CHARTA LEARNING · LESSON NOTES

Interest Rates

FRM Part I

1 A quoted rate is not the whole story

- Same quoted rate, different frequency, different money
- Compounding = interest earns interest
- Always ask: how often is interest added?
- Quotes with different frequencies cannot be mixed

IN THE LESSON

- Imagine two banks.
- Both advertise six per cent on a deposit.
- The first adds the interest once, at the end of the year.
- The second adds half of it after six months, and then earns interest on that interest for the rest of the year.
- Which is better?
- The second, even though the advertised number is the same.
- A quoted rate is only a label.
- To turn it into money you must also know how often interest is added, and that frequency is called the compounding frequency.
- Two quotes with different frequencies cannot be compared, added or swapped until they are put on the same footing.
- This is the first idea of the lesson, and a very common source of mistakes.

2 Growth with compounding

Periodic: (1 + R ÷ m)^(m × T)
Continuous: e^(R × T)

Quoted rate, R	Quoted rate for one year
Periods per year, m	How often interest is added
Years, T	Length of time in years
Growth factor	What one unit grows to

IN THE LESSON

- Here is the arithmetic.
- Let R be the quoted rate for a year, and let m be how many times a year interest is added.
- In each period you earn the quoted rate divided by m.
- Over T years there are m times T periods.
- So one unit of money grows to one plus R divided by m, raised to the power m times T.
- Now let m get bigger and bigger, with interest added every instant.
- In the limit this is called continuous compounding, and one unit grows to e to the power R times T, where e is the number two point seven one eight.
- Continuous compounding is a neat mathematical tool, and it is the easiest convention to work with when we later combine rates over different periods.

3 One quote, four real rates

Quoted rate 6% for one year. What does one unit really earn?

Compounding	Effective annual rate
Annual	6.000%
Semiannual	6.090%
Quarterly	6.136%
Continuous	6.184%

IN THE LESSON

- Let's see what the same six per cent quote really delivers over one year, under four conventions.
- Added once a year, one unit becomes one point zero six, so the effective annual rate is six per cent.
- Added twice a year, you earn three per cent twice, which gives six point zero nine per cent.
- Added four times a year, it rises to six point one four per cent.
- And with continuous compounding it is six point one eight per cent.
- The pattern is clear.
- For a given quoted rate, more frequent compounding gives a higher effective annual rate.
- The quote stayed the same; the money actually earned did not.
- The effective rate is the honest way to compare quotes.

4 Example 1: price of a zero, four ways

Zero pays 100 in 5 years. Quoted rate 6% each time

Annual	100 ÷ 1.06^5	**74.73**
Semiannual	100 ÷ 1.03^10	**74.41**
Quarterly	100 ÷ 1.015^20	**74.25**
Continuous	100 × e^(−0.06×5)	**74.08**

IN THE LESSON

- Now apply this to a bond's price.
- A zero coupon bond pays one hundred dollars in five years and nothing before.
- We discount it at a quoted rate of six per cent, but under four conventions.
- Annual: one hundred divided by one point zero six to the fifth gives seventy four dollars seventy three.
- Semiannual: divide by one point zero three to the tenth, seventy four forty one.
- Quarterly: divide by one point zero one five to the twentieth, seventy four twenty five.
- Continuous: one hundred times e to the minus point three, seventy four oh eight.
- The more often we compound, the faster the growth, the smaller the discount factor, and the lower the price.
- Same quote, four prices.

5 Example 2: converting a quote

Annual quote 6%. Which quote on another basis gives the same price?

Growth factor to match	1 + 6%	**1.0600**
Semiannual quote	2 × (√1.06 − 1)	**5.913%**
Continuous quote	ln 1.06	**5.827%**
Check	e^0.05827	**1.0600**

IN THE LESSON

- The reverse question is just as important.
- Suppose a price is fixed by an annual rate of six per cent.
- What quote on another convention gives the very same price?
- The rule is to equate the growth factors, so that the same money grows by the same amount.
- Semiannual first: one plus the rate over two, squared, must equal one point zero six.
- Take the square root, subtract one, and multiply by two: five point nine one three per cent.
- For a continuous quote, the natural logarithm of one point zero six is five point eight two seven per cent.
- Notice the order.
- Annual is highest, semiannual lower, continuous lowest.
- The usual error is taking the number from one convention and plugging it into a formula built for another.

6 Spot rates: lending with nothing in between

- Zero coupon bond: one payment, at maturity
- Spot (zero) rate = its annualised yield
- Return for lending from today to date T, nothing between
- All maturities together form the spot curve

IN THE LESSON

- So far we used one rate for every date.
- Real markets do not work like that.
- Lending for one year, for five years and for thirty years earn different rates, because lenders want different compensation for tying money up for longer.
- The cleanest way to describe this is with a zero coupon bond, which pays one amount at the end and nothing along the way.
- Its annualised yield is called the spot rate, or zero rate, for that maturity.
- It is the return for lending from today to a given date with no interim payments.
- A spot rate belongs to a maturity.
- Put the spot rates for every maturity on a graph and you get the spot curve, also called the zero curve.

7 Discount factors from spot rates

DF = 1 ÷ (1 + s)^T or e^(−s × T)
s = DF^(−1 ÷ T) − 1

Spot rate, s	Spot rate for that maturity
Years, T	Years until the payment
Discount factor	Price today of one unit paid at T

IN THE LESSON

- A discount factor is the price today of one unit paid at a future date.
- It answers the question: how much is a promised dollar worth now?
- With annual compounding, the discount factor for T years is one divided by one plus the spot rate, raised to the power T.
- With continuous compounding it is e to the minus the spot rate times T.
- You can also go backwards.
- Given a discount factor, the annual spot rate is the discount factor raised to the power minus one over T, minus one.
- The discount factor is the bridge between a rate and a price.
- Every price in fixed income is a sum of cash flows multiplied by their own discount factors.

8 A spot curve and its discount factors

Spot curve, annual compounding

Maturity: spot rate	Discount factor
1 year: 3.0%	0.9709
2 years: 3.5%	0.9335
3 years: 4.0%	0.8890
4 years: 4.5%	0.8386
5 years: 5.0%	0.7835

IN THE LESSON

- Here is the curve we will use for the rest of the lesson.
- The spot rate is three per cent for one year, three and a half for two years, four for three years, four and a half for four, and five per cent for five years, all with annual compounding.
- This is an upward sloping curve, the most common shape.
- Beside each rate is its discount factor.
- One year: one over one point zero three, which is point nine seven zero nine.
- Two years: point nine three three five.
- Three years: point eight eight nine zero.
- Four years: point eight three eight six.
- Five years: point seven eight three five.
- Notice that discount factors always fall as the date moves further out.
- A dollar later is worth less.

9 Example 3: pricing a coupon bond

3-year bond, face 100, annual coupon 5. Use each date's own discount factor

Coupons	5 × (0.9709+0.9335+0.8890)	**13.97**
Principal	100 × 0.8890	**88.90**
Price	13.97 + 88.90	**102.87**
Wrong: 4% for every date	5 × 2.7751 + 88.90	**102.78**

IN THE LESSON

- A coupon bond is just a bundle of zero coupon bonds.
- Take a three year bond with face value one hundred and an annual coupon of five.
- It pays five in year one, five in year two, and one hundred and five in year three.
- Each payment is multiplied by its own discount factor.
- The three coupons are worth five times the sum of the three factors, which is two point seven nine three four, giving thirteen point nine seven.
- The principal is one hundred times point eight eight nine zero, eighty eight point nine nine.
- The price is one hundred and two dollars eighty seven.
- If we wrongly used the three year rate of four per cent for every payment, we would get one hundred and two seventy eight.
- The mistake is small here only because the curve is gently sloped.

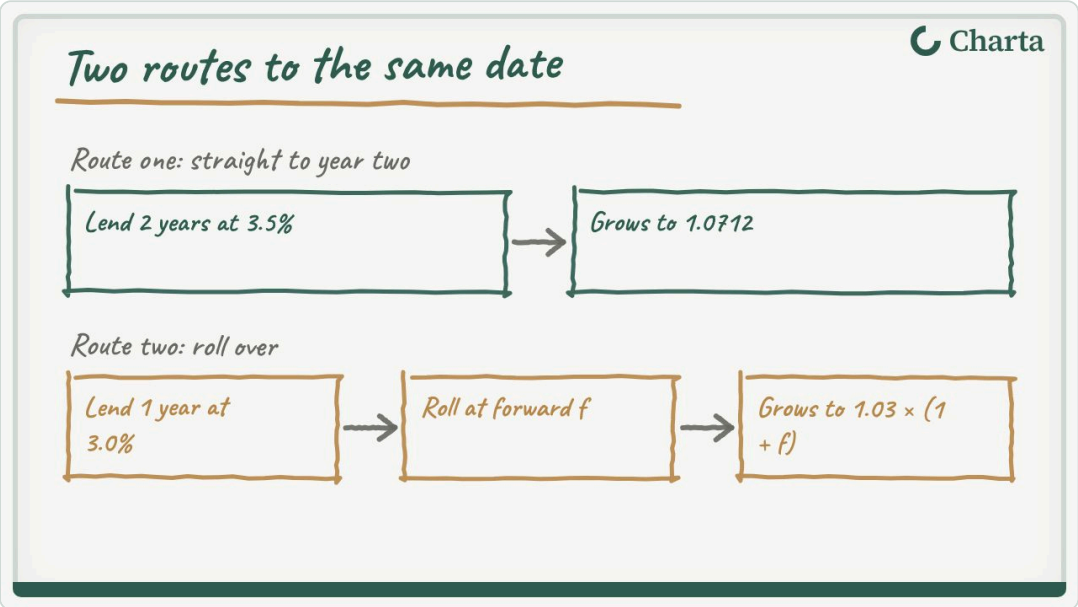
10 Forward rates: a rate fixed today for later

- Forward rate = rate fixed today for a future period
- It is implied by the spot curve, not guessed
- No arbitrage: two routes to the same date must match
- Long loan = short loan, then roll at the forward

IN THE LESSON

- Now a new question.
- Today you can fix a rate for borrowing or lending over a future period, say from year one to year two.
- That rate is called a forward rate.
- You do not need a crystal ball to find it, because it is already hidden inside the spot curve.
- The logic is called no arbitrage, which simply means there is no free lunch.
- Lending for two years must give the same result as lending for one year and then lending again for the second year at a rate agreed today.
- If the two routes paid differently, traders would borrow on the cheap route, lend on the dear one, and pocket a risk free profit until prices moved back into line.
- So the forward rate is whatever rate closes the gap.

11 Two routes to the same date



IN THE LESSON

- Look at the two routes.
- The first route lends for two years at the two year spot rate of three and a half per cent.
- One unit grows to one point zero three five, squared, which is one point zero seven one two.
- The second route lends for one year at three per cent, giving one point zero three, and then rolls the whole sum into a second year at the forward rate, f.
- Because no arbitrage forces the two outcomes to match, one point zero three times one plus f must equal one point zero seven one two.
- Divide, subtract one, and we find f.
- The forward rate is the break even rate for the second year.
- It is exactly the rate that makes the two routes equal.

12 The forward rate formula

$g = (1 + s_2)^{T_2} \div (1 + s_1)^{T_1}$, $f = g^{(1 \div (T_2 - T_1))} - 1$
Continuous: $f = (s_2 T_2 - s_1 T_1) \div (T_2 - T_1)$

Spot rate, s	Spot rates to T one and T two
Years, T	The two maturities in years
Forward rate, f	Rate for the period in between
Growth factor	Ratio of the two growth factors

IN THE LESSON

- Here is the general result.
- Let s one and s two be the annual spot rates to times T one and T two, and f the forward rate between them.
- The two routes give one plus s two to the power T two, equals one plus s one to the power T one, times one plus f to the power T two minus T one.
- Solve for f.
- Divide the long growth factor by the short one, then take the root that matches the length of the forward period, and subtract one.
- Do not forget that root, because the answer must be a rate per year.
- With continuous compounding the formula is simpler: s two times T two, minus s one times T one, all divided by T two minus T one.

13 Example 4: forward rates from the curve

Spot: 1y 3.0%, 2y 3.5%, 3y 4.0% (annual compounding)

Forward, year 1 to 2	$1.035^2 \div 1.03 - 1$	**4.00%**
Forward, year 2 to 3	$1.04^3 \div 1.035^2 - 1$	**5.01%**
Two-year forward from year 1	$(1.04^3 \div 1.03)^{\wedge}½ - 1$	**4.50%**
Continuous: s 3% and 3.5%	(7.0% − 3.0%) ÷ 1	**4.0%**

IN THE LESSON

- Let's compute forwards from our curve.
- From year one to year two: one point zero three five squared is one point zero seven one two, divided by one point zero three, and the answer, minus one, is four point zero zero per cent.
- From year two to year three: one point zero four cubed over one point zero three five squared gives five point zero one per cent.
- Now a two year forward starting in one year.
- Divide one point zero four cubed by one point zero three, take the square root, subtract one: four point five zero per cent.
- That square root is the annualising step.
- Finally, a continuous example: spot rates of three and three and a half per cent for one and two years give a forward of seven minus three, which is four per cent.

14 What a forward rate tells you

- Forward = break-even rate, not a forecast
- Future rate above forward: rolling short loans wins
- Future rate below forward: the long bond wins
- Common slip: forgetting the root when annualising

IN THE LESSON

- A common misreading is to treat the forward rate as a prediction of where rates will go.
- It is not.
- It is a break even rate.
- Suppose the forward rate for year two is four per cent.
- If the actual one year rate turns out higher than four per cent, then rolling short loans would have beaten the long loan.
- If it turns out lower, the long loan wins.
- Today's forward is simply the point of indifference, set by today's prices, and it also contains compensation for risk, so it is a poor forecast.
- Its real value is practical.
- It is the rate you can lock in today for a future period, and the benchmark against which any later outcome is judged.

15 Par rates: the coupon that prices at par

- Par rate: the coupon that prices the bond at face value
- At par, coupon rate = yield
- Price = c × sum of DFs + final DF = 1
- Solve for c

IN THE LESSON

- Our third rate answers a different question.
- Suppose you want to issue a bond that sells for exactly its face value, with no premium and no discount.
- What coupon should it carry?
- That coupon is the par rate for the maturity.
- At par, the coupon rate equals the yield.
- Here is the logic.
- The price must equal one unit per unit of face value.
- The price is the coupon times the sum of the discount factors, plus the principal times the last discount factor.
- Set that equal to one and solve for the coupon.
- This shows why par rates matter: market quotes for bonds and swaps are very often given as par rates, so we must be able to turn them back into discount factors.

16 The par rate formula

$$c = (1 - DF_n) \div (DF_1 + DF_2 + \ldots + DF_n)$$

Par rate, c	Par rate, annual coupon rate
Discount factor	Discount factors for each date
Years left, n	Years to maturity

IN THE LESSON

- Here is the result.
- For a bond paying n annual coupons, the par rate is one minus the last discount factor, divided by the sum of all the discount factors up to n.
- The top is the part of the face value that the coupons must make up for, because the principal alone is worth only the last discount factor.
- The bottom is the value today of receiving one unit each year, sometimes called the annuity factor.
- For semiannual coupons, the sum runs over every half year discount factor and the answer is multiplied by two.
- A bond with a coupon above the par rate trades above par, and one below it trades below.
- And the par rate is not the final spot rate, except on a flat curve.

17 Example 5: par rates from discount factors

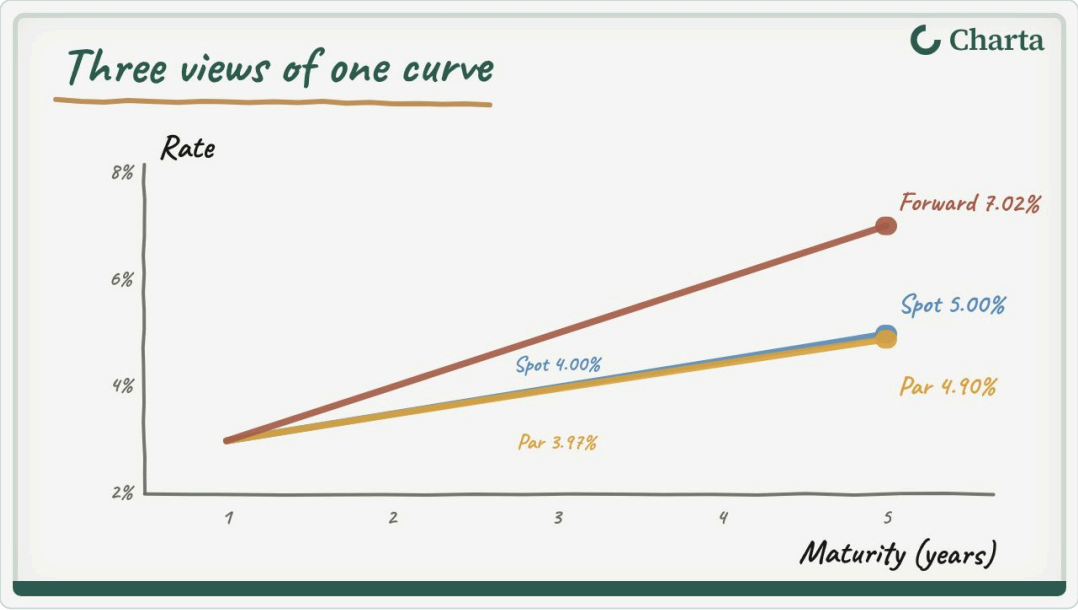
Use the discount factors from the table. Per 100 of face value

Three years: top	1 − 0.8890	**0.1110**
Three years: bottom	0.9709+0.9335+0.8890	**2.7934**
Par rate, 3 years	0.1110 ÷ 2.7934	**3.97%**
Par rate, 5 years	0.2165 ÷ 4.4155	**4.90%**

IN THE LESSON

- Use our curve.
- For a three year bond, the top of the formula is one minus point eight eight nine zero, which is point one one one zero.
- The bottom is the sum of the first three discount factors, two point seven nine three four.
- Divide: three point nine seven per cent.
- That is lower than the three year spot rate of four per cent.
- For five years, the top is one minus point seven eight three five, point two one six five, the bottom is four point four one five five, and the par rate is four point nine zero per cent, again below the five year spot rate of five per cent.
- As a test, a three year bond with a coupon of three point nine seven per cent prices at exactly one hundred.
- The par rate is not the spot rate.

18 Three views of one curve



IN THE LESSON

- Let's draw the three rates side by side.
- Along the bottom is maturity, from one to five years.
- Up the side is the rate.
- The middle line is the spot curve, rising from three to five per cent.
- Just underneath it is the par curve, which starts at three per cent and ends at four point nine.
- Far above both is the line of one year forward rates, which starts at three and climbs to seven point zero two per cent for the final year, from year four to year five.
- Notice the order at five years: forward on top, then spot, then par at the bottom.
- All three describe the same market, and each can be calculated from the others.
- They differ in what they average.

19 How spot, forward and par rates relate

- Spot = average of the forwards up to n (geometric)
- Par = discount-weighted average of those forwards
- Upward curve: forward > spot > par
- Inverted curve: the order reverses. Flat: all equal

IN THE LESSON

- These three rates are different ways of reading one set of discount factors.
- The spot rate to year n is an average of the one year forward rates up to year n.
- More exactly it is a geometric average, because growth factors multiply.
- With continuous compounding it becomes a simple arithmetic average.
- The par rate is also an average of those forwards, but weighted by the discount factors, so it lies close to the spot rate without being equal to it.
- Now the ordering.
- When the curve slopes up, the last forward is the marginal rate and sits above the spot rate, which sits above the par rate.
- When the curve is inverted, the order flips.
- On a flat curve, all three are the same.

20 Example 6: spot as an average of forwards

One-year forwards: 3.00%, 4.00%, 5.01%. Spot 3y 4.00%. Par 3y 3.97%

Multiply growth factors	1.03 × 1.0400 × 1.0501	**1.1249**
Cube root, minus one	1.1249^(1/3) − 1	**4.00%**
Ordering at 3 years	forward, spot, par	**5.01, 4.00, 3.97**
Inverted curve	order flips	**par > spot > fwd**

IN THE LESSON

- Let's prove the averaging with our numbers.
- The one year forwards are three per cent, four point zero zero, and five point zero one.
- Multiply their growth factors: one point zero three, times one point zero four zero zero, times one point zero five zero one.
- The product is one point one two four nine.
- Take the cube root and subtract one: four point zero zero per cent, exactly the three year spot rate.
- The geometric average of the forwards is the spot rate.
- Now apply the ordering at three years.
- The forward for year three is five point zero one, the spot rate is four point zero zero, and the par rate is three point nine seven.
- Forward above spot above par, just as the upward sloping curve predicts.

21 How price changes as time passes

Price = Par ÷ (1 + y)^n

Price	Price of the zero today
Yield, y	Yield, held constant
Years left, n	Years left to maturity

IN THE LESSON

- Now let time pass.
- A bond's price is the present value of what it has left to pay.
- Suppose the yield curve does not move.
- Take a zero coupon bond.
- Its price is the face value divided by one plus the yield, raised to the number of years left.
- Every year that passes, one year drops out of the power, and the price rises by a factor of one plus the yield.
- So a zero accretes steadily toward its face value, earning roughly the yield each year.
- At maturity, every bond equals its face value.
- This is not a bet on rates.
- It is simply the clock running, and it has to be kept apart from the separate effect of the yield curve moving.

22 Example 7: three years of pulling to par

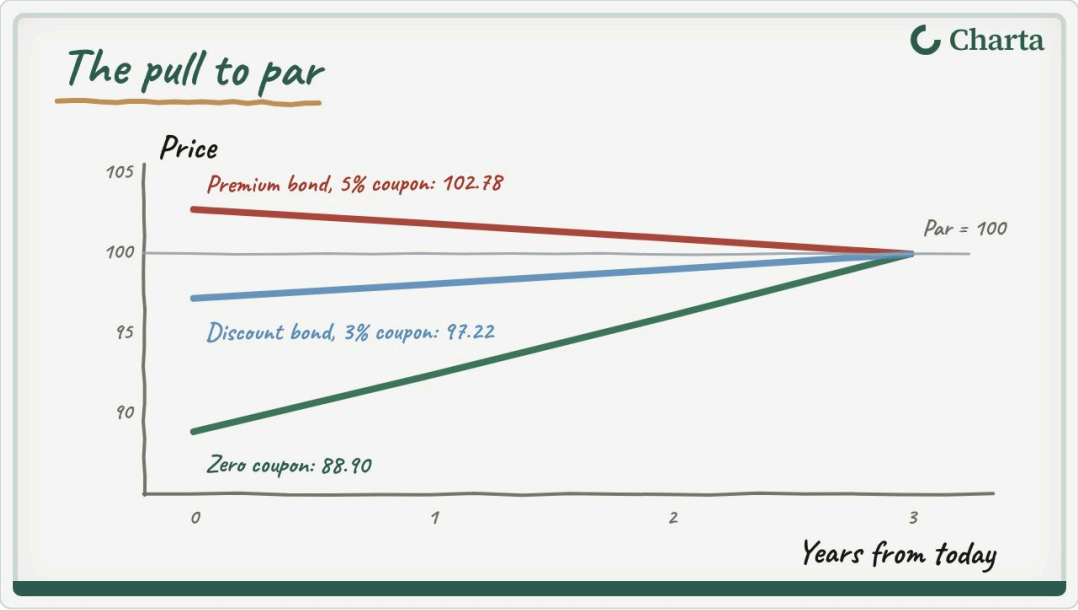
Yield stays 4% for all maturities; coupons reinvested at 4%. Face 100

Zero, 3 years left	100 ÷ 1.04^3	**88.90**
Zero, 2 years left	100 ÷ 1.04^2	**92.46**
5% coupon bond, 3 years left	priced at 4%	**102.78**
Same bond, 2 years left	priced at 4%	**101.89**
Coupon bond total return	(101.89 + 5) ÷ 102.78 − 1	**4.0%**

IN THE LESSON

- Use a flat yield of four per cent.
- A zero coupon bond with three years left costs one hundred divided by one point zero four cubed, eighty eight dollars ninety.
- One year later, with two years left, it is worth ninety two forty six.
- That is a gain of four per cent, with no change in yield at all.
- Now a bond with a five per cent coupon, priced at a premium.
- With three years left it costs one hundred and two seventy eight.
- A year later, after paying the five dollar coupon, the price has fallen to one hundred and one eighty nine.
- The price fell, but the total return is the new price plus the coupon, divided by the old price, minus one.
- That is four per cent, the yield.

23 The pull to par



IN THE LESSON

- This picture shows the pull to par for three bonds at a constant four per cent yield.
- Along the bottom is time, from today to maturity in three years.
- Up the side is price.
- The middle line is the zero coupon bond, climbing from about eighty nine to one hundred.
- The upper line is the five per cent coupon bond at a premium: it starts above par and sinks down toward one hundred.
- The lower line is a three per cent coupon bond at a discount, starting near ninety seven and rising to one hundred.
- All three meet at par on the maturity date.
- The price change you see over time is the time effect.
- If yields also move, that adds a second, separate yield effect on top.

24 Flatteners and steepeners

- Slope = long rate minus short rate, e.g. 10y minus 2y
- Gap narrows: flattens. Gap widens: steepens
- Steepening view: buy short bond, sell long bond
- Flattening view: sell short bond, buy long bond

IN THE LESSON

- Now let's use the curve.
- The slope of the curve is the gap between a long rate and a short rate, for example the ten year rate minus the two year rate.
- When that gap narrows, the curve flattens.
- When it widens, the curve steepens.
- Notice that nothing is said about whether rates are rising or falling.
- The whole curve can move up or down while its shape changes.
- Traders express a view on shape directly.
- If you expect steepening, you buy the short maturity bond and sell the long one.
- If you expect flattening, you do the opposite.
- Buying a bond means you gain if its yield falls, so the buy and sell legs work together to profit from the change in the gap.

25 Four ways a curve can change shape

Bull = yields fall. Bear = yields rise

Move	What happens
Bull flattener	Long yields fall more
Bear flattener	Short yields rise more
Bull steepener	Short yields fall more
Bear steepener	Long yields rise more

IN THE LESSON

- There are four named cases.
- A bull move means yields fall, a bear move means yields rise.
- In a bull flattener the long yields fall by more than the short ones, so the gap narrows.
- In a bear flattener the short yields rise by more than the long ones, and the gap narrows again.
- In a bull steepener the short yields fall by more than the long ones, so the gap widens.
- And in a bear steepener the long yields rise by more, and the gap widens.
- So flattener and steepener describe the change in shape, while bull and bear describe whether the general level of yields is falling or rising.
- Both labels together tell you what happened to the curve and which end led the move.

26 Example 8: a steepener that ignores the level

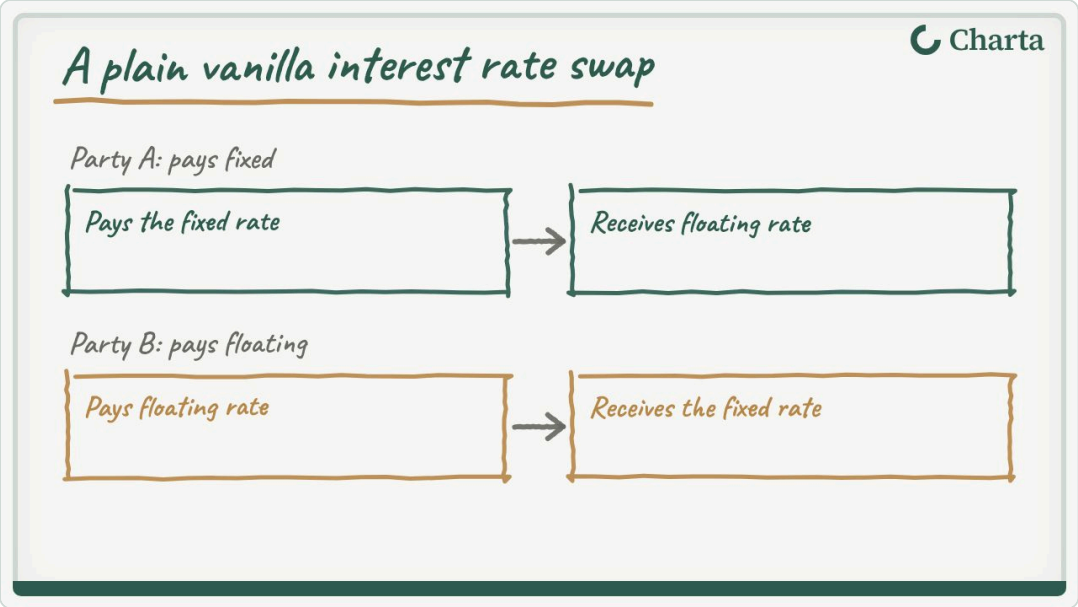
Sell 10m 10y (DV01 0.08 per 100), buy 2y (DV01 0.02)

10-year leg DV01	10m × 0.08 ÷ 100	**8,000 a bp**
2-year face to match	8,000 ÷ (0.02 ÷ 100)	**40m**
Gap widens 20 bp	80,000 + 80,000	**+160,000**
Parallel up 50 bp	−400,000 + 400,000	**0**
Equal face, 10m each, up 50 bp	−100,000 + 400,000	**+300,000**

IN THE LESSON

- Let's build a steepener.
- The two year yield is three per cent and the ten year yield is four per cent, a gap of one hundred basis points.
- We sell ten million of the ten year bond.
- Its D V oh one, the dollar value of one basis point, is eight thousand dollars.
- We must buy enough of the two year bond to match that.
- If its D V oh one is two cents per hundred of face value, we need forty million of face value.
- Now suppose the gap widens by twenty basis points, with the two year yield falling ten and the ten year rising ten.
- Both legs gain eighty thousand, a profit of one hundred and sixty thousand.
- A parallel rise of fifty basis points gives zero.
- Equal face amounts would instead leave a three hundred thousand dollar exposure.

27 A plain vanilla interest rate swap



IN THE LESSON

- Finally, the swap.
- An interest rate swap is an agreement between two parties to exchange interest payments on a notional principal.
- The notional is only a yardstick for the size of the payments, and it is never handed over.
- In a plain vanilla swap one party pays a fixed rate and receives a floating rate.
- The floating rate is a benchmark rate that is reset each period, for example an overnight or term rate.
- The other party does the reverse, paying floating and receiving fixed.
- On each payment date the two amounts are netted, so only the difference moves.
- Why do it?
- A borrower paying floating can convert to fixed, locking in certainty, while the other side takes the opposite exposure.

28 Valuing a swap and the par swap rate

- Floating leg is worth par just after each reset
- Fixed leg worth par makes the swap worth zero
- Par swap rate = (1 − DFn) ÷ (DF1 + … + DFn)
- Later the swap gains or loses as swap rates move

IN THE LESSON

- How do we value a swap?
- Break it into two legs.
- Just after each reset date the floating leg is worth par, because a floating rate loan that resets to the market rate is worth its face value.
- So the swap is worth zero at the start only if the fixed leg is also worth par.
- The fixed rate that makes it so is called the par swap rate.
- It is found with the same formula as the par bond rate: one minus the last discount factor, divided by the sum of all the discount factors.
- This is how par rates come to be defined in the swap market.
- Later, as market swap rates move away from the contract rate, the swap gains value for one party and loses it for the other.

29 Example 9: pricing and revaluing a swap

3-year swap, notional 100m. A year on: 2 years left, DFs now 0.9709, 0.9335 (rates fell)

Par swap rate, 3 years	(1 − 0.8890) ÷ 2.7934	**3.97%**
Yearly fixed payment	3.97% × 100m	**3.97m**
Market 2-year swap rate later	(1 − 0.9335) ÷ 1.9044	**3.49%**
Fixed leg worth, per 100m	3.9738% × 1.9044 + 0.9335	**100.919m**
Value to the fixed payer	100m − 100.919m	**−0.919m**

IN THE LESSON

- Let's price a three year swap with a notional of one hundred million, on our discount factors.
- The par swap rate is the same three point nine seven per cent we found for the par bond, so the fixed payer pays about three point nine seven million a year.
- Now fast forward one year.
- Suppose rates have fallen, so the next two years are discounted with the factors for one and two years from our table.
- The market two year swap rate is one minus point nine three three five, over one point nine zero four four, which is three point four nine per cent.
- Our contract pays more.
- The fixed leg is worth about one hundred point nine one nine million, so the payer of fixed has lost nine hundred and nineteen thousand.

30 Recap: the main ideas

- Quote + frequency; equate growth factors to convert
- DF = 1 ÷ (1+s)^T; bond = sum of cash flow × own DF
- Forward = break-even; par = coupon at par
- Up-sloping curve: forward > spot > par
- Pull to par; trade slope with DV01 weights; swap par rate

IN THE LESSON

- Let's pull it together.
- A quoted rate needs a compounding frequency, and more frequent compounding means a higher effective rate and a lower price for the same quote.
- Convert by equating growth factors.
- The spot rate is the yield on a zero coupon bond, and the discount factor turns it into a price.
- A coupon bond is a portfolio of zeros, each at its own discount factor.
- The forward rate is the break even rate for a future period, and the par rate is the coupon that prices a bond at face value.
- On an upward curve forward is above spot, above par.
- Bond prices pull to par with time.
- Steepeners and flatteners are traded with D V oh one weights.
- A swap's par rate follows the par bond formula.

31 Try three quick questions

1. 4% quoted semiannually. Equivalent continuous quote?
   Answer: 2 × ln(1.02) = 3.96%.
2. Spot 1y 4%, 2y 5%. Forward rate for year 2?
   Answer: $1.05^2 \div 1.04 - 1 = 6.01\%$.
3. DFs 0.96, 0.92, 0.88. Three-year par rate?
   Answer: 0.12 ÷ 2.76 = 4.35%.

IN THE LESSON

- Time for three quick questions.
- Pause the video, think each one through, then listen to the answer.
- Question one.
- A bank quotes four per cent with semiannual compounding.
- What is the equivalent continuous quote?
- Two times the natural log of one point zero two, which is three point nine six per cent.
- Question two.
- The one year spot rate is four per cent and the two year spot rate is five per cent.
- What is the forward rate for year two?
- One point zero five squared divided by one point zero four, minus one, which is six point zero one per cent.
- Question three.
- The discount factors for years one to three are point nine six, point nine two and point eight eight.
- What is the three year par rate?
- Point one two divided by two point seven six, which is four point three five per cent.