CHARTA LEARNING · LESSON NOTES

# Credit Derivatives

FRM Part II

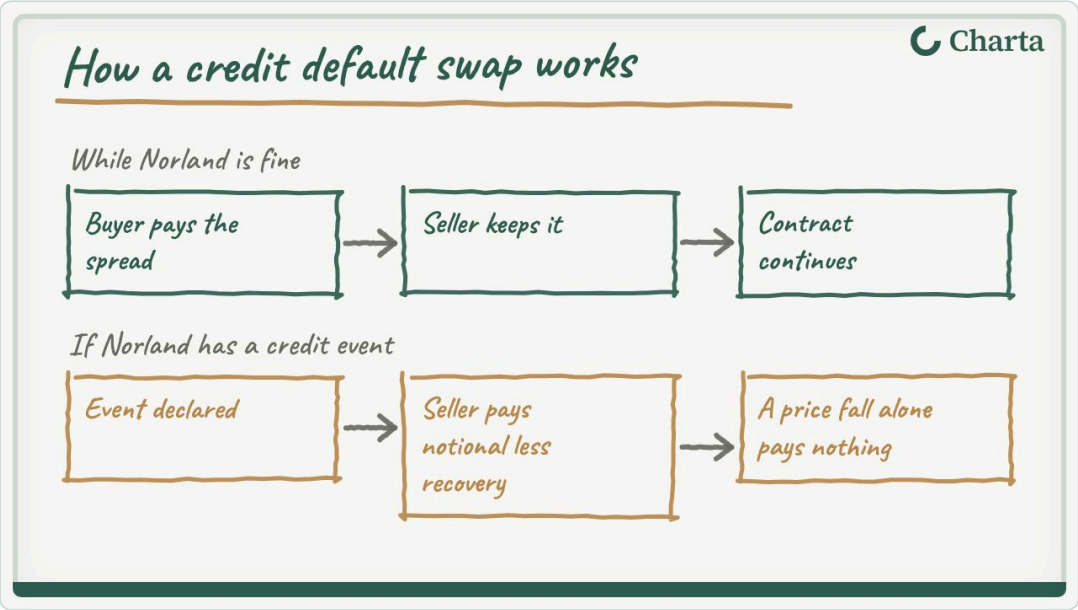
## 1 A story: Harbour Bank and Norland Steel

- Payoff depends on a reference entity's credit
- Moves credit risk without selling the loan or bond
- CDS: pays on a credit event
- TRS: passes on the whole return of an asset
- CDO: slices portfolio losses into tranches

IN THE LESSON

- Harbour Bank has lent twenty five million dollars to Norland Steel.
- The bank likes the customer and wants to keep the relationship, but it worries that too much of its money rides on one company.
- A credit derivative offers a way out.
- It is a contract whose payoff depends on the creditworthiness of a reference entity, here Norland Steel.
- So credit risk can be moved to someone else without selling the underlying loan or bond.
- Three contracts matter in this lesson.
- The credit default swap pays when the reference entity has a credit event.
- The total return swap passes on the whole return of an asset.
- The collateralised debt obligation slices up the losses of a whole portfolio.

## 2 How a credit default swap works




IN THE LESSON

- Start with the credit default swap, or C D S.
- Harbour Bank is the protection buyer.
- It pays a periodic spread, like an insurance premium, to a protection seller, say Atlas Insurance.
- If Norland Steel has a credit event, such as failing to pay or going bankrupt, the seller pays the buyer.
- The payout is the notional less the post default value of the reference obligation.
- Here is the point examiners love.
- A C D S pays on a credit event, not on a price fall.
- If Norland's bond drops in price because investors grow nervous, but no credit event occurs, the seller pays nothing.

## 3 The CDS payout

**Payout = (1 − R) × Notional**

| | |
|---|---|
| **Payout** | Cash paid on a credit event |
| **Recovery rate, R** | Post-default value / face |
| **Notional** | Face amount of protection bought |

IN THE LESSON

- Here is the payout.
- Let R be the post default value of the reference obligation, as a fraction of its face value.
- This is the recovery rate.
- The loss on each dollar is then one minus R.
- The payout is one minus R, multiplied by the notional, which is the face amount of protection bought.
- So the seller pays only the loss, not the full face value, because the buyer still owns something worth R on the dollar.
- If recovery is thirty five per cent, the seller pays sixty five cents on each dollar of notional.

## 4 Example 1: Harbour Bank's CDS

*Notional $25m, spread 1.6% a year, recovery 35% (illustration)*

| | | |
|---|---|---|
| **Annual premium** | 1.6% × 25 | **$0.4m** |
| **Bond value after the event** | 35% × 25 | **$8.75m** |
| **Loss on the bond** | 25 − 8.75 | **$16.25m** |
| **CDS payout** | 25 × (1 − 0.35) | **$16.25m** |
| **Price falls, no credit event** | no event, no payout | **$0** |

IN THE LESSON

- Harbour Bank buys twenty five million dollars of protection on Norland Steel at a spread of one point six per cent a year.
- The annual premium is point one six times twenty five, which is four hundred thousand dollars.
- Suppose Norland has a credit event and its bond is then worth thirty five cents on the dollar.
- The bond is worth eight point seven five million, so the bank has lost sixteen point two five million.
- The C D S pays twenty five times one minus point three five, which is exactly sixteen point two five million.
- The loss is covered.
- Now the trap.
- If instead Norland's bond price falls but there is no credit event, the payout is zero, however much the price has dropped.

## 5 The total return swap

- Receiver gets coupons plus price gains, and bears price falls
- Receiver pays the payer a funding rate plus a spread
- Transfers market risk as well as default risk
- Pays out with no default: a price fall is enough
- A CDS would pay nothing in that case

IN THE LESSON

- Now the total return swap, or T R S.
- The total return receiver gets the full return of a reference asset, such as a Norland bond: the coupons plus any price gains, and it also suffers any price falls.
- In exchange it pays the total return payer a funding rate plus a spread.
- Compare this with a C D S.
- A T R S passes on market risk as well as default risk.
- If the bond loses value because spreads widen, the receiver pays, even though nobody has defaulted.
- That is exactly the difference the exam tests.

## 6 Example 2: a TRS with no default

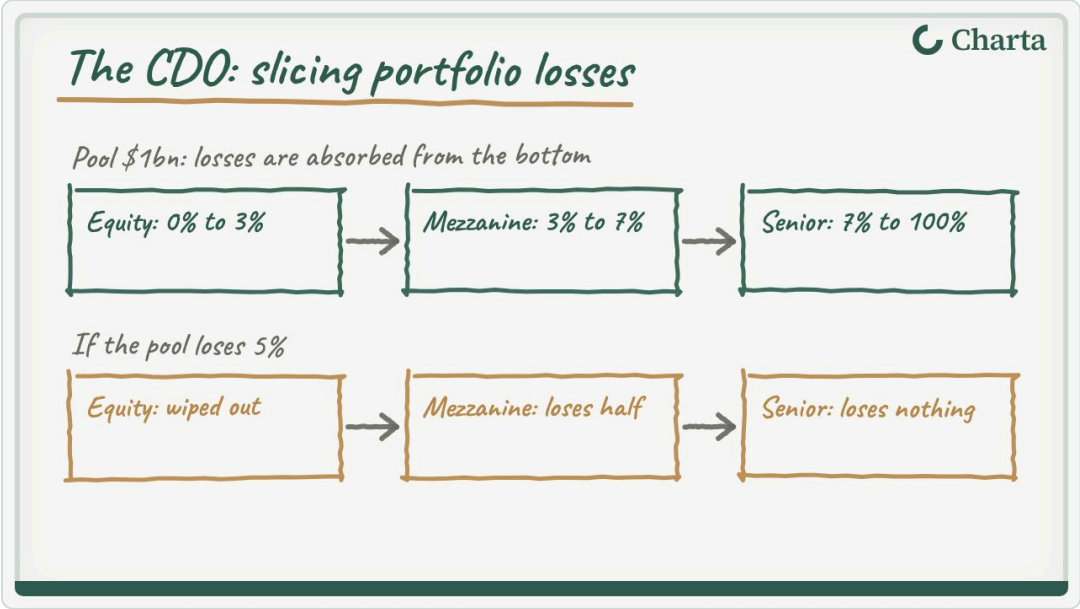
*Notional $20m, coupon 6%, funding 4% + 0.8%, price 100 to 94 (illustration)*

| | | |
|---|---|---|
| **Coupon received** | 6% × 20 | **+$1.20m** |
| **Price change** | (94 − 100)% × 20 | **−$1.20m** |
| **Funding paid** | 4.8% × 20 | **−$0.96m** |
| **Net result, no default** | 1.2 − 1.2 − 0.96 | **−$0.96m** |

IN THE LESSON

- A fund called Quartz Capital is the receiver on a twenty million dollar total return swap on a Norland bond.
- The bond pays a six per cent coupon.
- Quartz pays a funding rate of four per cent plus a spread of eighty basis points, which is four point eight per cent.
- Norland does not default, but its credit weakens, and the bond price falls from one hundred to ninety four.
- Quartz receives the coupon, one point two million.
- It bears the price fall of six per cent, which is one point two million.
- It pays funding of point nine six million.
- The net result is a loss of point nine six million dollars, with no default at all.

## 7 The CDO: slicing portfolio losses




IN THE LESSON

- The third contract is the collateralised debt obligation, or C D O.
- Take a portfolio of loans or bonds, say a pool of one billion dollars, and cut its losses into slices called tranches.
- The equity tranche takes the first losses, here the first three per cent.
- Then the mezzanine tranche takes the next four per cent, from three to seven.
- The senior tranche is hit only after seven per cent of the pool is lost.
- Suppose the pool loses five per cent.
- The equity tranche is wiped out.
- Mezzanine loses five minus three, over four, which is half of its value.
- The senior tranche loses nothing.

## 8 Valuing a CDS when the seller can fail

- Reference entity may default: the CDS hedges this
- Seller may default: the CDS does not hedge this
- Seller default costs money only if the CDS value is positive
- Exposure = positive market value, not notional
- Value = value ignoring seller risk − CVA

IN THE LESSON

- Now we value a C D S more carefully.
- Harbour Bank, the protection buyer, faces two risks.
- Norland may default, and the C D S hedges that.
- But Atlas Insurance, the protection seller, may default too, and the C D S does not hedge that.
- When does the seller's failure cost the bank money?
- Only when the C D S has positive value to the bank, because then the bank would have to replace the contract at today's higher price.
- So the exposure is the positive market value of the contract, never the notional.
- Valuation therefore starts from a risk free value and subtracts an adjustment for the seller's credit risk.

## 9 The credit valuation adjustment

**CVA = LGD × Σ ( PD × EPE × DF )**

| | |
|---|---|
| **Seller's loss given default** | Share of the claim not recovered |
| **Chance seller defaults in period** | Seller default in that period |
| **Expected positive exposure** | Average positive value then |
| **Discount factor** | Brings the amount to today |

IN THE LESSON

- The adjustment is called the credit valuation adjustment, or C V A.
- For each payment period, multiply three things.
- First, the probability that the seller defaults in that period.
- Second, the expected positive exposure, which is the average of the contract's positive value at that time, counting negative values as zero.
- Third, the discount factor, to bring the amount to today.
- Add these up over all periods, then multiply by the seller's loss given default, the share of the claim the bank would not recover from a failed seller.
- That is the C V A.

## 10 Example 3: CVA on Harbour Bank's CDS

*Seller LGD 60%. PD × EPE × DF each year (illustration)*

| | | |
|---|---|---|
| **Year 1** | 1.0% × 400k × 0.96 | **$3,840** |
| **Year 2** | 1.5% × 900k × 0.92 | **$12,420** |
| **Year 3** | 2.0% × 600k × 0.88 | **$10,560** |
| **Sum** | 3,840 + 12,420 + 10,560 | **$26,820** |
| **CVA** | 60% × 26,820 | **$16,092** |

IN THE LESSON

- Suppose three annual periods remain, and Atlas Insurance has a loss given default of sixty per cent.
- In year one, the seller's default chance is one per cent, expected positive exposure is four hundred thousand and the discount factor is point nine six.
- The term is three thousand eight hundred and forty dollars.
- Year two uses one and a half per cent, nine hundred thousand and point nine two, giving twelve thousand four hundred and twenty.
- Year three uses two per cent, six hundred thousand and point eight eight, giving ten thousand five hundred and sixty.
- The sum is twenty six thousand eight hundred and twenty.
- Multiplying by sixty per cent, the C V A is sixteen thousand and ninety two dollars.

## 11 Wrong-way risk

- Wrong-way: seller default correlated with the entity's
- Exposure is highest when the seller is weakest
- Year 2 term: 2.5% × $1.4m × 0.92 = $32,200
- CVA rises from $16,092 to $27,960
- CDS worth less, so the buyer should pay a lower spread

IN THE LESSON

- The most important risk in this calculation is wrong way risk.
- Suppose the seller's default is positively correlated with the default of the reference entity.
- Then the C D S gains value, and exposure rises, exactly when the seller is most likely to fail.
- Suppose that in year two this lifts the seller's default chance to two and a half per cent and expected exposure to one point four million.
- That term, two and a half per cent times one point four million times the same discount factor of point nine two, becomes thirty two thousand two hundred dollars, not twelve thousand four hundred and twenty, and the C V A rises from about sixteen thousand to about twenty eight thousand dollars.
- The C D S is worth less, so the buyer should pay a lower spread.

## 12 Two families of default probability

*Same name, two kinds of probability, two jobs*

| | Risk-neutral | Real-world |
|---|---|---|
| Backed out of | Bond yields, CDS spreads | Historical defaults |
| Level | Higher: has risk premium | Lower |
| Use for | Pricing, valuation | Credit VaR, capital |
| Trap | Not for capital | Not for pricing |

IN THE LESSON

- Valuing a C D S needs default probabilities, and there are two families.
- Risk neutral, or market implied, probabilities are backed out of bond yields or C D S spreads.
- They include the premium that investors demand for bearing default risk, so they are higher than real world probabilities, which are estimated from the historical default experience of rated firms.
- A C D S is valued with risk neutral probabilities, so that the model reproduces market prices.
- Real world probabilities belong in risk management, such as credit value at risk, economic capital and scenario analysis, where you want the expected frequency of actual losses.
- Never price with historical probabilities, or measure capital with market implied ones.

## 13 Spread, hazard rate and recovery

**Spread ≈ λ × (1 − R)**
**λ ≈ Spread ÷ (1 − R)**
**PD by t = 1 − e^(−λt)**

| | |
|---|---|
| **CDS spread** | Annual CDS spread |
| **Hazard rate, lambda** | Annual default chance |
| **Recovery rate, R** | Recovery rate |
| **Default probability by t** | Cumulative default probability |

IN THE LESSON

- There is a handy approximation that links the spread to the default probability.
- The hazard rate, written lambda, is the annual chance of default, given survival so far.
- The spread is approximately lambda times one minus the recovery rate.
- The seller is paid for the expected loss each year, which is the chance of default times the loss on default.
- Turn it around and the hazard rate is about the spread divided by one minus R, in annual terms.
- Over t years, the cumulative probability of default that follows from a constant hazard rate is one minus e to the power of minus lambda times t.

## 14 Example 4: market implied versus historical

*Spread 160 bp, recovery 35%, 5 years. Historical hazard 1% (assumed)*

| | | |
|---|---|---|
| **Implied hazard rate** | 1.6% ÷ (1 − 35%) | **2.46%** |
| **Risk-neutral 5-year PD** | 1 − e^(−5 × 2.4615%) | **11.58%** |
| **Historical hazard (assumed)** | from past defaults | **1.0%** |
| **Real-world 5-year PD** | 1 − e^(−5 × 1.0%) | **4.88%** |
| **Use for pricing the CDS** | the risk-neutral one | **11.58%** |

IN THE LESSON

- Norland's five year C D S trades at one hundred and sixty basis points, with recovery of thirty five per cent.
- The market implied hazard rate is point zero one six divided by point six five, which is two point four six per cent a year.
- The five year default probability is one minus e to the minus five lambda, or about eleven point five eight per cent.
- Now suppose the bank's historical data suggest a hazard rate of one per cent.
- That gives a five year probability of four point eight eight per cent, much lower.
- Both numbers are correct for their purpose.
- The market figure prices the C D S.
- The historical figure measures the losses the bank actually expects to suffer.

## 15 Credit indices and fixed coupons

- Index: 125 single-name CDSs, equally weighted
- Standard maturities; new series about every 6 months
- Fixed coupons (e.g. 100 or 500 bp) make contracts alike
- Coupon rarely equals spread: upfront settles it
- On a default: pay (1 − R) × name's share; notional falls

IN THE LESSON

- Single name C D S contracts are traded in a standard way.
- Credit indices such as C D X North America Investment Grade and i Traxx Europe are equally weighted portfolios of one hundred and twenty five single name C D S contracts.
- They have standard maturities, and a new series is rolled roughly every six months.
- The index pays a standardised fixed coupon, and single name contracts generally use fixed coupons such as one hundred or five hundred basis points.
- But the coupon rarely equals the market spread.
- The difference is settled by an upfront payment when the trade starts.
- When a name defaults, the index pays one minus R times that name's share of the notional, and the notional falls.

## 16 The upfront payment

**Upfront = (Market spread − Coupon) × Annuity × Notional**

| | |
|---|---|
| **Market spread** | Quoted market spread |
| **Fixed coupon** | Standard fixed coupon |
| **Risky annuity** | PV of $1 a year, if it survives |
| **Notional** | Face amount of protection |

IN THE LESSON

- To a good approximation, the upfront payment is the market spread minus the coupon, multiplied by the risky annuity, multiplied by the notional.
- The risky annuity is the present value of one dollar a year, paid only while the name survives.
- The sign matters.
- If the market spread is above the coupon, the protection is worth more than the coupon pays for, so the protection buyer pays the upfront.
- If the market spread is below the coupon, the buyer is overpaying every year, so the buyer receives the upfront.
- And remember the running payment is always the coupon, never the quoted spread.

## 17 Example 5: upfront on a $10m trade

*Coupon 100 bp, annuity 4.3, notional $10m. Index $500m, 125 names, R = 40%*

| | | |
|---|---|---|
| **Upfront, spread 160 bp** | 0.6% × 4.3 × 10m | **Buyer pays 258k** |
| **Upfront, spread 80 bp** | −0.2% × 4.3 × 10m | **Buyer gets 86k** |
| **Running payment each year** | coupon 1.0% × 10m | **$100,000** |
| **One name defaults: payout** | (1 − 0.4) × 4m | **$2.4m** |
| **Index notional afterwards** | 500 − 4 | **$496m** |

IN THE LESSON

- Take a ten million dollar trade with a standard coupon of one hundred basis points, and a risky annuity of four point three.
- If the market spread is one hundred and sixty basis points, the gap is sixty basis points.
- Point zero zero six times four point three times ten million is two hundred and fifty eight thousand dollars, paid by the buyer.
- If the spread is only eighty, the gap is minus twenty basis points, and the buyer receives eighty six thousand.
- In both cases the running payment is the coupon: one hundred thousand dollars a year, not the quoted spread.
- Now an index of five hundred million dollars with one hundred and twenty five names.
- Each name is four million.
- If one defaults with forty per cent recovery, the payout is two point four million, and the notional falls to four hundred and ninety six million.

## 18 Forward CDS

- Buy or sell protection from T1 to T2 at a spread set today
- Usually knocked out if the entity defaults before T1
- Replicate: buy T2 protection, sell T1 protection
- Default before T1: the two trades cancel, nothing owed
- So the forward spread follows from two CDS spreads

IN THE LESSON

- Now two derivatives on top of the C D S.
- A forward C D S is an agreement to buy or sell protection over a future period, from time T one to time T two, at a spread fixed today.
- It usually terminates with no payment if the reference entity defaults before T one, because then the protection period never starts.
- That knock out feature lets us replicate the forward.
- Buy protection to T two, and sell protection to T one.
- If the name defaults before T one, the two trades cancel exactly, and you owe nothing.
- If it survives to T one, you are left holding protection from T one to T two.

## 19 The forward CDS spread

$$F = (s_2A_2 - s_1A_1) \div (A_2 - A_1)$$

| | |
|---|---|
| **Spread to T1** | CDS spread to T1 |
| **Annuity to T1** | Risky annuity to T1 |
| **Spread to T2** | CDS spread to T2 |
| **Annuity to T2** | Risky annuity to T2 |

IN THE LESSON

- Let s one and s two be the C D S spreads for the two maturities, and A one and A two the matching risky annuities.
- The cost of the T two protection is s two times A two.
- The T one protection that we sell earns s one times A one.
- The difference is the cost of protection between the two dates.
- Dividing it by the difference in annuities, which is the value of one dollar a year over that period, converts it into a spread.
- So the forward spread F is s two A two minus s one A one, all over A two minus A one.

## 20 Example 6: a forward spread

*2-year: 110 bp, A = 1.85. 5-year: 160 bp, A = 4.30 (illustration)*

| | | |
|---|---|---|
| **Cost to T2** | 1.6% × 4.30 | **6.880%** |
| **Cost to T1** | 1.1% × 1.85 | **2.035%** |
| **Difference** | 6.880 − 2.035 | **4.845%** |
| **Annuity difference** | 4.30 − 1.85 | **2.45** |
| **Forward spread F** | 4.845% ÷ 2.45 | **197.8 bp** |

IN THE LESSON

- Suppose the two year C D S on Norland trades at one hundred and ten basis points with a risky annuity of one point eight five.
- The five year spread is one hundred and sixty basis points with an annuity of four point three.
- The T two cost is point zero one six times four point three, or six point eight eight per cent.
- The T one cost is point zero one one times one point eight five, or two point zero three five per cent.
- The difference is four point eight four five per cent.
- The annuity difference is two point four five.
- Divide, and the forward spread is one point nine eight per cent, about one hundred and ninety eight basis points, higher than either spot spread.

## 21 Options on a CDS: swaptions

**Payer payoff = max(spread − K, 0) × Annuity × Notional**
**Payer: right to buy protection. Receiver: right to sell**

| | |
|---|---|
| **Payer swaption payoff** | Value at expiry |
| **CDS spread** | Market CDS spread at expiry |
| **Strike spread** | Strike spread, set today |
| **Risky annuity** | Risky annuity then |

IN THE LESSON

- An option on a C D S, a credit default swaption, gives the right, but not the obligation, to enter a C D S at a strike spread.
- A payer swaption is the right to buy protection at the strike, so it gains when spreads widen.
- A receiver swaption is the right to sell protection at the strike, so it gains when spreads tighten.
- At expiry, the payer swaption pays the larger of the market spread minus the strike, and zero, multiplied by the annuity and the notional.
- If the spread stays below the strike, the payer expires worthless.

## 22 Example 7: a payer swaption pays off

*Strike 150 bp, annuity 3.9, notional $10m (illustration)*

| | | |
|---|---|---|
| **Payer, spread 190 bp** | (1.9% − 1.5%) × 3.9 × 10m | **$156,000** |
| **Payer, spread 130 bp** | max(−0.2%, 0) × 3.9 × 10m | **$0** |
| **Receiver, spread 130 bp** | (1.5% − 1.3%) × 3.9 × 10m | **$78,000** |

IN THE LESSON

- Harbour Bank buys a payer swaption on ten million dollars of Norland protection, with a strike of one hundred and fifty basis points.
- At expiry, the risky annuity is three point nine.
- Case one: spreads have widened to one hundred and ninety basis points.
- The gap is forty basis points, and point zero zero four times three point nine times ten million is one hundred and fifty six thousand dollars.
- Case two: spreads have tightened to one hundred and thirty.
- The gap is negative, so the payer pays nothing.
- A receiver swaption at the same strike would pay nothing in case one, but in case two it would pay point zero zero two times three point nine times ten million, which is seventy eight thousand dollars.
- The common error is to forget the floor at zero.

## 23 The synthetic CDO and the spread payments method

- Built from CDSs on a reference portfolio, not owned bonds
- Tranche investors are protection sellers
- They earn a spread on outstanding tranche principal
- Fair spread: PV premiums = PV expected losses
- Premiums shrink as losses write the principal down

IN THE LESSON

- Now back to the C D O, this time in its synthetic form.
- A synthetic C D O is built from C D S contracts on a reference portfolio rather than from owned bonds.
- The tranche investors act as protection sellers.
- They receive a spread on the outstanding tranche principal, and they absorb losses once the attachment point is passed.
- How do we find the fair spread?
- The spread payments method sets the present value of the expected premium payments equal to the present value of the expected tranche losses.
- The premium payments fall over time, because every loss writes down the principal on which the spread is paid.

## 24 Example 8: a fair tranche spread

*$40m tranche. New losses 5, 7, 8%; principal 100, 95, 88%. Discount 0.96, 0.92, 0.88*

| | | |
|---|---|---|
| **PV of premium, per 1.00 spread** | 0.96 + 0.92×0.95 + 0.88×0.88 | **2.6084** |
| **PV of expected losses** | 5%×0.96 + 7%×0.92 + 8%×0.88 | **18.28%** |
| **Fair spread** | 18.28% ÷ 2.6084 | **7.01%** |
| **Year 1 premium on $40m** | 7.01% × 40m | **$2.80m** |

IN THE LESSON

- Take a mezzanine tranche from three to seven per cent of a one billion dollar pool, so the tranche is forty million.
- Assume annual periods, with expected cumulative losses of five per cent after year one and twelve per cent after year two, and expected new losses of five, seven and eight per cent of the tranche in years one to three.
- Discount factors are point nine six, point nine two and point eight eight.
- Premiums are paid on the principal outstanding at the start of each year, which is one hundred, ninety five and eighty eight per cent.
- The present value of premiums per unit of spread is two point six zero eight four.
- The present value of expected losses is eighteen point two eight per cent.
- The fair spread is seven point zero one per cent a year.

## 25 The Gaussian copula for default times

**$x = \sqrt{\rho} \times M + \sqrt{(1 - \rho)} \times Z$**
**Default before T if $x < N^{-1}(Q(T))$**

| | |
|---|---|
| **Name's normal variable** | Compared with a threshold |
| **Common factor, M** | Standard normal, shared by all |
| **Own shock, Z** | Standard normal, one per name |
| **Correlation, rho** | Correlation between names |
| **Default probability by T** | Name's default probability by T |

IN THE LESSON

- To get the joint distribution of default times, the Gaussian copula links each name's default time to a standard normal variable.
- For each name, let M be a common factor shared by every name, like the state of the economy.
- Let Z be an independent factor for that name only.
- Both are standard normal, so x is too.
- With correlation rho, the name's variable is x equals root rho times M plus root one minus rho times Z.
- The name defaults before time T if x is below the inverse standard normal of Q of T, where Q of T is that name's default probability by T.
- Names with a bad common factor tend to default together.

## 26 Example 9: default chance in a bad economy

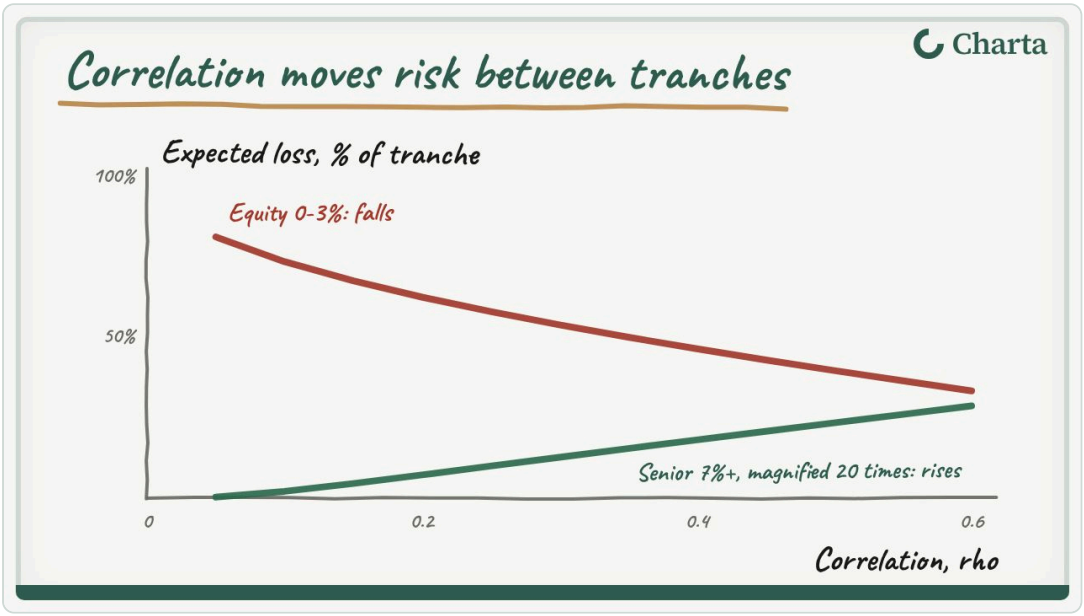
*Q(5y) = 3%, ρ = 0.30, bad economy M = −2.5 (illustration)*

| | | |
|---|---|---|
| **Default threshold** | $N^{-1}(3\%)$ | **−1.881** |
| **Common part** | $\sqrt{0.30} \times (-2.5)$ | **−1.369** |
| **Own shock must be below** | $(-1.881 + 1.369) \div \sqrt{0.70}$ | **−0.611** |
| **Chance of default** | $N(-0.611)$ | **27.0%** |

IN THE LESSON

- Take a name with a three per cent chance of default by year five.
- Without knowing the economy, it defaults if x falls below inverse N of point zero three, which is minus one point eight eight one.
- Let rho be point three, and suppose the economy is in a bad state, with M equal to minus two point five.
- The common part is root point three, about point five four eight, times minus two point five, giving minus one point three six nine.
- For x to fall below the threshold, the own shock must be below minus one point eight eight one plus one point three six nine, all over root point seven, which is minus point six one one.
- The chance of that is about twenty seven per cent, nine times the three per cent average.

## 27 Correlation moves risk between tranches




IN THE LESSON

- This chart shows what correlation does to the tranches.
- Along the bottom is the copula correlation, from zero to sixty per cent.
- Up the side is the expected loss of each tranche as a share of its own size, for a large pool with a five per cent default probability and forty per cent recovery.
- The red line is the equity tranche, the first three per cent.
- As correlation rises, its expected loss falls, from over eighty per cent to about a third.
- The green line is the senior tranche, from seven per cent up.
- Its expected loss is small, so I have magnified it twenty times.
- It rises, because bunched defaults can reach the senior slice.

## 28 Implied correlation: compound and base

*Two ways to turn a tranche price into a correlation*

| | Compound | Base |
|---|---|---|
| Applies to | One tranche, flat ρ | Equity 0 to K |
| Mezzanine | Solved directly | Equity(K2) − Equity(K1) |
| Problem | May be none, or several | Well behaved |
| Pattern | No simple pattern | Rises with K: skew |
| Equity tranche | Same number | Same number |

IN THE LESSON

- Implied correlation is the Gaussian copula correlation that reproduces a quoted tranche price.
- There are two forms.
- Compound, or tranche, correlation applies one flat correlation to a single tranche, such as three to seven per cent, and solves for the value that matches that tranche's market spread.
- Base correlation instead prices equity tranches from zero up to each detachment point K.
- A mezzanine tranche from K one to K two is the equity tranche zero to K two at its base correlation, minus the equity tranche zero to K one at its own.
- Compound correlation can be non unique or absent for mezzanine tranches, because their value is not monotonic in correlation.
- Base correlation is well behaved and typically rises with detachment point, a pattern called a skew.
- For the equity tranche, the two measures coincide.

## 29 Example 10: a mezzanine from base correlations

*Pool EL as % of pool. Base ρ: 0-3% is 0.20, 0-7% is 0.25 (illustration)*

| | | |
|---|---|---|
| **Equity 0-7% at ρ = 0.25** | expected loss, % of pool | **2.53%** |
| **Equity 0-3% at ρ = 0.20** | expected loss, % of pool | **1.88%** |
| **Mezzanine 3-7%** | 2.53 − 1.88 | **0.65% of pool** |
| **As share of tranche** | 0.65 ÷ 4 | **about 16%** |

IN THE LESSON

- Take the three to seven per cent tranche.
- Suppose the base correlation for the zero to three tranche is twenty per cent, and for zero to seven it is twenty five per cent, the rising skew.
- Using the copula pool from the chart, the expected loss of the zero to seven equity tranche at twenty five per cent is two point five three per cent of the pool.
- The zero to three equity tranche at twenty per cent has one point eight eight per cent.
- The mezzanine loss is the difference, point six five per cent of the pool.
- Dividing by its width of four per cent, the tranche is expected to lose about sixteen per cent of its principal.

## 30 Default correlation

**Corr = $(P_{12} - P_1P_2) \div \sqrt{( P_1(1-P_1) \times P_2(1-P_2) )}$**

| | |
|---|---|
| **Default correlation** | Chance they default together |
| **Default probability, name 1** | Chance name 1 defaults |
| **Default probability, name 2** | Chance name 2 defaults |
| **Joint default probability** | Both default in the period |

IN THE LESSON

- Finally, other ways to estimate default correlation.
- First, what is it?
- Default correlation measures how closely two names default together.
- Let P one and P two be the individual default probabilities and P one two the joint default probability.
- If defaults were independent, the joint probability would be P one times P two.
- Default correlation is the excess, P one two minus P one P two, divided by the product of the two standard deviations of a yes or no default event.
- Two warnings.
- Default correlation is usually much lower than asset or equity correlation, and it is hard to estimate, because defaults are rare.

## 31 Example 11: a small default correlation

*P1 = 2%, P2 = 3%, joint P12 = 0.15% (illustration)*

| | | |
|---|---|---|
| **Joint if independent** | 2% × 3% | **0.06%** |
| **Excess joint probability** | 0.15% − 0.06% | **0.09%** |
| **Variance terms** | 2%×98%; 3%×97% | **0.0196; 0.0291** |
| **Denominator** | √(0.0196 × 0.0291) | **2.39%** |
| **Default correlation** | 0.09% ÷ 2.39% | **3.77%** |

**IN THE LESSON**

- Take two names.
- The first has a two per cent chance of default over the year, the second three per cent.
- Suppose the chance that both default is point one five per cent.
- If they were independent, the joint chance would be two per cent times three per cent, which is point zero six per cent.
- The excess is point zero nine per cent.
- The variance terms are two per cent times ninety eight per cent, which is point zero one nine six, and three per cent times ninety seven per cent, which is point zero two nine one.
- The denominator is the square root of their product, about two point three nine per cent.
- So the default correlation is about three point seven seven per cent.
- That looks tiny, yet the joint default is two and a half times what independence would give.
- A small default correlation can hide a large dependence.

## 32 Other methods for estimating default correlation

*Three proxies, none as forward-looking as implied correlation*

| Method | How it works |
|---|---|
| Historical | Correlate yearly default rates |
| Equity proxy | Equity returns proxy assets (Merton) |
| Factor model | Corr = $\text{loading}_1 \times \text{loading}_2$ ($\sqrt{R^2}$) |
| Example | √0.36 × √0.49 = 0.6 × 0.7 = 0.42 |

**IN THE LESSON**

- There are three alternatives to implied correlation.
- The first uses historical data: correlate annual default rates across sectors or ratings over many years.
- The second uses equity return correlations as a proxy for asset correlation in a structural, Merton type model.
- The third fits a factor model, in which each firm loads on a common factor, so the pairwise correlation equals the product of the two loadings.
- A loading is the square root of the firm's R squared.
- If the R squared values are point three six and point four nine, the loadings are point six and point seven, and the correlation is point four two.
- Each method is a proxy, and none matches the market implied approach in being forward looking.

## 33 Recap: the main ideas

- CDS pays (1 − R) × notional on a credit event only
- Value = risk-free value − CVA; exposure is positive value
- Price risk-neutral, capital real-world; λ ≈ spread ÷ (1 − R)
- Upfront = (spread − coupon) × annuity × notional
- Copula prices tranches; base correlation beats compound

**IN THE LESSON**

- Let us pull it together.
- A credit derivative moves credit risk without selling the asset.
- A C D S pays one minus R times notional on a credit event, not on a price fall.
- A T R S passes on market risk as well, and a C D O slices losses into tranches.
- Valuing a C D S means subtracting a C V A, which depends on positive exposure and is worst with wrong way risk.
- Price with risk neutral probabilities, measure capital with real world ones, and link spread to hazard rate by spread divided by one minus R.
- Indices use fixed coupons and upfront payments.
- Forwards and swaptions build on the C D S.
- The copula prices tranches, implied correlation comes in compound and base forms, and default correlation has other estimates.

## 34 Try three quick questions

1. CDS notional $12m; obligation worth 25% of face after the event. Payout?
   **Answer:** (1 − 0.25) × 12m = $9m.
2. CDS spread 240 bp, recovery 40%. Hazard rate?
   **Answer:** 0.024 ÷ 0.6 = 4% a year.
3. $10m, coupon 500 bp, spread 250 bp, annuity 4. Upfront?
   **Answer:** −2.5% × 4 × 10m: buyer receives $1m.

**IN THE LESSON**

- Now try three quick questions.
- Pause the video, think, then listen to the answers.
- Question one.
- A C D S has a notional of twelve million dollars, and the reference obligation is worth twenty five per cent of face after a credit event.
- What does the seller pay?
- Twelve times seventy five per cent, which is nine million dollars.
- Question two.
- A C D S spread is two hundred and forty basis points and recovery is forty per cent.
- What is the approximate hazard rate?
- Point zero two four divided by point six, which is four per cent a year.
- Question three.
- A ten million dollar trade has a fixed coupon of five hundred basis points, a market spread of two hundred and fifty, and a risky annuity of four.
- Who pays the upfront, and how much?
- The spread is below the coupon, so the buyer receives it: minus point zero two five times four times ten million is minus one million dollars, so the buyer receives one million.