CHARTA LEARNING · LESSON NOTES

Modeling Non-Parallel Term Structure Shifts and Hedging

FRM Part I

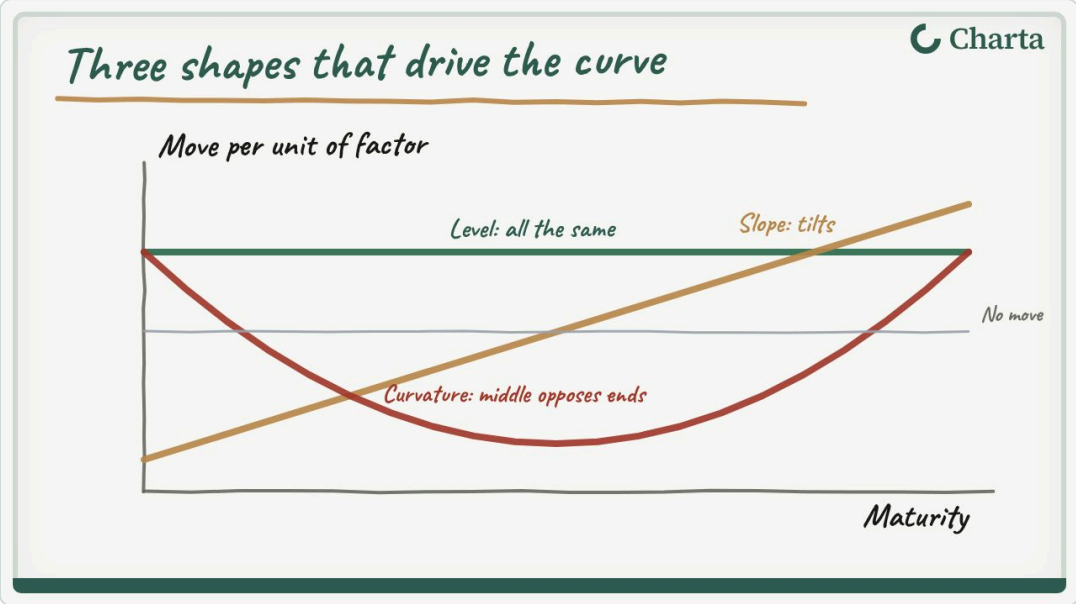
1 A curve that moves in pieces

- Yield curve: the rate for each length of lending
- Parallel shift: every rate moves by the same amount
- Real moves are uneven: short, middle and long differ
- Safe against a parallel shift is not safe overall

IN THE LESSON

- Start with a picture.
- A yield curve is simply the interest rate you earn for lending for one year, two years, five years, ten years, and so on.
- Imagine a bond portfolio.
- The easy case is a parallel shift, where every rate rises by the same amount.
- One number, the dollar value of a basis point, tells you the damage.
- But real curves rarely behave.
- Short rates may rise while long rates fall, and the middle may move on its own.
- A portfolio that is safe against a parallel shift can still lose money.
- To manage this we need a way to describe the shapes that curve movements actually take.

2 Three shapes that drive the curve



IN THE LESSON

- Here is the surprise.
- Although the curve looks free to move in many ways, a few patterns explain nearly all of its daily changes.
- The first is level, drawn flat: every maturity moves the same way.
- The second is slope, a line that tilts: short rates move one way and long rates move the opposite way, so the curve steepens or flattens.
- The third is curvature, a bowl: the middle of the curve moves against both ends.
- On this chart the horizontal axis is maturity and the vertical axis is how much each maturity moves per unit of the factor.
- Level is usually the biggest, then slope, then curvature.

3 What principal components analysis does

- Input: covariance of past rate changes at many maturities
- Eigenvector = a factor; its loadings = the shape
- Eigenvalue = the variance of that factor
- Factors are uncorrelated and ranked, biggest first

IN THE LESSON

- Principal components analysis, or P C A, finds those shapes from data rather than from theory.
- You collect a long history of daily changes in rates at many maturities and compute how those changes vary together, which is the covariance matrix.
- P C A then breaks that matrix into pieces called eigenvectors.
- Each eigenvector is one factor.
- Its loadings tell you how much each maturity moves per unit of that factor, and that is exactly the shape we just drew.
- Each factor also has an eigenvalue, which is the variance of that factor.
- The factors come out uncorrelated with each other by construction, and they are ranked from the largest variance to the smallest.

4 How much does each factor explain?

Share = Eigenvalue of the factor ÷ Sum of all eigenvalues

Eigenvalue of one factor	Variance of that one factor
Sum of all eigenvalues	Total variance, all factors
Share of variance explained	Fraction explained, adds to 100%

IN THE LESSON

- To rank the factors we need their share of the total movement.
- The share of variance a factor explains equals its eigenvalue divided by the sum of all the eigenvalues.
- The eigenvalue is that factor's variance.
- The sum of all the eigenvalues is the total variance of the rate changes, so each share is a fraction of the whole, and all the shares add to one hundred per cent.
- For government curves the first three factors, level, slope and curvature, usually account for the great majority of the variance.
- That is why risk teams often model only three factors instead of tracking a dozen separate maturities.

5 Example 1: variance explained

Eigenvalues (bp squared): 120, 22.5, 6, 1.5

Total variance	120 + 22.5 + 6 + 1.5	**150**
Level share	120 ÷ 150	**80%**
Slope share	22.5 ÷ 150	**15%**
Curvature share	6 ÷ 150	**4%**
First three together	80% + 15% + 4%	**99%**

IN THE LESSON

- Suppose a P C A on a government curve gives four eigenvalues, measured in basis points squared: one hundred and twenty, twenty two point five, six, and one point five.
- Add them: the total is one hundred and fifty.
- The first factor, level, explains one hundred and twenty over one hundred and fifty, which is eighty per cent.
- Slope explains twenty two point five over one hundred and fifty, fifteen per cent.
- Curvature explains six over one hundred and fifty, four per cent.
- The last explains the remaining one per cent.
- The first three together explain ninety nine per cent, so modelling just those three loses very little.

6 Reading P C A results with care

- Factors are found from data, then named by their shape
- Uncorrelated factors, but the rates themselves are linked
- High explained share describes the curve, not each portfolio
- A portfolio can hide in the small leftover factors

IN THE LESSON

- Three exam points.
- First, the factors are data driven.
- The analyst does not choose that the first one will be called level.
- It emerges from the history, and we name it afterwards by looking at its loadings.
- A different sample period can give slightly different shapes.
- Second, uncorrelated is a statement about the factors, not about the individual rates, which remain strongly linked to each other.
- Third, a high explained share describes the historical curve as a whole.
- It does not mean every portfolio is well described by three factors.
- A portfolio built to be neutral to level and slope may sit mostly in the small leftover factors.

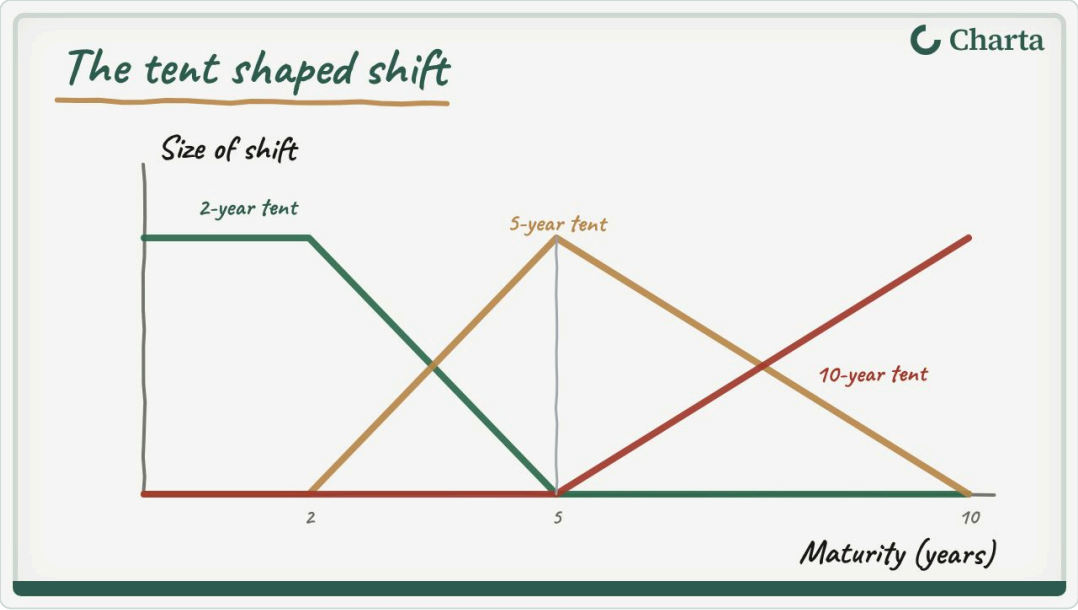
7 Key rates: measuring risk in pieces

- Pick key maturities, for example 2, 5 and 10 years
- KR01: value change when one key rate falls 1 basis point
- The other key rates stay unchanged
- Rates in between follow by straight-line interpolation

IN THE LESSON

- Now we turn to measuring a portfolio's exposure piece by piece.
- Choose a few key maturities, say two years, five years and ten years.
- A key rate zero one, written K R zero one, is the change in the portfolio's value when one chosen key rate falls by one basis point while the other key rates stay put.
- A basis point is one hundredth of one per cent.
- Rates between key maturities are found by interpolation, drawing straight lines between the key points.
- So if the five year rate falls, the rates near five years fall by nearly as much, and the effect fades away towards the neighbouring key maturities.

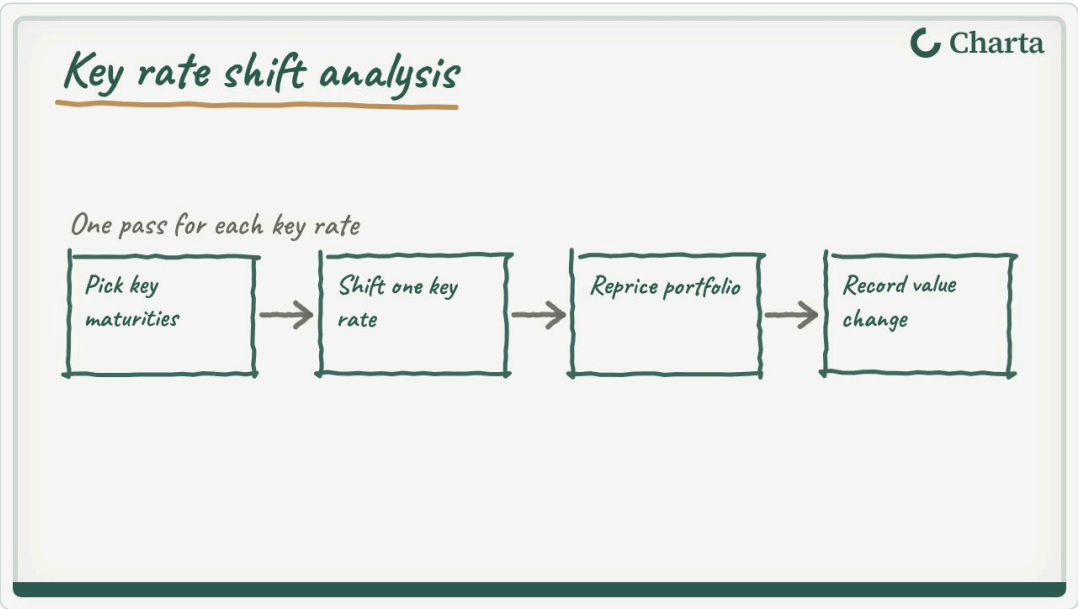
8 The tent shaped shift



IN THE LESSON

- This chart shows the shifts behind three K R zero ones.
- Each one is a tent.
- For the five year key rate, the shift is a full basis point at five years and fades in a straight line to zero at two years and at ten years.
- The two year shift is a full basis point out to two years, then fades to zero at five years.
- The ten year shift is zero up to five years and then climbs to a full basis point at ten years and beyond.
- Look at what happens if you stack the three tents.
- At every maturity they add up to exactly one basis point, which is a parallel shift.
- That is the design.

9 Key rate shift analysis



IN THE LESSON

- Key rate shift analysis is a simple routine, and it is worth knowing in order.
- First, pick the set of key maturities.
- Second, shift one key rate down by one basis point, with the tent shape spreading that shift to the neighbouring rates.
- Third, reprice the whole portfolio at the shifted curve.
- Fourth, record the change in value.
- That number is the K R zero one for that maturity.
- Then reset the curve and repeat for the next key rate.
- The output is a short list of numbers, one per key maturity, showing where along the curve the portfolio is exposed, rather than a single figure for the whole curve.

10 What KR01s tell you

Key rates and interpolation shape the split, not the total

Feature	Meaning
Positive KR01	Gains when that rate falls
Negative KR01	Net short at that maturity
Sum of KR01s	Equals the ordinary DV01
Individual KR01s	Depend on keys and interpolation

IN THE LESSON

- Here is how to read a list of K R zero ones.
- With the usual convention, a long bond has positive K R zero ones, because it gains when rates fall.
- If a K R zero one at some maturity is negative, the portfolio is net short at that point and loses when that rate falls.
- Because the tents add up to a parallel shift, the K R zero ones add up to the ordinary dollar value of a basis point, or D V zero one.
- One caution.
- The individual K R zero ones depend on which key rates and which interpolation you picked.
- Change those and the split changes, but the sum does not.

11 Splitting a cash flow between key rates

KR01 piece = PV × t × 0.0001
Weight on lower key = (Upper − t) ÷ (Upper − Lower)

PV	Present value of the cash flow
Years to the cash flow	Years until it is paid
Distance weights	Nearer key gets the larger weight
KR01 by key rate	Add pieces across all flows

IN THE LESSON

- To compute K R zero ones, treat the portfolio as a set of cash flows.
- A cash flow with present value P V, due in t years, gains about P V times t times point zero zero zero one when its own rate falls one basis point, with continuous compounding.
- If t sits exactly on a key maturity, all of that goes to that key rate.
- If t sits between two key maturities, split the amount by distance.
- The nearer key rate gets the bigger share.
- The weight on the lower key is the upper key minus t, over the gap between the keys, and the upper key gets the rest.
- The two weights always add to one.

12 Example 2: splitting each cash flow

Keys 2, 5, 10. Cash flow 1: PV $1,000,000 due in 3 years

Total of flow 1	1,000,000 × 3 × 0.0001	**$300**
Weights at 2y and 5y	(5−3)÷3 and (3−2)÷3	**2/3, 1/3**
Flow 1 split	$200 to 2y, $100 to 5y	**200, 100**
Flow 2: $2,000,000 at 5y	2,000,000 × 5 × 0.0001	**$1,000**
Flow 2 split	all on the 5y key	**1,000**

IN THE LESSON

- Take a portfolio with three cash flows and key rates at two, five and ten years.
- First, a long position with present value of one million dollars, due in three years.
- Its total is one million times three times point zero zero zero one, which is three hundred dollars.
- Three sits between two and five, and is one third of the way along.
- So the two year key gets two thirds, two hundred dollars, and the five year key gets one third, one hundred dollars.
- Second, two million dollars due at five years: two million times five times point zero zero zero one is one thousand dollars, and all of it goes to the five year key.

13 Example 3: the whole portfolio

Flow 3: PV −$500,000 due in 8 years, a short position

Total of flow 3	−500,000 × 8 × 0.0001	**−$400**
Split 5y and 10y	2/5 and 3/5	**−160, −240**
KR01s: 2y, 5y, 10y	200 | 100+1000−160 | −240	**200, 940, −240**
Sum of KR01s	200 + 940 − 240	**900**
Check: DV01	300 + 1000 − 400	**900**

IN THE LESSON

- The third cash flow is a short position, present value of minus five hundred thousand dollars, due in eight years.
- Its total is minus five hundred thousand times eight times point zero zero zero one, which is minus four hundred dollars.
- Eight lies between five and ten.
- Its distance from five is three of the five years, so the five year key gets two fifths, minus one hundred and sixty, and the ten year key gets three fifths, minus two hundred and forty.
- Now add by key rate.
- Two year: two hundred.
- Five year: one hundred plus one thousand minus one hundred and sixty is nine hundred and forty.
- Ten year: minus two hundred and forty.
- The total is nine hundred, matching the D V zero one.

14 Hedging key rate risk

- One hedge per key rate; every KR01 must reach zero
- Each instrument touches several keys, not just its own
- Each key: portfolio KR01 + positions × KR01s = 0
- Solve all the equations together

IN THE LESSON

- Once we know the K R zero ones, we can hedge them.
- The idea is to pick one hedging instrument for each key rate and choose positions so that the hedged portfolio has a K R zero one of zero at every key rate.
- The complication is that an instrument usually has exposure at several key rates, not only its own.
- A five year coupon bond, for example, pays coupons at one, two, three and four years as well, so it also feels the shorter key rates.
- That means we cannot hedge one key rate at a time without disturbing the others.
- Instead we write one equation for each key rate and solve them together.

15 The hedging instruments

Stylised KR01 per $1 million of face, at the 2y, 5y and 10y keys

Instrument	KR01 at 2y | 5y | 10y
2-year bond	200 | 0 | 0
5-year bond	40 | 480 | 0
10-year bond	20 | 80 | 960
Portfolio to hedge	200 | 940 | −240

IN THE LESSON

- Here are three hedging instruments, with the K R zero one of one million dollars of face value at each key rate.
- The two year bond only has exposure at the two year key.
- The five year bond has exposure at two and five.
- The ten year bond touches all three keys, a little at two, more at five, and mostly at ten.
- This triangular pattern is common, and it makes the solving easy, because we can start with the longest instrument, which is the only one that reaches the ten year key, and work downwards.
- The portfolio we are hedging is the one from the last example: two hundred, nine hundred and forty, and minus two hundred and forty.

16 Example 4: solving for the hedge

Work from the longest instrument down; positions in millions of face

10y key	−240 + 960 × x10 = 0	**buy 0.25m**
5y key	940 + 80×0.25 + 480 × x5 = 0	**sell 2.0m**
2y key	200 + 5 − 80 + 200 × x2 = 0	**sell 0.625m**
Result	all three KR01s are now	**zero**

IN THE LESSON

- Start at the ten year key, where only the ten year bond helps.
- The portfolio has minus two hundred and forty, so we need plus two hundred and forty from the hedge.
- Each million of the ten year bond gives nine hundred and sixty, so we buy a quarter of a million.
- Next, the five year key.
- The portfolio has nine hundred and forty, and the ten year position adds a quarter times eighty, which is twenty.
- That makes nine hundred and sixty to cancel.
- The five year bond gives four hundred and eighty per million, so we sell two million.
- Last, the two year key: two hundred, plus five from the ten year bond, minus eighty from the five year bond, is one hundred and twenty five.
- Sell six hundred and twenty five thousand of the two year bond.

17 Common hedging errors

- Do not ignore cross exposures at shorter key rates
- Positive portfolio KR01: short the positive-KR01 hedges
- Divide by KR01 per unit of face; keep the unit straight
- Check: every key rate should net to zero

IN THE LESSON

- Three mistakes appear again and again.
- The first is ignoring cross exposures, treating each instrument as if it touched only its own key rate.
- That leaves residual risk at the shorter keys.
- The second is the sign.
- A positive portfolio K R zero one means you gain when the rate falls, so you need a short position in instruments with positive K R zero ones, and a negative one needs a long position.
- The third is the unit of face.
- Positions come out as face amounts, found by dividing by the instrument's K R zero one per unit of face, so be sure whether that is per dollar, per hundred, or per million before you answer.

18 Combining key rates with P C A

- Express each factor as a bp move at every key rate
- Factor exposure = sum of KR01 × that factor's move
- Factors are uncorrelated: no covariance terms needed
- So the factor risks simply add up as squares

IN THE LESSON

- Now we bring the two tools together to estimate a portfolio's volatility.
- Key rates tell us the exposure to each part of the curve.
- P C A tells us how the curve actually moves, in terms of a few uncorrelated factors.
- The bridge is simple.
- For each factor, work out the basis point move it causes at every key rate when that factor moves by one standard deviation.
- Then the portfolio's exposure to that factor is the sum, over the key rates, of the K R zero one times the move.
- Because the factors are uncorrelated, there are no covariance terms to worry about.
- That is the great convenience of using P C A.

19 Portfolio volatility from factor exposures

Exposure of factor = Σ KR01 × move at that key rate
Volatility = √(sum of squared exposures)

KR01 by key rate	Gain per bp fall, each key rate
Basis point move per key rate	Move for a one sigma factor shock
Exposure to a factor	Dollar change per one sigma shock
Portfolio volatility	Root of summed squares

IN THE LESSON

- The recipe has two steps.
- First, the exposure to each factor is the sum over key rates of the K R zero one times the basis point move of that factor at that key rate, for a one standard deviation move.
- Second, portfolio variance is the sum of the squared exposures, and volatility is the square root of that.
- If the loadings are given as unit length eigenvectors instead, first multiply each exposure by that factor's own standard deviation, then square.
- To turn a daily volatility into a value at risk, multiply by a z value and by the square root of the number of days in the horizon.

20 Example 5: portfolio volatility

Daily one-sigma moves in bp at 2y, 5y, 10y: level 6,6,6; slope −4,0,4; curvature 2,−4,2

Level exposure	900 × 6	**5,400**
Slope exposure	200×(−4) + 0 + (−240)×4	**−1,760**
Curvature exposure	400 − 3,760 − 480	**−3,840**
Variance	$5{,}400^2 + 1{,}760^2 + 3{,}840^2$	**47,003,200**
Volatility	√47,003,200	**$6,856**

IN THE LESSON

- Use the portfolio with K R zero ones of two hundred, nine hundred and forty, and minus two hundred and forty.
- Suppose a one standard deviation move in level is six basis points at every key rate, slope is minus four, zero and plus four, and curvature is plus two, minus four and plus two.
- Level exposure: the K R zero ones sum to nine hundred, times six, is five thousand four hundred dollars.
- Slope exposure is minus eight hundred minus nine hundred and sixty, which is minus one thousand seven hundred and sixty.
- Curvature exposure is four hundred minus three thousand seven hundred and sixty minus four hundred and eighty, which is minus three thousand eight hundred and forty.
- Squaring and adding gives about forty seven million, and the square root is about six thousand eight hundred and fifty six dollars.

21 Example 6: zero DV01 is not riskless

KR01s 300, 0, −300 at the 2y, 5y, 10y keys; same factor moves

DV01	300 + 0 − 300	**0**
Level exposure	0 × 6	**0**
Slope exposure	300×(−4) + (−300)×4	**−2,400**
Curvature exposure	300×2 + (−300)×2	**0**
Volatility	$\sqrt{(0 + 2{,}400^2 + 0)}$	**$2,400**

IN THE LESSON

- Now a warning.
- Consider a portfolio with K R zero ones of three hundred at two years, nothing at five years, and minus three hundred at ten years.
- The D V zero one is three hundred minus three hundred, which is zero.
- A parallel shift does nothing, and the level exposure is zero.
- It looks perfectly hedged.
- But apply the slope factor, minus four at two years and plus four at ten years.
- The exposure is three hundred times minus four plus minus three hundred times four, which is minus two thousand four hundred dollars.
- The curvature exposure cancels to zero, so the volatility is two thousand four hundred dollars.
- Using only the level factor would have said zero, which understates the risk badly.

22 Example 7: from volatility to V a R

Daily volatility $6,856; 95% z-value 1.645; 10-day horizon

Square root of days	√10	**3.162**
Scale factor	1.645 × √10	**5.2019**
Ten-day V a R	6,856 × 5.2019	**$35,664**

IN THE LESSON

- Finally, turn the daily volatility from Example five into a value at risk.
- Suppose we want a ten day horizon at ninety five per cent confidence, so the z value is one point six four five.
- The square root of ten is about three point one six two.
- Multiply them: one point six four five times three point one six two is about five point two zero.
- Then multiply by the daily volatility of six thousand eight hundred and fifty six dollars.
- The answer is about thirty five thousand six hundred and sixty four dollars.
- Read it as: about one ten day period in twenty should lose more than that much, under these assumptions of normal factor moves.

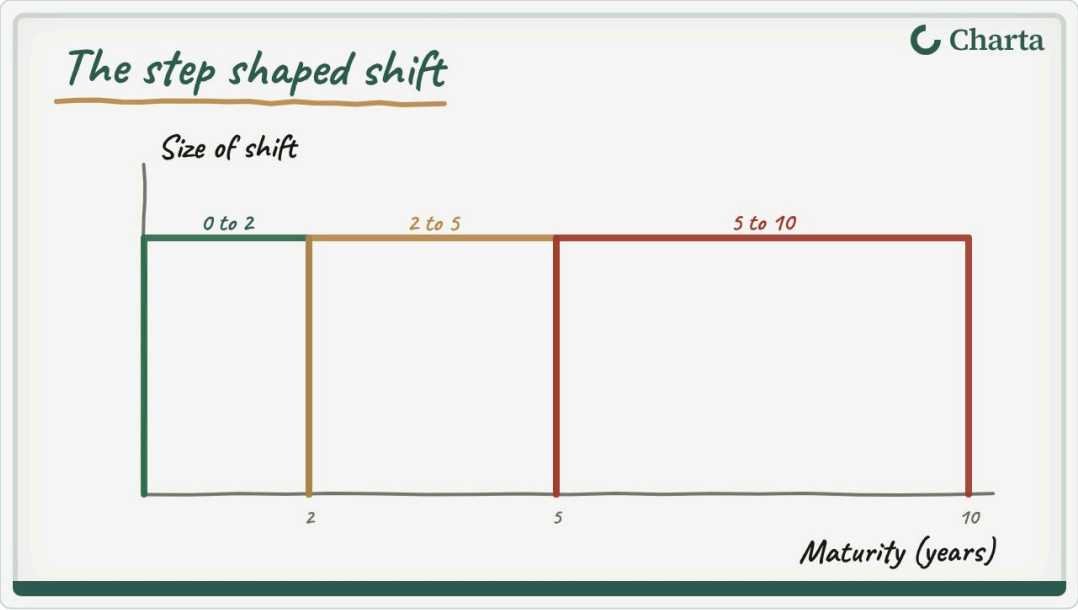
23 Buckets: a different way to cut the curve

- Split the curve into non-overlapping maturity buckets
- Shift the forward rates in one bucket by 1 basis point
- Result is a step shaped shift, not a tent
- The forward bucket 01s sum to the DV01

IN THE LESSON

- The key rate approach spreads each shift across neighbouring key rates in tents.
- The bucketing approach cuts the curve differently.
- It splits the curve into adjacent ranges that do not overlap, such as zero to two years, two to five years, and five to ten years.
- Then it shifts the forward rates inside one bucket by one basis point, and leaves every other forward rate alone.
- A forward rate is the rate for borrowing over a future period, for example the rate from year two to year five.
- The resulting shift is a step, not a tent.
- The change in portfolio value is called the forward bucket zero one, and the forward bucket zero ones also add up to the D V zero one.

24 The step shaped shift



IN THE LESSON

- Compare this chart with the tents.
- Here the three buckets are zero to two years, two to five years, and five to ten years.
- In the first bucket, all the forward rates fall by one basis point, and nothing else moves.
- In the second bucket the forward rates from two to five years fall, and the rest stay where they are.
- The third bucket works the same way.
- The shifts are flat steps with sharp edges, and they never overlap.
- If you laid the three steps side by side, they would add up to a single flat shift of one basis point at every maturity, which is the parallel shift again.
- So the sum rule holds here too.

25 Forward bucket 01 of a zero coupon bond

FB01 = PV × 0.0001 × years of the bucket before maturity

PV	Present value of the bond
Years of bucket before maturity	Zero if the bucket starts after it
Forward bucket 01	Gain per bp fall in the bucket

IN THE LESSON

- For a zero coupon bond with continuous compounding there is a neat shortcut.
- A zero coupon bond pays one amount at its maturity and nothing before.
- Its value depends on the sum of all the forward rates up to maturity.
- So a one basis point fall in the forward rates of a bucket raises the value by the present value, times point zero zero zero one, times the number of years of that bucket which fall before the bond's maturity.
- A bucket that lies entirely after maturity contributes nothing.
- A bucket that the maturity cuts through contributes only the part before maturity.
- A bucket completely before maturity contributes its full width.

26 Example 8: buckets for a seven year zero

Zero coupon bond, PV $1,000,000, matures in 7 years

Bucket 0 to 2	1,000,000 × 0.0001 × 2	**$200**
Bucket 2 to 5	1,000,000 × 0.0001 × 3	**$300**
Bucket 5 to 10	1,000,000 × 0.0001 × 2	**$200**
Sum of buckets	200 + 300 + 200	**$700**
Check: DV01	1,000,000 × 7 × 0.0001	**$700**

IN THE LESSON

- Take a zero coupon bond with a present value of one million dollars that matures in seven years.
- Use buckets zero to two, two to five, and five to ten.
- The first bucket lies fully before maturity, two years, so its forward bucket zero one is one million times point zero zero zero one times two, which is two hundred dollars.
- The second bucket, three years wide, also lies fully before maturity, giving three hundred.
- The third bucket runs from five to ten, but maturity cuts it at seven, so only two years count, giving two hundred.
- The sum is seven hundred dollars, which equals one million times seven times point zero zero zero one, the D V zero one.

27 Buckets compared with key rates

Same bond, PV $1,000,000, maturity 7 years

Method	Exposure, $ per bp
KR01 at 2y | 5y | 10y	0 | 420 | 280
Bucket 0-2 | 2-5 | 5-10	200 | 300 | 200
Sum of either method	700 = DV01
Forward start, years 2 to 5	only the 2-5 bucket

IN THE LESSON

- Now compare the two on the same bond.
- With key rates at two, five and ten, the seven year zero sits between the five and ten keys, two fifths of the way along.
- The five year key gets three fifths, four hundred and twenty dollars, and the ten year key gets two fifths, two hundred and eighty.
- Both methods sum to seven hundred, but the pictures differ.
- The key rates spread the exposure over neighbouring keys, so a seven year bond shows risk at five and at ten.
- The buckets show it where the forward rates actually matter.
- A forward start position, for instance one that runs only from year two to year five, shows risk only in the two to five bucket.

28 From a KR01 to a duration

Key rate duration = KR01 × 10,000 ÷ Market value
Dollar duration = KR01 × 10,000

KR01 by key rate	Gain from a 1 bp fall
Market value	Net market value, not face
Key rate duration	Percent change per 1 point

IN THE LESSON

- A K R zero one or a forward bucket zero one is a change in value per basis point.
- A duration is the percentage change in value for a one percentage point change in rate.
- To convert, divide the K R zero one by the portfolio's market value and scale to a one unit change in rate, which means multiplying by ten thousand.
- That gives the key rate duration.
- The forward bucket duration is found the same way.
- Multiplying the K R zero one by ten thousand, before dividing by value, gives the dollar duration.
- A duration of two means the value rises about point zero two per cent when that rate falls one basis point.

29 Example 9: durations of the portfolio

Net market value = 1,000,000 + 2,000,000 − 500,000 = $2,500,000

2y key rate duration	200 × 10,000 ÷ 2,500,000	**0.80**
5y key rate duration	940 × 10,000 ÷ 2,500,000	**3.76**
10y key rate duration	−240 × 10,000 ÷ 2,500,000	**−0.96**
Sum of durations	0.80 + 3.76 − 0.96	**3.60**
Effective duration	900 × 10,000 ÷ 2,500,000	**3.60**

IN THE LESSON

- Return to the portfolio from Examples two and three.
- Its present values are one million, two million and minus half a million, so the net market value is two and a half million dollars.
- Not the face value, and not the long side alone.
- The key rate durations are the K R zero ones times ten thousand over two and a half million.
- Two year: two hundred times ten thousand is two million, over two and a half million, point eight.
- Five year: nine hundred and forty gives three point seven six.
- Ten year: minus two hundred and forty gives minus point nine six.
- They add to three point six, which equals nine hundred times ten thousand over two and a half million, the effective duration.

30 Mistakes to avoid

- Split by distance; the sums must match the DV01
- Hedge: solve the system, mind sign and unit of face
- Volatility: add squares; zero DV01 is not zero risk
- Durations: divide by net market value, not face

IN THE LESSON

- Gather the main traps.
- In P C A, remember that the factors are found from data and describe history, not every portfolio.
- With K R zero ones, split a cash flow by distance and give the nearer key the larger weight, never all to the nearest key, and never with the weights reversed.
- As a check, the pieces must add to the D V zero one.
- In hedging, solve the whole system, mind the sign, and mind the unit of face.
- In volatility, add squares of factor exposures, not the exposures themselves, and do not trust a zero D V zero one.
- In durations, divide by market value, net for longs and shorts, not by face value.

31 Recap: the main ideas

- P C A: level, slope, curvature; share = eigenvalue ÷ total
- KR01: tent shift, sums to DV01, split by distance
- Hedge: one instrument per key; solve from the longest down
- Volatility: root of summed squared factor exposures
- Buckets use steps; duration = zero-ones × 10,000 ÷ value

IN THE LESSON

- Let us pull the lesson together.
- A few factors, level, slope and curvature, drive the curve, and P C A finds them from data, ranked by the share of variance each explains.
- A K R zero one measures the value change from a one basis point fall in one key rate, with tent shaped interpolation, and the K R zero ones sum to the D V zero one.
- They are found by splitting each cash flow by distance.
- To hedge them we solve a system with one instrument per key rate.
- With P C A, exposures to uncorrelated factors give volatility and V a R.
- Forward bucket zero ones use steps instead of tents, and dividing by market value turns either into a duration.

32 Try three quick questions

1. Eigenvalues 60, 30, 10. Share of the second factor?
   Answer: 30 ÷ 100 = 30%.
2. PV $200,000 due in 4 years, keys at 2 and 6. Total and split?
   Answer: 200,000 × 4 × 0.0001 = $80; halfway, so $40 to each key.
3. KR01 $300, net market value $1,000,000. Key rate duration?
   Answer: 300 × 10,000 ÷ 1,000,000 = 3.

IN THE LESSON

- Now try three quick questions yourself.
- Pause the video after each question, think, then listen to the answer.
- Question one.
- Eigenvalues are sixty, thirty and ten.
- What share does the second factor explain?
- Thirty over one hundred, which is thirty per cent.
- Question two.
- A cash flow with present value of two hundred thousand dollars is due in four years, with keys at two and six.
- What is its total gain from a one basis point fall in its own rate, and how is it split?
- Two hundred thousand times four times point zero zero zero one is eighty dollars, forty to each key, since four is halfway.
- Question three.
- A K R zero one of three hundred dollars on a net value of one million dollars.
- What is the key rate duration?
- Three hundred times ten thousand over one million, which is three.