, and we get seven point one one per cent a year.
- Notice we compound, we do not divide twenty two point nine by three.
- Next, suppose six hundred of the nine hundred fifty was borrowed at three per cent a year, with the interest paid in cash.
- Three years of interest on six hundred is fifty four.
- Subtract it from the earnings: two hundred seventeen point three less fifty four, over nine hundred fifty, is seventeen point one nine per cent.
- Annualised, the net figure is five point four three.

5 Spread: the extra the bond pays over the curve

- Benchmark curve: one spot rate per maturity
- Spread: one constant added to every spot rate
- Chosen so the present value equals the market price
- Positive spread pays for credit and liquidity risk

IN THE LESSON

- A benchmark curve, such as government bonds, gives a spot rate for every maturity: the rate for money lent until that date.
- Suppose we discount a corporate bond's promised cash flows at those benchmark rates.
- We get a value higher than the market price, because buyers demand extra for credit risk, trading difficulty and other things the benchmark lacks.
- The spread is the single constant that, added to every spot rate, makes the discounted cash flows equal the market price.
- A bond priced exactly on the curve has a spread of zero, and a positive spread means the market pays less than the curve value.

6 The spread pricing equation

Price = Σ CFt ÷ (1 + rt + s)^t

Cash flow	Promised payment at time t
Benchmark spot rate	Benchmark spot rate for time t
Spread	The same spread on every date
Market price	What the market pays today

IN THE LESSON

- The equation reads as follows.
- The price equals the sum, over every payment date, of the cash flow on that date, divided by one plus the benchmark spot rate for that date plus the spread, raised to the power of the number of years.
- There is no neat algebra for the spread, so it is found numerically, by trying values until the price matches.
- The classic error is adding the spread to only one rate.
- It belongs on every spot rate in the sum.
- And do not confuse it with the simple yield spread, which just subtracts one benchmark yield from the bond's yield.

7 Example 3: finding a spread

2-year bond, coupon 5, face 100. Spot rates 3% and 4%. Market price 100

Value on the curve (s = 0)	$5 \div 1.03 + 105 \div 1.04^2$	**101.93**
Try s = 1.00%	$5 \div 1.04 + 105 \div 1.05^2$	**100.05**
Try s = 1.10%	$5 \div 1.041 + 105 \div 1.051^2$	**99.86**
Solved spread	between 1.00% and 1.10%	**about 1.02%**

IN THE LESSON

- Take a two year bond paying a coupon of five a year on a face of one hundred.
- The benchmark spot rates are three per cent for one year and four per cent for two.
- On the curve it is worth five over one point zero three, plus one hundred five over one point zero four squared, which is one hundred one point nine three.
- The market price is only one hundred.
- Try a spread of one per cent: the value is one hundred point zero five, still too high.
- Try one point one per cent: it falls just below one hundred.
- So the spread is between them, about one point zero two per cent.

8 Spread against the simple yield spread

Two ways to measure the extra return over a benchmark

Measure	What it does
Spread	One number added to every spot rate
Yield spread	Bond yield minus one benchmark yield
Zero-coupon bond	Spread = yield − spot rate
Spread of zero	Priced exactly on the curve
Positive spread	Paid for credit and liquidity risk

IN THE LESSON

- Let me set the ideas side by side.
- The spread uses the whole curve, adding one number to every spot rate.
- The yield spread is cruder: it takes the bond's yield to maturity and subtracts a single benchmark yield, ignoring the shape of the curve.
- The two agree in one special case, a zero coupon bond, which pays a single amount at the end.
- There the spread equals the bond's yield minus the spot rate for that maturity.
- Read the sign carefully.
- Positive means extra reward for extra risk, zero means priced on the curve, and negative would mean the bond is richer than the benchmark.

9 Yield to maturity: one rate for the whole bond

- One discount rate that reproduces the market price
- The bond's internal rate of return
- Price falls when yield rises, and the reverse
- Not guaranteed: needs hold, no default, reinvest at y

IN THE LESSON

- A bond has many cash flows, so how do we describe its return with one number?
- The answer is yield to maturity, written Y T M.
- It is the single discount rate that makes the present value of all the bond's promised payments equal to its market price.
- Think of it as the bond's internal rate of return.
- A high price means a low yield and a low price means a high yield, so price and yield move in opposite directions.
- Y T M is a promise only under three conditions: you hold to maturity, the issuer pays every cash flow, and each coupon is reinvested at the Y T M itself.

10 Pricing a bond from its yield

Price = Σ CFt ÷ (1 + y ÷ m)^(m × t)

Cash flow	Payment at time t
Yield, y	Quoted yearly yield
Payments a year, m	Two for semi-annual bonds
Market price	One rate for every payment

IN THE LESSON

- With m payments a year, the price is the sum of each cash flow divided by one plus the yield over m, raised to the power of m times the number of years.
- In plain terms, discount every coupon and the face value at the same rate.
- For a bond paying twice a year, m is two.
- The yield is quoted as twice the half-year rate, which is called the bond equivalent basis.
- The classic mistake is forgetting this.
- For a semi-annual bond, halve the yield and the annual coupon, and double the number of years to get periods.
- If the coupon equals the yield, the bond sells at par.

11 Example 4: price from a yield

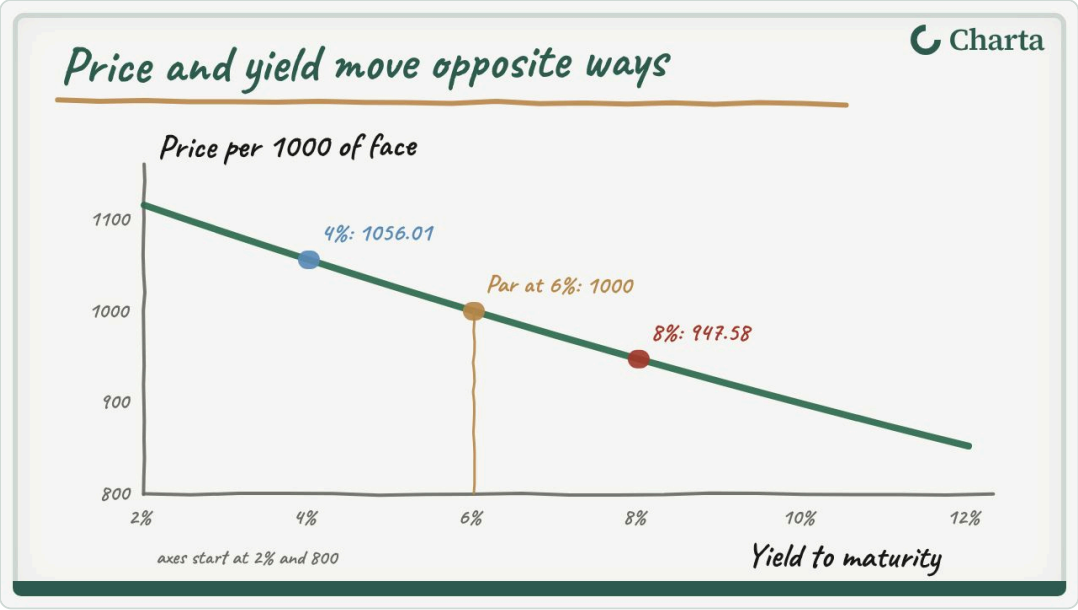
Face 1000, coupon 6% paid twice a year, 3 years. Yield to maturity 8%

Per period	coupon 30, 6 periods, rate 4%	
Value of the coupons	$30 \times (1 - 1.04^{-6}) \div 0.04$	**157.26**
Value of the face	$1000 \div 1.04^{6}$	**790.31**
Price	157.26 + 790.31	**947.58**

IN THE LESSON

- Price a three year bond with face value one thousand and a six per cent coupon paid twice a year, when the yield to maturity is eight per cent.
- First convert to periods.
- Each coupon is thirty, there are six half-year periods, and the rate per period is four per cent.
- The six coupons are worth one hundred fifty seven point two six today.
- The face value, discounted six periods at four per cent, is worth seven hundred ninety point three one.
- Add them: nine hundred forty seven point five eight.
- The price is below one thousand because the coupon rate of six is below the yield of eight.

12 Price and yield move opposite ways



IN THE LESSON

- Here is the same bond at different yields.
- Along the bottom is the yield to maturity, and up the side is the price.
- The line slopes downward: as yield rises, price falls.
- At a yield of six per cent, equal to the coupon, the price is exactly one thousand, which is par.
- Move left to four per cent and the bond is worth one thousand fifty six, a premium.
- Move right to eight per cent and it is worth nine hundred forty eight, a discount.
- The curve bends, steeper on the left than on the right, so a fall in yield adds more than a rise of the same size takes away.

13 How a yield to maturity is found

- No algebraic formula for a bond with several coupons
- Guess, price, adjust. Or interpolate between two trials
- Calculator: N, PMT, PV (negative), FV, then I/Y
- Semi-annual: the answer is per period, so double it

IN THE LESSON

- Pricing from a yield is easy.
- The reverse, finding the yield from a price, is harder.
- For a bond with several coupons there is no algebraic formula, because the equation is a polynomial whose degree is the number of periods.
- So in practice we iterate.
- Guess a yield, compute the price, and adjust until it matches the market price, or interpolate between two trial yields.
- A financial calculator does this for you.
- Enter the number of periods, the coupon per period, the price as a negative number, and the face value, then ask for the interest rate.
- Typical mistakes are mixing annual and semi-annual inputs, and giving price and face value the same sign.

14 Example 5: solving by interpolation

3-year bond, coupon 5, face 100, market price 96. Find the yield

Try y = 6%	price	**97.33**
Try y = 7%	price	**94.75**
Share of the gap	(97.33 − 96) ÷ (97.33 − 94.75)	**0.52**
Interpolated yield	6% + 0.52 × 1%	**6.52%**

IN THE LESSON

- A three year bond pays a coupon of five a year on a face of one hundred, and its price is ninety six.
- Guess six per cent.
- The price at six per cent is ninety seven point three three, too high, so the yield must be higher.
- Try seven per cent: ninety four point seven five, too low.
- The target lies between.
- The price must fall one point three three out of a total gap of two point five eight, about fifty two per cent of the way, so we add point five two to six.
- That gives six point five two per cent.
- The exact answer is six point five one, so interpolation is very close.

15 Example 6: a zero-coupon bond

5-year zero-coupon bond, face 100, price 78.35

Face ÷ price	100 ÷ 78.35	**1.2763**
Fifth root	1.2763^(1/5)	**1.0500**
Yield	1.0500 − 1	**5.00%**
Check: price	$100 \div 1.05^5$	**78.35**

IN THE LESSON

- A zero coupon bond is the exception, because it has only one cash flow and the yield has a formula.
- A five year zero with face value one hundred costs seventy eight point three five.
- Divide the face by the price to get one point two seven six three.
- Take the fifth root, which means raising to the power of one fifth, and subtract one.
- The yield is five per cent.
- We can check it by going forward: one hundred over one point zero five to the fifth power is seventy eight point three five.
- If the bond paid twice a year, the same method gives a rate per half-year, and you would double it for the quoted yield.

16 Annuities and perpetuities

- Annuity: equal payments, fixed number of periods
- Perpetuity: equal payments forever, finite value
- Due: paid at the start. Deferred: starts later
- Rate must match the payment frequency

IN THE LESSON

- Bond coupons are really an annuity: the same payment at regular intervals for a fixed number of periods.
- Valuing one by discounting each payment works, but there is a shortcut.
- An ordinary annuity pays at the end of each period, and a perpetuity pays forever.
- Strange as that sounds, its value is finite, because far-off payments are worth almost nothing today.
- An annuity due pays at the start of each period, so it is worth more, and a deferred annuity starts later.
- Always match the rate to the payment frequency, so a monthly payment needs a monthly rate, and never use the perpetuity formula for a stream that ends.

17 Annuity and perpetuity prices

Annuity = C × [1 − (1 + r)^(−n)] ÷ r
Perpetuity = C ÷ r Due = Annuity × (1 + r)

Payment, C	Same amount each period
Rate per period, r	Match the payment frequency
Number of payments, n	How many payments in all

IN THE LESSON

- The ordinary annuity price is the payment times one minus one plus r to the power of minus n, all over r.
- Here r is the rate per period and n the number of payments.
- The expression is just the sum of the n discounted payments, packaged.
- For a perpetuity, the discount term vanishes because n is infinite, and the price is simply the payment divided by r.
- An annuity due is the ordinary annuity times one plus r, since every payment comes one period earlier.
- A coupon bond, then, is an annuity of coupons plus the discounted face value.

18 Example 7: four kinds of stream

Payment 200, rate 6%. Annuities last 5 payments

Ordinary annuity	$200 \times (1 - 1.06^{-5}) \div 0.06$	**842.47**
Perpetuity	200 ÷ 0.06	**3333.33**
Annuity due	842.47 × 1.06	**893.02**
Deferred: first pay in year 4	$842.47 \div 1.06^{3}$	**707.36**

IN THE LESSON

- Payments of two hundred, with a rate of six per cent.
- As an ordinary annuity for five years, the price is two hundred times one minus one point zero six to the minus five, over point zero six: eight hundred forty two point four seven.
- As a perpetuity, it is two hundred over point zero six, three thousand three hundred thirty three point three three.
- As an annuity due, multiply the annuity by one point zero six: eight hundred ninety three point zero two.
- Finally, if the first payment arrives at the end of year four, value the annuity at year three, one period before the first payment, then discount three years: seven hundred seven point three six.

19 Spot rates and their link to YTM

- Spot rate: yield on a zero-coupon bond of that maturity
- Coupon bond = bundle of zero-coupon payments
- Price: each flow at its own spot rate
- YTM: roughly a weighted average of those spot rates
- Upward curve: YTM below final spot. Inverted: above

IN THE LESSON

- A spot rate is the yield on a zero coupon bond of a given maturity.
- It is the rate that discounts a single cash flow on a single date.
- A zero coupon bond's yield to maturity is therefore equal to the spot rate for that maturity.
- A coupon bond is a bundle of zero coupon payments, so its price is the sum of each cash flow discounted at its own spot rate.
- Its yield to maturity is the one rate that reproduces that same price, which makes it roughly an average of the spot rates, weighted by how much of the price each flow supplies.
- On an upward sloping curve a coupon bond's yield is below the spot rate for its final maturity, and on an inverted curve it is above.
- If the curve is flat, every yield equals the spot rate.

20 Example 8: price and YTM from spot rates

Spot rates 3%, 4%, 5% for 1, 2, 3 years. Bond: coupon 6, face 100, 3 years

Year 1 coupon	$6 \div 1.03$	**5.83**
Year 2 coupon	$6 \div 1.04^2$	**5.55**
Year 3 coupon plus face	$106 \div 1.05^3$	**91.57**
Price; then solved YTM	sum; solve for y	**102.94; 4.92%**

IN THE LESSON

- Spot rates are three per cent for one year, four for two years and five for three.
- A three year bond pays a coupon of six a year on a face of one hundred.
- Discount each flow at its own rate.
- The first coupon is worth five point eight three.
- The second is five point five five.
- The last, the final coupon plus the face value, one hundred six, over one point zero five cubed, is ninety one point five seven.
- The price is one hundred two point nine four.
- The yield to maturity that gives that same price is four point nine two per cent, between the spot rates but below the three year spot of five.

21 Example 9: bootstrapping a spot rate

1-year spot 3%. A 2-year bond, coupon 5, face 100, trades at 101.93

PV of the first coupon	$5 \div 1.03$	**4.85**
Left for the last payment	$101.93 - 4.85$	**97.08**
Growth over two years	$105 \div 97.08$	**1.0816**
Two-year spot rate	$\sqrt{1.0816} - 1$	**4.00%**

IN THE LESSON

- How do we find spot rates in the first place?
- We extract them from coupon bond prices, step by step, which is called bootstrapping.
- Say the one year spot rate is three per cent, from a one year zero.
- A two year bond with a coupon of five and face one hundred trades at one hundred one point nine three.
- Its first coupon is worth five over one point zero three, or four point eight five.
- That leaves ninety seven point zero eight for the final payment of one hundred five.
- Divide, take the square root, subtract one: the two year spot rate is four per cent.
- Repeat the idea for each longer maturity.

22 The coupon effect

- Same maturity, different coupons, different YTMs
- High coupon: more value arrives early
- Upward curve: higher coupon, lower YTM
- Inverted curve: the order reverses

IN THE LESSON

- Now compare bonds with the same maturity but different coupons.
- If the spot curve is flat they share one yield, but when it slopes they do not.
- A high coupon bond hands back more of its value early, so its yield leans toward the lower, earlier spot rates.
- On an upward sloping curve it therefore has a lower yield to maturity than a low coupon or zero coupon bond of the same maturity.
- On an inverted curve the order reverses.
- This is the coupon effect, and it says the yield to maturity alone is a poor way to compare bonds with different coupons.

23 Coupon effect on an upward curve

Spot rates 3%, 4%, 5%. Three-year bonds, face 100, annual coupons

Coupon rate	YTM
0% (zero-coupon)	5.00%
6%	4.92%
10%	4.88%
Reference: 3-year spot rate	5.00%

IN THE LESSON

- Here are the numbers, using the same spot rates of three, four and five per cent, and three year bonds with face one hundred.
- The zero coupon bond has a yield of exactly five per cent, the three year spot rate.
- The bond with a six coupon has a yield of four point nine two, as we found.
- The ten coupon bond comes in at four point eight eight.
- Each step up in coupon pulls the yield further down, because more of the price comes from early flows, which are discounted at lower rates.
- Notice the effect on yield is small.

24 Coupon, yield and price

Three-year bond, face 1000, 6% coupon paid twice a year

Yield to maturity	Price
4% (coupon above yield)	1056.01, premium
6% (coupon equals yield)	1000.00, par
8% (coupon below yield)	947.58, discount
8%, one year left	981.14, nearer par

IN THE LESSON

- A separate relationship links the coupon, the yield and the price, and it should not be confused with the coupon effect.
- If the coupon equals the yield the bond trades at par.
- If the coupon is above the yield it trades at a premium, and below, at a discount.
- Take our three year bond with a six per cent coupon.
- At a yield of four it is worth one thousand fifty six, at six exactly one thousand, at eight nine hundred forty eight.
- Hold the yield at eight and let time pass: with one year left the price pulls toward par, to nine hundred eighty one point one four.
- Lower coupon bonds also move more in percentage terms for the same change in yield.

25 Breaking down profit and loss

- Carry roll-down: income if nothing changes
- Rate change: the benchmark curve moves
- Spread change: the bond's spread moves
- Splitting shows which risk made or lost the money

IN THE LESSON

- Suppose you own a bond position and, over a month or a year, it makes or loses money.
- A single number hides why.
- Risk managers therefore split the profit and loss into drivers.
- The first is carry roll-down: what you earn if nothing in the market changes.
- It is the coupon income, less the cost of financing the position, plus the price gain from simply aging, as the bond rolls down the unchanged curve to a shorter maturity.
- The second is the rate change effect, from movement in the benchmark curve.
- The third is the spread change effect, from movement in the bond's own spread over that curve.

26 The profit and loss decomposition

Total P&L = Carry roll-down + Rate effect + Spread effect
Rate effect ≈ −DV01 × change in yield (bp)

Carry roll-down	Income if nothing changes
Rate effect	Benchmark curve moves
Spread effect	Bond's own spread moves
D V zero one	Money per basis point

IN THE LESSON

- The pieces simply add up.
- Total profit and loss equals carry roll-down, plus the rate change effect, plus the spread change effect.
- For the rate effect we use D V zero one, the money gained or lost for each one basis point move in yield, a basis point being one hundredth of one per cent.
- The rate effect is minus D V zero one times the change in yield in basis points.
- The sign matters: when yields rise, bond prices fall and you lose money.
- The spread effect has the same form with the bond's sensitivity to its spread.
- A hedge against the benchmark removes the rate effect and leaves the spread.

27 Example 10: a year of profit and loss

Position 10 million. DV01 4,500; spread sensitivity 4,400 per basis point

Carry roll-down	450,000 − 250,000 + 60,000	**260,000**
Rate effect	−4,500 × 12 bp	**−54,000**
Spread effect	−4,400 × 5 bp	**−22,000**
Total P&L	260,000 − 54,000 − 22,000	**184,000**

IN THE LESSON

- A position worth ten million has a D V zero one of four thousand five hundred.
- Over the year it earns coupons of four hundred fifty thousand and costs two hundred fifty thousand to finance.
- Aging adds a roll-down price gain of sixty thousand, so carry roll-down is two hundred sixty thousand.
- Benchmark yields then rise twelve basis points, giving a rate effect of minus four thousand five hundred times twelve, minus fifty four thousand.
- The bond's spread widens five basis points, with a spread sensitivity of four thousand four hundred, costing twenty two thousand.
- The total is one hundred eighty four thousand.
- Hedging the rate move would leave two hundred thirty eight thousand.

28 Carry roll-down: two assumptions

- Unchanged curve: each spot rate stays put
- Upward curve: roll-down gain. Inverted: roll-down loss
- Forwards realized: every bond earns the financing rate
- The gap between them is a view on forwards

IN THE LESSON

- Carry roll-down depends on a path assumed for rates, and two assumptions are standard.
- The first is that the term structure is unchanged.
- Every spot rate stays where it is, so the bond ages and is repriced on today's curve at its shorter maturity.
- On an upward sloping curve the roll-down is positive, and on an inverted curve it is negative.
- The second is that forward rates are realized: the future curve equals the forwards implied by today's curve.
- Then every bond earns exactly the short term financing rate, and net carry roll-down is zero.
- The gap between the two is the view that forwards will not come true.

29 Example 11: an unchanged curve

Bond: coupon 6, 3 years, price 102.94. Spots 3%, 4%, 5%. Finance at 3%

Price at horizon (2 years left)	6 ÷ 1.03 + 106 ÷ 1.04^2	**103.83**
Gross return	(103.83 + 6 − 102.94) ÷ 102.94	**6.69%**
Financing cost	3% × 102.94	**3.09**
Net carry roll-down	(6.89 − 3.09) ÷ 102.94	**3.69%**

IN THE LESSON

- Use the earlier bond and spot rates of three, four and five per cent.
- The bond costs one hundred two point nine four.
- After one year it is a two year bond.
- On an unchanged curve, discount six over one point zero three, plus one hundred six over one point zero four squared: one hundred three point eight three.
- The gain from aging is point eight nine.
- Add the coupon of six received, for a total of six point eight nine, which is six point six nine per cent.
- Financing at the one year rate of three per cent costs three point zero nine.
- Net carry roll-down is three point eight zero, or three point six nine per cent.

30 Example 12: forwards realized

Same bond. Next year's curve equals today's implied forward rates

Forward 1-year rate in a year	1.04^2 ÷ 1.03 − 1	**5.01%**
Forward 2-year rate in a year	(1.05^3 ÷ 1.03)^(1/2) − 1	**6.01%**
Price at horizon	6 ÷ 1.0501 + 106 ÷ 1.0601^2	**100.03**
Net carry roll-down	100.03 + 6 − 102.94 × 1.03	**0**

IN THE LESSON

- Now assume forwards are realized.
- The curve in a year equals the forward rates.
- The one year rate one year ahead is one point zero four squared over one point zero three, minus one: five point zero one per cent.
- The two year rate one year ahead comes from one point zero five cubed over one point zero three, then the square root, minus one: six point zero one per cent.
- Reprice the bond with those, and it is worth one hundred point zero three.
- Add the coupon of six and the total is one hundred six point zero three.
- Today's price grown at three per cent is the same.
- Net carry roll-down is zero.

31 Common mistakes to avoid

- Returns: keep reinvestment, base on price paid, compound
- Yields: halve the rate, double the periods
- Spread goes on every spot rate, not one
- P&L: rising yields lose money; roll-down only once

IN THE LESSON

- Let me collect the traps.
- In realized return, people drop the reinvestment income, use face value instead of the price paid, average instead of compounding, or report gross when net is asked.
- In yields, they forget to halve the rate and double the periods for a semi-annual bond, and treat yield to maturity as guaranteed.
- With spreads, they add the spread to a single rate.
- With annuities, they use an annual rate for monthly payments or discount a deferred annuity once too often.
- In profit and loss, they make sign errors, since rising yields lose money, and count the roll-down twice.

32 Recap: the main ideas

- Realized: price + coupons + reinvested, ÷ price paid
- Spread: one constant on all spot rates
- Annuity: C × [1 − (1 + r)^(−n)] ÷ r
- Coupon effect: yields differ when the curve slopes
- P&L = carry roll-down + rate effect + spread effect

IN THE LESSON

- To recap.
- Realized return counts price change, coupons and reinvestment, over the price paid, and net deducts financing.
- The spread is the constant added to every spot rate to hit the market price.
- Yield to maturity is the single rate that reproduces the price, found by iteration, except for a zero coupon bond.
- Annuities have a closed form, and a perpetuity is the payment over the rate.
- A coupon bond is a bundle of zero coupon payments, so its yield is a blend of spot rates, which creates the coupon effect.
- Profit and loss splits into carry roll-down, rate effect and spread effect.

33 Try three quick questions

1. Pay 100 a year for 4 years, rate 5%. Annuity value?
   Answer: 100 × (1 − 1.05^{-4}) ÷ 0.05 = 354.60.
2. 2-year zero, face 100, price 90. Yield?
   Answer: √(100 ÷ 90) − 1 = 5.41%.
3. Carry 120, DV01 30 per bp, yields +4 bp, spread effect +10. Total P&L?
   Answer: 120 − 30 × 4 + 10 = 10.

IN THE LESSON

- Now try three quick questions yourself.
- Pause the video after each one, think, then listen.
- Question one.
- A bond pays one hundred a year for four years, and the rate is five per cent.
- What is the annuity worth?
- One hundred times one minus one point zero five to the minus four, over point zero five, is three hundred fifty four point six zero.
- Question two.
- A two year zero coupon bond with face one hundred costs ninety.
- What is its yield?
- The square root of one hundred over ninety, minus one, is five point four one per cent.
- Question three.
- Carry roll-down is one hundred twenty, D V zero one is thirty per basis point, yields rise four basis points, and the spread effect is plus ten.
- What is the total profit and loss?
- One hundred twenty, minus thirty times four, plus ten, is ten.