CHARTA LEARNING · LESSON NOTES

Collateral and Wrong-Way Risk in CVA

FRM Part II

1 Collateral: the cushion with a gap

- Variation margin is posted as value moves
- At default the bank keeps the collateral
- Exposure = only what is not covered at close-out
- The margin period of risk (MPoR) leaves a gap

IN THE LESSON

- Recall Harbourline Bank and its swap with Corvane Industries.
- Suppose the two sides sign a collateral agreement.
- Each day the swap is valued, and if Corvane owes Harbourline money, Corvane hands over collateral called variation margin.
- If Corvane fails, Harbourline keeps the collateral.
- So exposure is no longer the whole value of the swap, only the part not covered at close out.
- But there is always a gap.
- Between the last margin call and the day the bank has actually closed out and replaced the trade, the market keeps moving, and that gap is called the margin period of risk, or M P O R.
- The value in the collateral account lags the true value by about that long.

2 Collateralised exposure

VM = max(L − Threshold , 0)
Exposure = max(V − VM − IM , 0)

Portfolio value one MPoR earlier, L	Value one MPoR ago
Threshold	Unsecured amount
Variation margin held	Margin held at time t
Initial margin held	Extra buffer
Collateralised exposure	What the bank can lose

IN THE LESSON

- Here is the arithmetic.
- Variation margin held at time t reflects the portfolio value one margin period of risk earlier, which we call L, less any threshold.
- A threshold is an amount of exposure that is left unsecured.
- So variation margin is the larger of L minus the threshold, and zero.
- Exposure at time t is the larger of V, the value now, minus the variation margin held, minus any initial margin, and zero.
- Initial margin is an extra buffer posted at the start and held against future moves.
- Compute the collateralised expected exposure first, then apply loss given default, discounting and default probabilities exactly as in the uncollateralised formula.

3 Example 1: one date, with margin

L = $7m, threshold $2m, V = $8.5m. Amounts in $ millions

Variation margin	max(7 − 2, 0)	**5.0**
Exposure, no initial margin	max(8.5 − 5, 0)	**3.5**
Exposure, IM of 1.5	max(8.5 − 5 − 1.5, 0)	**2.0**
If V had fallen to 4	max(4 − 5, 0)	**0**

IN THE LESSON

- Take one date.
- Seven million dollars was the portfolio value one margin period of risk ago.
- The threshold is two million.
- So variation margin is seven minus two, five million dollars.
- The value today has risen to eight and a half million.
- With no initial margin, exposure is eight and a half minus five, which is three and a half million dollars.
- If Corvane also posted one and a half million of initial margin, exposure falls to two million.
- And if the value had instead fallen to four million, the bank holds five million against a four million value, and the exposure is zero.
- Exposure is never negative, because the formula takes the larger of the result and zero.

4 Example 2: collateral lowers CVA

Same swap: rate 3%, CDS 1.5%, R 40%. DF = e^(−3%×t), PD from the curve

Uncollateralised EE ($m)	1.8, 2.1, 1.2	**CVA $69,906**
Collateralised EE ($m)	0.3, 0.4, 0.2	**same DF, PD**
Collateralised CVA	60% × Σ DF × EE × PD	**$12,331**
Ratio to uncollateralised	12,331 ÷ 69,906	**17.6%**

IN THE LESSON

- Now carry the collateral through to C V A.
- The uncollateralised swap had expected exposures of one point eight, two point one and one point two million dollars, and C V A of sixty nine thousand nine hundred and six dollars, using Corvane's one point five per cent spread and forty per cent recovery.
- Suppose the collateral agreement brings the expected exposures down to three hundred, four hundred and two hundred thousand dollars.
- We do not change anything else: the same discount factors, the same marginal default probabilities, the same sixty per cent loss given default.
- The new C V A is twelve thousand three hundred and thirty one dollars, about eighteen per cent of the original.

5 What moves collateralised CVA

Direction of the effect on CVA

Feature	Effect on CVA
Larger threshold	Raises it
Longer MPoR	Raises it: EE ∝ √MPoR
Initial margin	Lowers it further
Minimum transfer amount	Small residual exposure

IN THE LESSON

- Each feature of the agreement has a clear direction.
- A larger threshold leaves more exposure uncollateralised, and raises C V A.
- A longer margin period of risk lets more value change occur without margin, and raises C V A.
- With a zero threshold and no initial margin, collateralised exposure is roughly proportional to the square root of the margin period of risk, so a margin period four times longer doubles it.
- Initial margin lowers exposure further.
- And a minimum transfer amount, the smallest call that can be made, leaves a small residual exposure even in a well margined portfolio, because small moves go uncalled.

6 Example 3: the square root rule

10-day MPoR gives EE $0.40m. Zero threshold, no IM

MPoR 10 days	base case	**$0.40m**
MPoR 5 days	× √(5 ÷ 10) = 0.707	**$0.28m**
MPoR 20 days	× √2 = 1.414	**$0.57m**
MPoR 40 days	× √4 = 2	**$0.80m**

IN THE LESSON

- Let us check the square root rule.
- Assume the portfolio's value changes are normally distributed, the threshold is zero and there is no initial margin.
- This is an approximation, but a useful one.
- Suppose a ten day margin period of risk gives collateralised expected exposure of four hundred thousand dollars.
- If the period is five days, the exposure is multiplied by the square root of one half, about point seven oh seven, so it is about two hundred and eighty three thousand.
- At twenty days it is multiplied by the square root of two, about five hundred and sixty six thousand.
- At forty days it is multiplied by the square root of four, which is exactly two, so eight hundred thousand dollars.

7 Wrong-way and right-way risk

- Wrong-way: high exposure when default is likely
- Right-way: low exposure when default is likely
- Specific: from the nature of one trade
- General: from broad economic factors
- Standard CVA assumes independence

IN THE LESSON

- The first lesson's formula assumed that exposure and default are independent.
- Often they are not.
- Wrong way risk is positive dependence between the exposure and the counterparty's default probability: exposure tends to be high when the counterparty is more likely to default.
- Right way risk is the opposite: exposure is low when default is likely.
- There are two kinds of wrong way risk.
- Specific wrong way risk comes from the nature of one trade, for example buying a put on a company's shares from that very company.
- General wrong way risk comes from broad economic factors that affect both exposure and default, such as a recession.
- The standard formula uses the unconditional expected exposure, which ignores all of this.

8 How wrong-way risk bends CVA

Conditional exposure decides the sign of the error

Dependence	Standard CVA
None (independent)	Correct
Wrong-way	Understated
Right-way	Overstated
Test	Where is the bank owed money?

IN THE LESSON

- What does wrong way risk do to the number?
- The right exposure to use in C V A is the exposure conditional on default, because that is the exposure the bank would face if the counterparty fails.
- Under independence, the conditional figure equals the unconditional one, and the standard formula is correct.
- Under wrong way risk, exposure conditional on default exceeds the unconditional figure, so standard C V A is understated.
- Under right way risk, the conditional figure is lower, so standard C V A is overstated.
- The exam point is the sign of the dependence.
- Check it from the bank's own exposure: in which state is the bank owed money, and how healthy is the counterparty then?

9 Example 4: two states of the world

Weak: EE $5m, PD 6%. Strong: EE $1m, PD 2%. LGD 60%, DF 0.95

Independence: 3m × 4%	0.6 × 0.95 × 3m × 4%	**$68,400**
Exposure given default	(½×5×6% + ½×1×2%) ÷ 4%	**$4.0m**
Wrong-way CVA	0.6 × 0.95 × 4.0m × 4%	**$91,200**
Right-way: exposure given default	(½×1×6% + ½×5×2%) ÷ 4%	**$2.0m**
Right-way CVA (reversed)	0.6 × 0.95 × 2.0m × 4%	**$45,600**

IN THE LESSON

- Here is a simple model, with two equally likely states.
- In a weak economy, the exposure is five million dollars and the counterparty's default probability is six per cent.
- In a strong economy, the exposure is one million and the default probability is two per cent.
- Average exposure is three million and average default probability is four per cent.
- A model that assumes independence multiplies three million by four per cent.
- Using sixty per cent loss given default and a discount factor of point nine five, C V A is sixty eight thousand four hundred dollars.
- But exposure conditional on default is four million, so the true C V A is ninety one thousand two hundred, and the standard figure is understated by a quarter.
- Reverse the exposures: one million in the weak state, five in the strong.
- Exposure given default is half times one times six per cent, plus half times five times two per cent, over four per cent, which is two million, and C V A is forty five thousand six hundred.

10 Examples of wrong-way risk

Each is a wrong-way example

Trade	Why wrong-way
Put on a firm's shares, from the firm	Pays when issuer is distressed
Forward: EM bank pays $	Devaluation raises exposure and stress
CDS bought from linked seller	Seller fails if the reference fails

IN THE LESSON

- Now the real life illustrations.
- First, buying a put on a company's shares from that company.
- The put pays out when the share price falls, and the share price is lowest when the issuer is in distress, so the bank's claim is largest just when the seller is least able to pay.
- Second, a forward in which a bank in an emerging market pays dollars against local currency.
- If the currency is devalued, the dealer's exposure rises and the bank is stressed at the same moment.
- Third, buying C D S protection from a seller whose credit is linked to the reference entity, for example a domestic bank selling protection on its own sovereign.
- If the sovereign fails, the seller probably fails too.

11 Examples of right-way risk

Genuine hedgers give right-way risk

Client and trade	Why right-way
Airline buys oil forward	Owed when oil falls: client healthy
Producer sells oil forward	Owed when oil rises: client healthy
Warning	Speculation or over-hedging reverses it

IN THE LESSON

- Right way risk is the other side of the coin.
- Consider a company that hedges genuinely, such as an airline buying oil forward, or an oil producer selling oil forward.
- When oil rises, the airline's hedge pays, so the dealer owes the airline and has no exposure to it.
- The dealer is owed money only when oil falls, and then the airline's costs are low and its credit is stronger.
- For the producer the dealer is owed when oil rises, and then its revenues are high.
- So the exposure is high when the counterparty is healthy and low when it is weak.
- Be careful, because speculating or over hedging can reverse the dependence.
- Always ask in which state the bank is owed money.

12 Example 5: a jump at default

Bank pays $10m, receives LC 200m. Rate 20 then 25 per dollar

Value to dealer at 20 per $	10m − 200m ÷ 20	**$0**
LC 200m after devaluation	200m ÷ 25	**$8.0m**
Dealer's exposure at default	10m − 8.0m	**$2.0m**
Share of the notional	2 ÷ 10	**20%**

IN THE LESSON

- One illustration deserves numbers.
- A bank in an emerging market agrees a forward with a dealer.
- The bank pays ten million dollars and receives two hundred million units of local currency, at an exchange rate of twenty units per dollar.
- At that rate the two legs are equal and the forward is worth nothing to the dealer.
- Now the currency is devalued to twenty five units per dollar, and the bank defaults at the same moment.
- The two hundred million units are worth only eight million dollars.
- The dealer is owed ten million dollars and pays what is now worth eight, so its exposure jumps to two million dollars, a fifth of the notional, exactly when the counterparty fails.

13 How collateral bears on wrong-way risk

- Daily margin limits how far exposure can build
- Threshold and minimum transfer amount remain
- MPoR gap between last call and close-out
- Jumps at default outrun margin calls
- Collateral can be wrong-way itself

IN THE LESSON

- Collateral reduces the effect of wrong way risk, because daily margin calls limit how far exposure can build up while a counterparty weakens.
- But it does not remove it.
- Four things remain.
- The threshold and the minimum transfer amount leave exposure uncollateralised.
- The margin period of risk allows a gap between the last call and the close out, during which a weakening counterparty may fail to pay.
- Jumps at default, such as a sudden devaluation, happen faster than any margin call.
- And collateral can carry wrong way risk itself.
- If its value falls when the counterparty's credit falls, as with the counterparty's own bonds or shares, it loses value exactly when the bank needs it.

14 Two places wrong-way risk hides

Exposure side and collateral side

Where	What happens	Response
In the exposure	Trade gains as credit falls	Margin
In the collateral	Value falls with credit	Haircuts
In the collateral	Own securities posted	Eligibility
In the collateral	One kind dominates	Limits

IN THE LESSON

- An exam answer must separate two things.
- There is wrong way risk in the exposure: the trade itself gets more valuable to the bank as the counterparty weakens, and collateral, meaning margin, mostly deals with that.
- And there is wrong way risk in the collateral: the value of what the counterparty has posted falls when its credit falls.
- The two can add up.
- There are responses to the second.
- Use larger haircuts for collateral that is correlated with the counterparty.
- Restrict eligibility so that the counterparty's own securities are not accepted.
- And set concentration limits, so that the bank does not hold too much of one kind of collateral.

15 Collateral exposed to wrong-way risk

Does the collateral fall when the poster fails?

Collateral posted	Link to the poster
Own bonds or shares; affiliates	Direct
Oil shares from an oil producer	Same sector
Home sovereign bonds from a bank	Bank and sovereign credit linked
Securitisation of own loans	Same pool
Cash in a major currency	None: least exposed

IN THE LESSON

- Collateral is exposed to wrong way risk when its value is positively related to the posting counterparty's default.
- The clearest cases are the counterparty's own bonds or shares, bonds of its parent or affiliates, and securities from the same sector, such as oil company shares posted by an oil producer.
- A bank posting its home government's bonds is a standard illustration, because bank and sovereign credit are linked.
- A firm posting securitisations of loans it originated is another.
- So is a gold miner posting gold.
- The least exposed collateral is cash in a major currency, or highly rated, liquid, diversified securities with no link to the counterparty.

16 Example 6: a haircut that is too thin

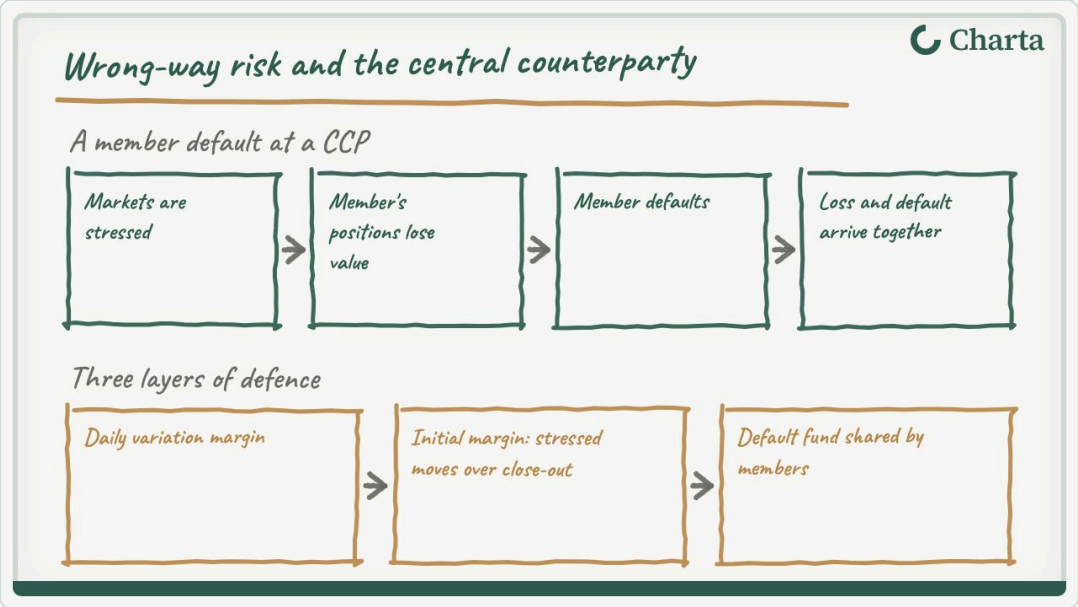
Norlake posts own bonds worth $10m against a $9m claim. Haircut 10%

Value credited	10m × (1 − 10%)	**$9.0m**
Bonds after default, −40%	10m × (1 − 40%)	**$6.0m**
Shortfall	9.0m − 6.0m	**$3.0m**
Haircut that would cover	1 − 6.0 ÷ 10	**40%**

IN THE LESSON

- Now numbers.
- Norlake Mining owes Harbourline nine million dollars, and posts its own bonds, worth ten million, as collateral.
- Harbourline applies a ten per cent haircut, calibrated to normal price volatility, and so credits nine million dollars, just covering the claim.
- Then Norlake defaults.
- The market reads the default as bad news for its bonds, which fall forty per cent.
- They are now worth six million, so Harbourline faces a shortfall of three million dollars.
- The haircut that would have been needed is the full forty per cent.
- A haircut calibrated to normal volatility will not cover the correlated fall seen at default.
- Ask what the counterparty's default says about the collateral price.

17 Wrong-way risk and the central counterparty



IN THE LESSON

- Wrong way risk matters just as much for a central counterparty, or C C P.
- A clearing member is more likely to default in the same stressed markets in which its positions lose value, so the C C P's loss and the default arrive together.
- Follow the boxes.
- Markets are stressed, the member's positions lose value, and it fails.
- The C C P must now close out a portfolio that is losing money, in a market that is stressed.
- The defences are three layers.
- Daily variation margin keeps the value current.
- Initial margin is sized for stressed moves over the close out period.
- And a default fund is mutualised among the members, so that the survivors share what is left.

18 Wrong-way risk inside a CCP

Four channels, one theme: stress arrives together

Channel	Why it matters
Member clears CDS on itself	Pays out when it fails
Crisis margin increases	Drain members' liquidity
Concentrated portfolio	Costly to liquidate in stress
Surviving members	Weakened by the same event

IN THE LESSON

- Wrong way risk shows up at a central counterparty in several ways.
- A member may clear trades that pay out when it fails, such as C D S on itself or its affiliates.
- C C Ps restrict these trades or margin them heavily.
- Margin increases in a crisis drain members' liquidity, so the defence can itself make a member weaker.
- Liquidating a large, concentrated portfolio is costly in a stressed market, because the prices the C C P needs are the ones that have moved against it.
- And the surviving members that fund the default fund are weakened by the same event that caused the default, so the cushion is thinnest when it is most needed.

19 Four ways to model wrong-way risk

Each method gives exposure conditional on default

Approach	How dependence enters
Hazard rate	Stochastic spread, correlated
Structural	Firm value below a barrier
Parametric	Multiplier on exposure or CVA
Jump	Discrete move in exposure at default

IN THE LESSON

- There are four ways to model wrong way risk, and the result of each is exposure conditional on default.
- In the hazard rate approach, the counterparty's hazard rate, its credit spread, is stochastic and correlated with the market variables that drive exposure, so default is more likely in high exposure states.
- In the structural approach, default occurs when firm value falls below a barrier, and the firm value is correlated with the exposure driver.
- The parametric approach imposes the dependence directly, for example by applying a multiplier to exposure or to C V A.
- The jump approach adds a discrete move in exposure at default, such as a currency devaluation when a sovereign defaults.

20 Example 7: hazard rate and parametric

Two-state example: independence CVA $68,400

Hazard rate: cond. exposure	4.0m ÷ 3.0m	**= 1.33**
Hazard rate CVA	0.6 × 0.95 × 4.0m × 4%	**$91,200**
Parametric multiplier	91,200 ÷ 68,400	**1.33**
Parametric CVA	68,400 × 4/3	**$91,200**

IN THE LESSON

- Go back to the two state example.
- The hazard rate idea is exactly what we built: default probability rises in the weak state, when exposure is high.
- It gave exposure conditional on default of four million dollars, and C V A of ninety one thousand two hundred, against sixty eight thousand four hundred under independence.
- The parametric approach would reach the same figure more crudely, by imposing a multiplier.
- Ninety one thousand two hundred divided by sixty eight thousand four hundred is one point three three, so a multiplier of one and a third on the independent C V A reproduces it.
- The jump approach is the devaluation example, where exposure leaps to two million at default.

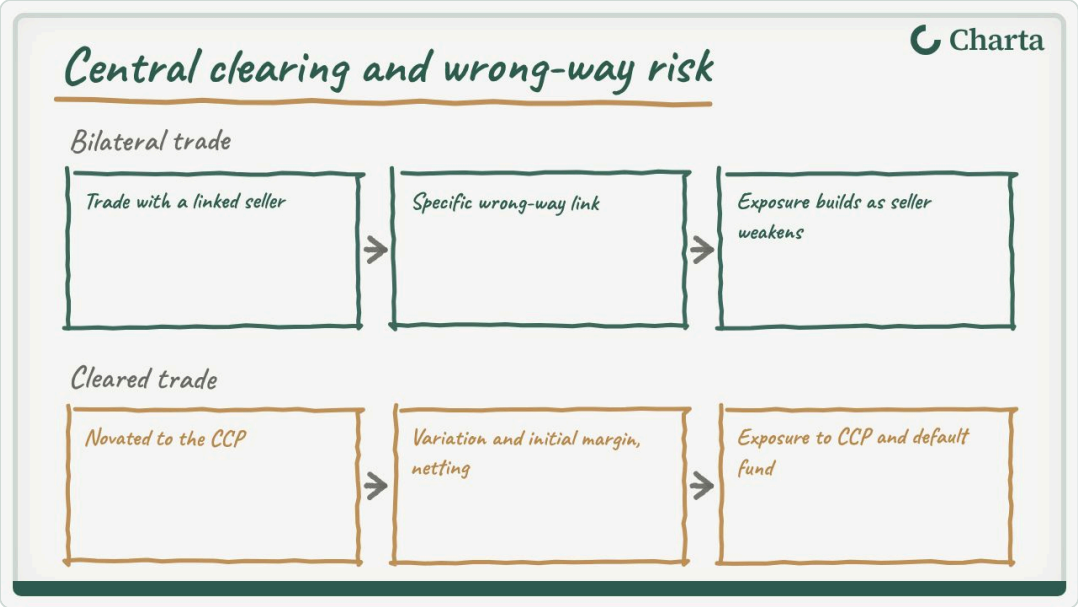
21 Which model for which problem

- Smooth correlation: hazard rate or structural
- Gap at default: jump approach
- Parametric: quick multiplier when little is known
- Output: exposure conditional on default

IN THE LESSON

- Choose the approach by the problem.
- A smooth correlation between exposure and credit, such as a recession hitting both, suits the hazard rate or structural models.
- A gap on default, such as a devaluation that happens at the moment of default, needs the jump approach, because no smooth correlation can produce it.
- The parametric approach is the quick method, useful for sizing the effect when little is known about the dependence.
- Whatever the method, what the model must deliver is exposure conditional on default, and that conditional figure goes into the C V A formula in place of the unconditional one.
- If the model returns the unconditional figure, wrong way risk has not been captured.

22 Central clearing and wrong-way risk



IN THE LESSON

- Central clearing changes the shape of wrong way risk without removing it.
- Look at the two rows.
- In the top row, a bilateral trade carries a specific link between the trade and a correlated seller.
- When the trade is cleared, it is novated to the C C P: the C C P becomes the counterparty to both sides, and the link to the seller is replaced by exposure to a margined C C P.
- Daily variation margin, initial margin and multilateral netting greatly reduce the exposure that can build up.
- What remains is exposure to the C C P and its default fund, which is where the next table picks up.

23 What remains after clearing

Clearing mitigates wrong-way risk; it does not eliminate it

What remains	Source
Exposure to CCP, default fund	Other members fail in stress
Procyclical margin	Drains liquidity in a crisis
Concentration in large members	One failure is large
Collateral wrong-way risk	E.g. home sovereign's bonds

IN THE LESSON

- What remains is exposure to the C C P and its default fund, and that depends on four things.
- Other members may default in stressed markets, and the default fund pays for their losses.
- Margin is procyclical, rising in a crisis and draining liquidity.
- Concentration in large members means that one failure is a big failure.
- And wrong way risk in the collateral still applies, for example a member posting its home sovereign's bonds.
- C C Ps manage all of this with haircuts, concentration limits and restrictions on trades with obvious dependence.
- The exam point is simple: clearing mitigates wrong way risk, it does not eliminate it.

24 Recap: the main ideas

- Exposure = max(V − VM − IM, 0); VM lags by one MPoR
- Larger threshold or longer MPoR raises CVA
- Wrong-way: CVA understated. Right-way: overstated
- Collateral helps but leaves gaps; can be wrong-way
- CCPs: mitigated, not eliminated; use conditional exposure

IN THE LESSON

- Let us pull it together.
- Collateral reduces exposure to the amount not covered at close out.
- Variation margin reflects value one margin period of risk ago, less the threshold, and exposure is value minus margin minus initial margin, floored at zero.
- Bigger thresholds and longer margin periods raise C V A.
- Wrong way risk is exposure high when default is likely, and it makes standard C V A too low, while right way risk makes it too high.
- Collateral helps but leaves residual gaps and can itself be wrong way.
- Central counterparties face the same dependence.
- Four models capture it, and clearing reshapes it without removing it.

25 Try three quick questions

1. L = $12m, V = $13m, threshold $3m, no IM. Exposure?
   Answer: VM = 12 − 3 = 9. Exposure = 13 − 9 = $4m.
2. CVA $80,000; conditional EE 25% above the one used. Corrected CVA?
   Answer: Wrong-way, so understated: 80,000 × 1.25 = $100,000.
3. A bank posts its home government's bonds. Wrong-way?
   Answer: Yes: bank and sovereign credit are linked.

IN THE LESSON

- Now try three quick questions.
- Pause the video, think, then listen to the answers.
- Question one.
- The portfolio was worth twelve million dollars one margin period of risk ago and is worth thirteen million today.
- The threshold is three million and there is no initial margin.
- What is the exposure?
- Variation margin is twelve minus three, nine million, so exposure is thirteen minus nine, four million dollars.
- Question two.
- Standard C V A is eighty thousand dollars, but exposure conditional on default is a quarter higher than the figure used.
- Is C V A over or understated, and what is the corrected figure?
- Wrong way risk, so understated, and the corrected figure is one hundred thousand dollars.
- Question three.
- A bank posts its own government's bonds.
- Is the collateral wrong way?
- Yes, because bank and sovereign credit are linked.