Option Sensitivity Measures: The Greeks

FRM Part I

1 A story: the insurance seller

- Option writer collects a premium, takes on an obligation
- Profit is capped at the premium
- Loss can be large
- Question: how bad, and how do we hedge it?

IN THE LESSON

- Think of an insurance company.
- It sells a policy, collects the premium, and hopes nothing bad happens.
- If a flood arrives, it pays out.
- A person who sells, or writes, an option is in the same seat.
- The buyer pays a fee, called the premium, and gets a right.
- The writer takes on an obligation, and if the market moves the wrong way the writer pays.
- So a sensible writer asks how badly it can go wrong, and how to reduce that danger.

2 Naked and covered positions

- Naked: written option, nothing else held
- Naked call: loss unlimited in theory
- Naked put: loss capped at strike minus premium
- Covered call: long stock plus short call

IN THE LESSON

- A naked option position is a written option with nothing else held to protect it.
- A covered position pairs the written option with the underlying asset.
- Start with the naked call.
- The writer must deliver the stock at the strike price if the buyer asks.
- If the stock soars, the writer must buy at the market price and deliver at the strike.
- A stock can rise without any limit, so the loss is, in theory, unlimited.
- The naked put is different.
- The stock cannot fall below zero, so the loss is large but capped at the strike price minus the premium.
- A covered call means owning the stock and writing a call on it.
- The shares are there to hand over, so a rally no longer hurts.

3 What each position can lose or gain

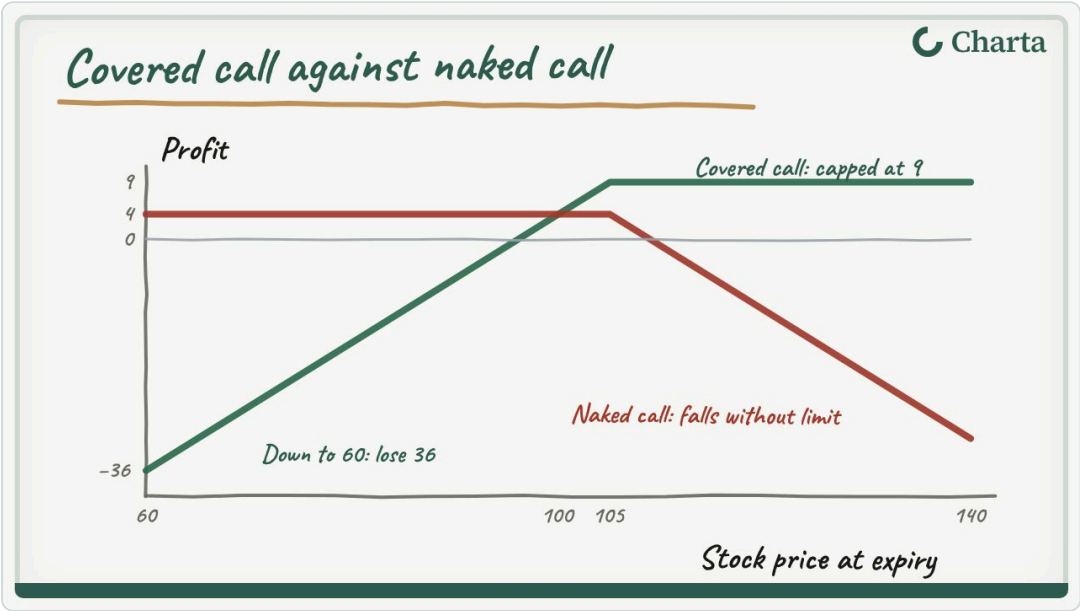
Four positions, and the worst thing that can happen to each

Position	Worst case
Naked call	Loss unlimited in theory
Naked put	Loss up to strike less premium
Covered call: rally	Capped at strike plus premium
Covered call: fall	Almost the whole stock loss

IN THE LESSON

- Here are the four cases side by side.
- The naked call writer earns at most the premium, and the loss grows without limit as the stock rises.
- The naked put writer also earns at most the premium, and the loss is bounded because the stock can only fall to zero.
- The covered call does something interesting.
- Your proceeds are capped, at the strike price plus the premium you received, because the shares will be called away.
- But you still own the stock, so you carry almost the full fall in the stock price, cushioned only by the premium.
- The most common mistake is to call a covered call risk free.
- Covering removes the rally risk.
- It does not remove the market risk.

4 Covered call against naked call



IN THE LESSON

- Here is the profit at expiry.
- Along the bottom is the stock price at the end, and up the side is profit.
- Assume we bought the stock at one hundred and wrote a call with a strike of one hundred and five for a premium of four.
- The green line is the covered call.
- It rises with the stock until one hundred and five, then goes flat at a profit of nine, because the shares are handed over.
- On the left it slopes down steeply: a fall to sixty loses thirty six.
- The red line is the naked call.
- It earns four if the stock stays low, then plunges as the stock rises.
- Notice the green shape: flat on the right, sloping on the left.
- By put call parity, that is a short put.

5 The stop-loss idea

- Written call: buy the stock when price rises above K
- Sell it when price falls back below K
- Covered when in the money, naked when out
- Attraction: simple, needs no volatility estimate

IN THE LESSON

- Suppose you have written a call and want protection without paying for another option.
- The simplest plan is called a stop loss strategy.
- When the stock price rises above the strike, you buy the stock.
- When it falls back below the strike, you sell it.
- The logic is neat.
- If the call ends in the money, you already own the stock and can hand it over.
- If it ends out of the money, you own nothing and keep the premium.
- So you are covered when the option is in the money and naked when it is out of the money.
- The big attraction is simplicity: you do not need any estimate of volatility, and the rule fits on one line.

6 Example 1: the ideal stop-loss cost

Strike 50, stock 52, 1 year, vol 20%, rate 5%. Call price about 6.57

Buy stock at the start	In the money, so cover	**Pay 52**
Deliver or sell at the strike	Receive 50	**Get 50**
Ideal cost	max(52 − 50, 0)	**2**
Option price received	From the model	**6.57**

IN THE LESSON

- In the perfect world, what does the hedge cost?
- Take a call with a strike of fifty, and a stock now at fifty two, so the call is already in the money.
- The rule says to buy the stock straight away, at fifty two.
- If the stock ends above fifty, you hand it over for fifty and lose two on the exchange.
- If it ends below fifty, you sell it at fifty and again lose two.
- In both cases the cost is two.
- In general the ideal cost is the larger of the start price minus the strike, and zero.
- Using a standard pricing model, with a volatility of twenty per cent and a rate of five per cent, the call is worth about six point five seven.
- So the ideal cost of two is well below the price received.

7 Why stop-loss disappoints

- Cannot trade exactly at K: slippage
- Discrete monitoring and interest add error
- Price hovering near K: repeated losing trades
- Easy but unreliable: delta hedging replaces it

IN THE LESSON

- So why is the real strategy poor?
- The answer is that you can never trade exactly at the strike.
- The price jumps, and you only trade once you see that it has crossed, so you buy a little above and sell a little below.
- That gap is called slippage.
- Checking the price only at intervals adds more error.
- Worse, if the price hovers around the strike, the rule keeps telling you to buy and sell, and every round trip loses a little.
- The result is a hedging cost that is uncertain and often large, with results that vary widely.
- The lesson for the exam: stop loss is easy but unreliable, and delta hedging replaces it.

8 What delta means

- Delta = change in option price per small change in stock
- Call delta between 0 and 1
- Put delta between −1 and 0
- Delta is also the hedge ratio, in shares

IN THE LESSON

- Now the first and most important Greek.
- Delta answers this question: if the stock price moves by one small unit, how much does the option price move?
- Picture a call option with a delta of point six.
- If the stock rises by one dollar, the call gains about sixty cents.
- Delta is therefore the slope of the option price against the stock price.
- For a call it lies between zero and one.
- For a put it lies between minus one and zero, because a put gains when the stock falls.
- Delta is also a hedge ratio.
- An option with a delta of point six behaves like point six of a share, for small moves.
- That is the key to hedging.

9 Delta of a European option

Δcall = e^(−qT) · N(d₁)
Δput = e^(−qT) · [N(d₁) − 1]

Delta	Call delta, between 0 and 1
Dividend factor	Dividend factor, e^(−qT)
d one	Needs r − q in place of r
Put delta	N(d1) − 1, times dividend factor

IN THE LESSON

- For a European call on a stock that pays no dividends, delta equals N of d one.
- Here N is the cumulative normal probability, a number between zero and one.
- The quantity d one comes from the standard pricing model: the log of the stock price over the strike, plus the interest rate plus half the variance, all times the time to expiry, divided by the volatility times the square root of time.
- The put delta is the call delta minus one.
- With a continuous dividend yield, replace the rate by the rate minus the yield inside d one, and multiply each delta by a dividend factor, e to the minus the yield times time.
- Do not confuse N of d one with N of d two, which is the risk neutral chance of exercise.

10 Example 2: delta of a call and a put

S 100, K 100, r 5%, volatility 20%, T 1 year, no dividends

d one	(0 + 0.05 + 0.02) ÷ 0.20	**0.35**
N of d one	From the normal table	**0.6368**
Call delta	N(d1)	**0.6368**
Put delta	0.6368 − 1	**−0.3632**

IN THE LESSON

- Let's compute it.
- A stock trades at one hundred, the strike is one hundred, the rate is five per cent, volatility twenty per cent, one year to expiry, and no dividends.
- First, d one.
- The log of one hundred over one hundred is zero.
- Add the rate plus half the variance: point zero five plus point zero two, which is point zero seven.
- Divide by point two times the square root of one, which is point two.
- That gives d one of point three five.
- Next, N of point three five from the normal table is point six three six eight.
- That is the call delta.
- The put delta is that minus one: minus point three six three two.
- Notice the put delta is negative.
- And the answer is not N of d two, which would be point five five nine six.

11 Example 3: delta with a dividend yield

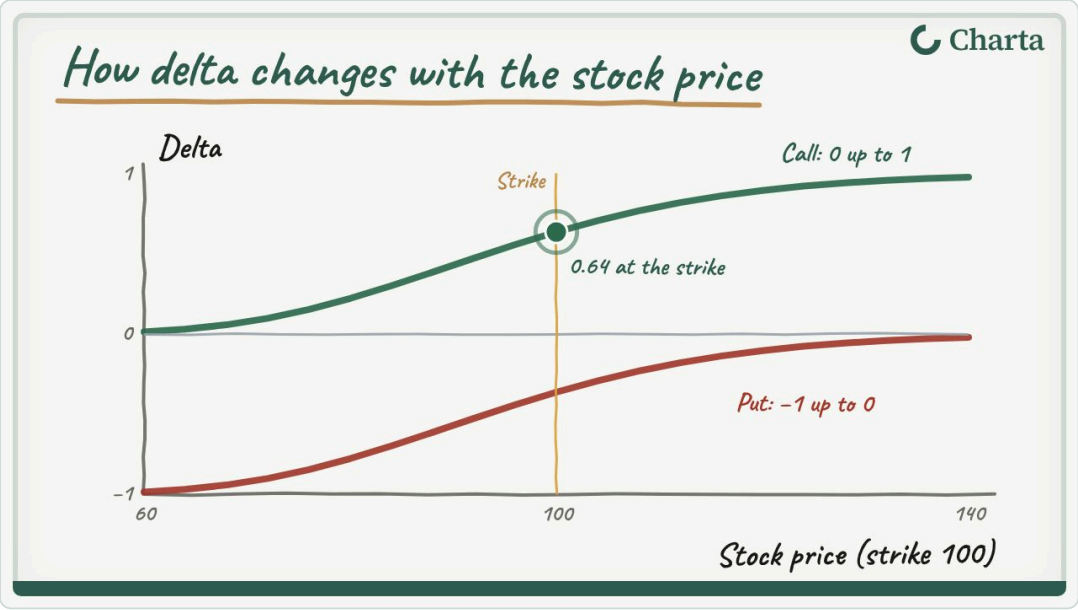
Same inputs, plus a dividend yield of 2%

d one	(0.03 + 0.02) ÷ 0.20	**0.25**
Dividend factor	e^(−0.02)	**0.9802**
Call delta	0.9802 × 0.5987	**0.5869**
Put delta	0.9802 × (0.5987 − 1)	**−0.3933**

IN THE LESSON

- Now add a continuous dividend yield of two per cent, with everything else unchanged.
- Two changes.
- First, d one now uses the rate minus the yield: point zero five minus point zero two, which is point zero three, plus point zero two, giving point zero five.
- Divided by point two, d one is point two five.
- Second, N of point two five is point five nine eight seven.
- Third, the dividend factor, e to the minus point zero two, is point nine eight zero two.
- The call delta is point nine eight zero two times point five nine eight seven, equal to point five eight six nine.
- The put delta is point nine eight zero two times the bracket, point five nine eight seven minus one, which is minus point three nine three three.
- A dividend lowers a call's delta.

12 How delta changes with the stock price



IN THE LESSON

- This picture shows delta against the stock price, for a call and a put with a strike of one hundred.
- The green curve is the call.
- When the stock is far below the strike, the call is nearly worthless and delta is close to zero.
- As the price approaches the strike, delta climbs, and at the strike it is about point six four.
- Far above it, delta approaches one.
- The red curve is the put, the same shape shifted down by one, running from minus one on the left to zero on the right.
- That bend, the fact that delta itself changes, is why hedging needs constant attention.

13 Delta hedging: the idea

- Hold the underlying in proportion to delta
- Short calls: buy delta shares per option
- Short puts: sell shares (a short put has positive delta)
- Small price moves then cancel out

IN THE LESSON

- Now we can do better than stop loss.
- A delta hedge makes a position insensitive to small price moves.
- Say you have written ten thousand calls, each with a delta of point six four.
- Together they behave like short six thousand four hundred shares, so you buy that many shares.
- Now a rise in the stock loses money on the calls and earns the same on the shares.
- A trader who is short calls buys shares.
- A trader who is short puts does the opposite.
- A short put has positive delta, so the trader sells shares to cancel it.

14 Why the hedge must keep moving

- Delta moves with the price (gamma)
- Delta also moves with time, with no price move
- So the hedge is rebalanced again and again
- Short calls: buy after rises, sell after falls

IN THE LESSON

- There is a catch.
- Delta is not a constant.
- As we just saw, it changes with the stock price.
- That rate of change is called gamma.
- Delta also changes as time passes, even when the stock does not move, because the option drifts toward expiry.
- So a hedge that is right this morning is slightly wrong tonight.
- The trader must rebalance, either continuously in theory or at set intervals in practice.
- A trader short calls ends up buying after rises and selling after falls.
- That is a cost, and it is the price of protection.
- The common error is to treat the hedge as set and forget.

15 Example 4: the incremental trade

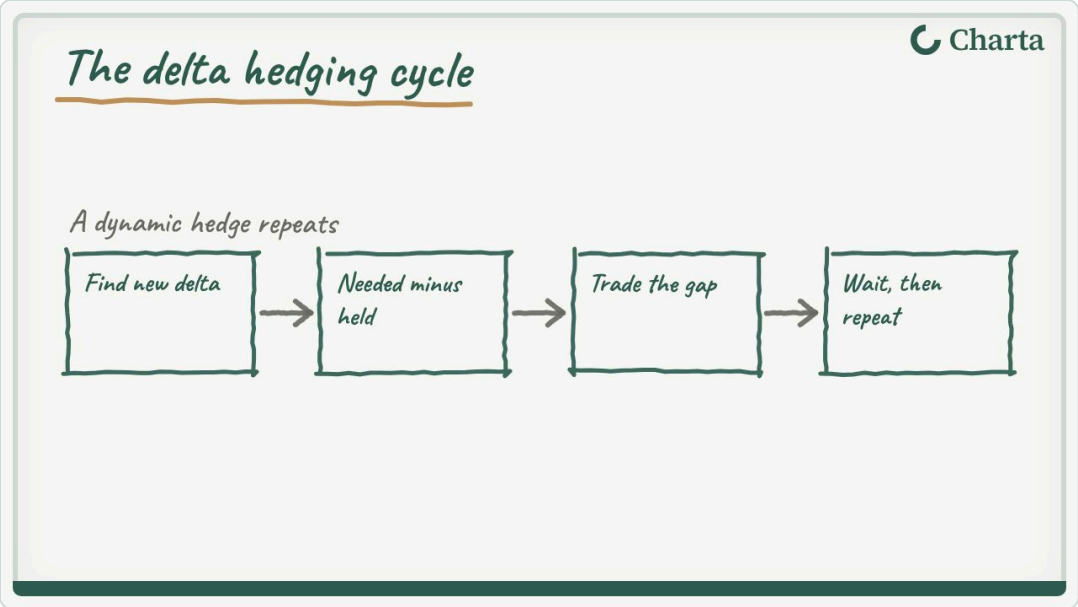
Short 10,000 calls. Stock 100, delta 0.6368. Next day stock moves

Opening hedge	10,000 × 0.6368	**Hold 6,368**
Stock up to 102	Delta 0.6732: target 6,732	**Buy 364**
Stock down to 98	Delta 0.5981: target 5,981	**Sell 387**
Rule	New holding − current	**Every day**

IN THE LESSON

- A dealer is short ten thousand calls, strike one hundred.
- At the start the stock is one hundred and delta is point six three six eight, so the dealer buys six thousand three hundred and sixty eight shares.
- The next day the stock closes at one hundred and two, and with one day less to run the delta is point six seven three two.
- The new target is six thousand seven hundred and thirty two shares.
- The trade is the new required holding minus the current holding: three hundred and sixty four more shares to buy.
- Suppose instead the stock had fallen to ninety eight.
- Delta would be point five nine eight one, the target five thousand nine hundred and eighty one, and the dealer would sell three hundred and eighty seven shares.

16 The delta hedging cycle



IN THE LESSON

- Read the boxes from left to right.
- First, calculate the delta of the option position with the latest stock price, volatility and time left.
- Second, compare the shares you need with the shares you hold.
- Third, trade just the difference, the incremental trade, and not the whole position.
- Then wait for the market to move or for time to pass, and the cycle starts again.
- That is why the hedge is called dynamic.
- In theory, with continuous trading and no costs, the hedge would be perfect.
- In practice, trades happen at intervals and carry costs, so a small leftover risk remains, and larger price jumps cause bigger errors.

17 Gamma, vega, theta and rho

- Gamma: how fast delta changes (same for call and put)
- Vega: sensitivity to volatility (same for call and put)
- Theta: time decay, normally negative when long
- Rho: interest rates, positive for calls, negative for puts

IN THE LESSON

- Delta is only the first Greek.
- Gamma is how fast delta changes when the stock moves, the curvature we saw on the chart.
- It is the same for a call and a put with the same strike and expiry.
- Vega is the sensitivity to volatility, also the same for calls and puts.
- Theta is time decay: the change in value as a day passes.
- For a long option it is normally negative, because the option loses time value.
- Rho is the sensitivity to the interest rate, positive for calls and negative for puts.
- A simple summary: long options have positive gamma and positive vega, and short options have the opposite.

18 Gamma and vega for a non-dividend stock

$\Gamma = N'(d_1) \div (S \cdot \sigma \cdot \sqrt{T})$
$\text{Vega} = S \cdot \sqrt{T} \cdot N'(d_1)$

Gamma	Same for calls and puts
Vega	Divide by 100 per point
Density	$N'(d_1)$: the bell curve height
Stock price	S: the stock price

IN THE LESSON

- Both formulas use the same ingredient, but it is easy to mistake.
- For gamma, take N prime of d one and divide by the stock price times the volatility times the square root of time.
- For vega, multiply the stock price by the square root of time and by N prime of d one.
- Now the trap.
- N prime is the normal density, the height of the bell curve, a quantity near point four at its peak.
- It is not N, the cumulative probability used for delta.
- Vega is usually quoted per one percentage point of volatility, so divide the raw figure by one hundred.

19 Example 5: gamma and vega

S 100, K 100, r 5%, volatility 20%, T 1, d one 0.35

Density $N'(0.35)$	From the bell curve	**0.3752**
Gamma	0.3752 ÷ (100 × 0.20 × 1)	**0.0188**
Vega, raw	100 × 1 × 0.3752	**37.52**
Vega per point	37.52 ÷ 100	**0.3752**

IN THE LESSON

- Use the same option: stock one hundred, strike one hundred, five per cent rate, twenty per cent volatility, one year, d one of point three five.
- The density at point three five is point three seven five two, found from the bell curve formula.
- Gamma is point three seven five two divided by one hundred times point two times one, which is twenty.
- That gives point zero one eight eight.
- A one dollar rise in the stock lifts delta by about point zero one nine.
- Vega is one hundred times one times point three seven five two, thirty seven point five two.
- Per percentage point, divide by one hundred: point three seven five two.
- So a rise in volatility from twenty to twenty one per cent adds about thirty seven cents to the option.

20 Long and short options compared

Signs of the Greeks by position (theta normally negative when long)

Position	Sign of each Greek
Long call	Δ + Γ + Vega + Θ −
Long put	Δ − Γ + Vega + Θ −
Short call	Δ − Γ − Vega − Θ +
Short put	Δ + Γ − Vega − Θ +

IN THE LESSON

- Here is a summary to carry into the exam.
- A long call has positive delta, positive gamma and positive vega, and negative theta.
- A long put has negative delta, positive gamma and positive vega, and also negative theta.
- Short positions flip every sign: a short call has negative delta, negative gamma and negative vega, with positive theta, and a short put has positive delta with the same gamma, vega and theta signs as the short call.
- Gamma and vega are the same for calls and puts, so only delta and rho differ in sign between them.

21 Delta-neutral and gamma-neutral

- Delta-neutral: total delta is zero
- Gamma-neutral: total gamma is zero
- Shares have zero gamma, so only options change gamma
- Step 1: options for gamma. Step 2: shares for delta

IN THE LESSON

- Delta hedging protects against small moves, but it still suffers when moves are larger, because delta changes.
- The cure is to hedge the gamma as well.
- A position is delta neutral when its total delta is zero.
- It is gamma neutral when its total gamma is zero, so the delta stays near zero even as the stock moves.
- Here is the problem: shares have a delta of one but a gamma of zero.
- Trading shares can never change your gamma.
- To fix gamma you need another option, a traded option, with gamma of its own.
- So the procedure has two steps, and the order matters.
- First deal with gamma using the option.
- Then repair delta using the shares.

22 The two-step recipe

Options = −Γp ÷ ΓT
Shares = −(Δp + Options · ΔT)

Options	Options to buy (+) or sell (−)
Portfolio gamma	Γp: gamma of the portfolio
Portfolio delta	Δp: delta of the portfolio
Shares	Shares to buy (+) or sell (−)

IN THE LESSON

- Step one.
- The number of traded options to buy is minus the portfolio gamma, divided by the gamma of one traded option.
- The minus sign just means we trade in the direction that cancels.
- If the portfolio has negative gamma, we buy options.
- Step two.
- The options we just added carry their own delta, so the portfolio delta has changed.
- The number of shares to buy is minus the old portfolio delta plus the number of options times the delta of one traded option, all taken together.
- The usual mistake is to size the share trade before adding the options.
- Do that and the final position is not delta neutral, because the option trade has moved delta after your share trade.

23 Example 6: building the neutral position

Portfolio delta −1,000 and gamma −3,000. Traded option: delta 0.62, gamma 1.5

Options to buy	−(−3,000) ÷ 1.5	**2,000**
Delta after the options	−1,000 + 2,000 × 0.62	**+240**
Shares to trade	−240	**Sell 240**
Final gamma and delta	Both zero for now	**0 and 0**

IN THE LESSON

- A portfolio has a delta of minus one thousand and a gamma of minus three thousand.
- A traded option has a delta of point six two and a gamma of one point five.
- Step one: options equals minus the portfolio gamma over the option gamma, which is three thousand over one point five, so buy two thousand options.
- Now the new delta.
- The options add two thousand times point six two, which is twelve hundred and forty.
- The portfolio delta moves from minus one thousand to plus two hundred and forty.
- Step two: shares equals minus that new delta, so sell two hundred and forty shares.
- The final position has zero gamma and zero delta.
- It is neutral only locally, and must be rebalanced.

24 The Greeks are tied together

- One equation links theta, delta and gamma
- Delta neutral: large gamma goes with large time decay
- Long gamma pays theta; short gamma collects it
- Vega is not in this equation

IN THE LESSON

- The Greeks are not independent.
- Every option price must satisfy one equation, the Black Scholes Merton differential equation.
- In Greek terms it says that theta equals the interest rate times the value, minus the interest rate times the stock price times delta, minus a half times volatility squared times the stock price squared times gamma.
- Read it as a trade off.
- This is the economic bargain behind every option: a holder who likes large moves must pay for them in time decay, and a writer who collects decay is exposed to large moves.

25 Theta, delta and gamma

Θ = rV − rSΔ − ½ σ^2 S^2 Γ
Delta-neutral, V ≈ 0: Θ ≈ −½ σ^2 S^2 Γ

Theta	Time decay per year
Volatility	σ squared, in annual terms
Stock price	S squared: price squared
Gamma	Γ: portfolio gamma

IN THE LESSON

- Here is the relationship, and the simplified version you will use most.
- In full: theta equals the rate times the portfolio value, minus the rate times the stock price times delta, minus a half times volatility squared times stock price squared times gamma.
- Now make the portfolio delta neutral, so the delta term vanishes.
- If the value is also close to zero, the first term drops out too.
- What remains is theta approximately equal to minus a half times volatility squared times stock price squared times gamma.
- The two exam slips: forgetting the one half, and using volatility rather than volatility squared.
- Use annual units, then divide by the number of days if a daily figure is wanted.

26 Example 7: theta from gamma

S 50, volatility 30% a year, gamma −2,000, delta neutral, value near zero

Half sigma squared	0.5 × 0.09	**0.045**
Times S squared	0.045 × 2,500	**112.5**
Theta per year	−112.5 × (−2,000)	**+225,000**
Theta per day	225,000 ÷ 365	**+616.44**

IN THE LESSON

- A delta neutral portfolio with a value close to zero is short gamma.
- The stock price is fifty, volatility is thirty per cent a year, and the portfolio gamma is minus two thousand.
- Use theta equals minus a half times volatility squared times price squared times gamma.
- Volatility squared is point zero nine.
- Price squared is two thousand five hundred.
- Multiply: point five times point zero nine times two thousand five hundred is one hundred and twelve point five.
- Times the gamma of minus two thousand, with the leading minus sign, gives plus two hundred and twenty five thousand per year.
- Divide by three hundred and sixty five for a daily figure: about six hundred and sixteen dollars a day.
- Short gamma earns a steady daily income, but it is the reward for taking the risk of a large move.

27 Greeks of a whole portfolio

Δp = Σ quantity × Δ (also Γp, Vega p)
ΔV ≈ Δ·dS + ½·Γ·dS² + Vega·dσ

Position sum	Long positive, short negative
Price move	dS: stock price move
Volatility move	dσ: volatility move, in points
Profit estimate	Estimated change in value

IN THE LESSON

- A trader seldom holds one option, so we need the Greeks of a portfolio.
- The rule is simple: add up quantity times the Greek, position by position.
- Long positions have positive quantity, short positions negative.
- A put has negative delta per contract, and the position is short, so it contributes positive delta.
- Shares have a delta of one each, with zero gamma and zero vega.
- Once you have the totals, you can estimate profit quickly.
- For a price move, the value changes by delta times the move, plus a half times gamma times the move squared.
- For a volatility move, it changes by vega times the move in volatility, in the same units as vega.

28 Example 8: a multi-position table

Long 4,000 calls, short 3,000 puts, 1,000 shares: delta/gamma/vega

Calls	× 0.55 / 0.04 / 0.20	**2,200/160/800**
Short puts	× −0.35 / 0.05 / 0.18	**1,050/−150/−540**
Shares	× 1 / 0 / 0	**1,000/0/0**
Portfolio total	Delta / gamma / vega	**4,250/10/260**

IN THE LESSON

- Let's build a portfolio.
- The trader is long four thousand calls, each with delta point five five, gamma point zero four and vega point two per point.
- The trader is short three thousand puts, each with delta minus point three five, gamma point zero five and vega point one eight.
- And the trader holds one thousand shares.
- Delta: four thousand times point five five is two thousand two hundred.
- Short three thousand puts times minus point three five is plus one thousand and fifty.
- Shares add one thousand.
- Total delta is four thousand two hundred and fifty.
- Gamma: one hundred and sixty minus one hundred and fifty, which is ten.
- Vega: eight hundred minus five hundred and forty, which is two hundred and sixty.
- Shares add nothing to gamma or vega.

29 Example 9: estimating the profit

Totals: delta 4,250, gamma 10, vega 260 per point

Up 2, volatility up 1	8,500 + 20 + 260	**+8,780**
Down 3, volatility flat	−12,750 + 45	**−12,705**
Gamma term, down 3	½ × 10 × 3²	**45**
Check of units	Vega per point × points	**Same units**

IN THE LESSON

- Now use the totals to estimate profit and loss.
- Suppose the stock rises two dollars and volatility rises one point.
- Delta part: four thousand two hundred and fifty times two, eight thousand five hundred.
- Gamma part: a half times ten times two squared, which is twenty.
- Vega part: two hundred and sixty times one, two hundred and sixty.
- The estimated gain is eight thousand seven hundred and eighty dollars.
- Now suppose the stock falls three dollars and volatility is unchanged.
- Delta part: four thousand two hundred and fifty times minus three, minus twelve thousand seven hundred and fifty.
- Gamma part: a half times ten times nine, which is forty five.
- The estimate is a loss of twelve thousand seven hundred and five.
- Dropping the one half doubles the gamma term.

30 Portfolio insurance

- Goal: a floor under an equity portfolio
- Replicate a put instead of buying one
- Sell a fraction equal to the put's delta
- Sell more as the market falls, buy back as it rises

IN THE LESSON

- Now a famous use of the same ideas.
- A fund manager holds a large equity portfolio and wants a floor under its value.
- The obvious route is to buy a put option, but suppose a suitable put is too costly or does not exist.
- Instead the manager creates the put by trading.
- A put has a negative delta, so the manager sells a fraction of the portfolio, or of index futures, equal to the put's delta.
- If the market falls, the put delta becomes more negative, so the manager sells more.
- If the market rises, the manager buys back.
- This is called portfolio insurance.
- It tries to copy the put through continuous trading, with no option bought.

31 Example 10: insurance by trading

Portfolio $100m, put delta −0.30. Market then falls 5%, put delta −0.45

Start: sell 30%	Stocks 70m, cash 30m	**70 / 30**
After a 5% fall	66.5m stocks + 30m cash	**96.5m**
New target in stocks	55% × 96.5m	**53.075m**
Further sale	66.5m − 53.075m	**13.425m**

IN THE LESSON

- A manager has a portfolio worth one hundred million dollars.
- The put being copied has a delta of minus point three, so the manager sells thirty per cent, thirty million, and holds the cash.
- Seventy million stays in stocks.
- Now the market falls five per cent.
- The stocks are worth sixty six point five million, so the portfolio is worth ninety six point five million.
- Suppose the put delta is now minus point four five.
- The manager should hold fifty five per cent in stocks, which is fifty three point zero seven five million.
- So the trade is to sell a further thirteen point four two five million.
- Falling prices produce selling, and rising prices would produce buying back.

32 Insurance versus delta hedging

Same trades, different purpose, same weakness

	Insurance | Delta hedge
Purpose	Floor on value | Cancel a liability
After a fall	Sell more | Sell more
After a rise	Buy back | Buy back
Both need	Liquid, continuous trading
Both fail in	Gaps and crashes

IN THE LESSON

- Portfolio insurance and delta hedging use the same trades.
- The dealer who sold a put and delta hedges it also sells as the market falls and buys as it rises.
- The difference is purpose.
- The insurer wants a floor on the portfolio value.
- The dealer wants only to cancel a liability.
- Both depend on continuous trading in liquid markets.
- And both can fail.
- In a sudden price gap or crash, the sales cannot happen at planned prices, so the floor is broken.
- Worse, when many insurers sell at once as prices drop, their selling can deepen the fall.
- The exam points are the direction of trading, selling in falls, and the failure under gaps.

33 Recap: the main ideas

- Naked call: unlimited loss. Covered call: market risk stays
- Delta = N(d1) for a call, N(d1) − 1 for a put
- Gamma and vega use the density N′(d1)
- Neutral: options for gamma, then shares for delta
- Portfolio Greeks are sums; insurance fails in a gap

IN THE LESSON

- Let's pull it together.
- Naked options can lose without limit for a call, and a covered call removes only the rally risk.
- Stop loss hedging is simple but unreliable, and delta hedging replaces it.
- Delta is the slope of the option price: N of d one for a call, minus one for a put, adjusted by a dividend factor.
- A delta hedge is dynamic, and the trade is the new holding minus the old.
- Gamma and vega use the density, and are the same for calls and puts.
- A delta and gamma neutral position needs options first, then shares.
- Theta, delta and gamma are tied together.
- Portfolio Greeks are sums.
- And portfolio insurance copies a put by trading, which fails in a crash.

34 Try three quick questions

1. N(d1) = 0.62, no dividends. Put delta?
   Answer: 0.62 − 1 = −0.38. Not N(d2), and not positive.
2. Short 2,000 calls, delta 0.45. Hedge?
   Answer: Buy 2,000 × 0.45 = 900 shares.
3. Portfolio gamma −1,200, option gamma 0.8. Options for gamma neutral?
   Answer: Buy 1,200 ÷ 0.8 = 1,500 options, then fix delta with shares.

IN THE LESSON

- Now try three quick questions.
- Pause the video, think, then listen.
- Question one.
- A European call on a non dividend stock has N of d one equal to point six two.
- What is the put delta?
- Point six two minus one, which is minus point three eight.
- Question two.
- A dealer is short two thousand calls with a delta of point four five.
- What is the hedge?
- Buy two thousand times point four five, which is nine hundred shares.
- Question three.
- A portfolio has gamma of minus one thousand two hundred, and an option has a gamma of point eight.
- How many options make it gamma neutral?
- Buy twelve hundred divided by point eight, which is fifteen hundred options, then fix delta with shares.