CHARTA LEARNING · LESSON NOTES

Structured Credit Risk

FRM Part II

1 The families of structured products

Named by what sits in the pool

Family	Backed by
ABS	Auto loans, credit cards, student loans
MBS	Residential or commercial mortgages
CDO	A pool of bonds or loans
CLO	Leveraged loans
CDO squared	Tranches of other securitisations

IN THE LESSON

- Structured credit takes the credit risk of a pool of debt and repackages it into securities with different risk profiles.
- The families are named by what sits in the pool.
- Asset-backed securities, or A B S, are backed by consumer loans such as auto loans, credit cards or student loans.
- Mortgage-backed securities, or M B S, are backed by mortgages, residential or commercial.
- A collateralised debt obligation, or C D O, is backed by a pool of bonds or loans.
- A C L O, a collateralised loan obligation, is a C D O backed by leveraged loans.
- And a C D O squared has as its collateral tranches of other securitisations.
- So ask first: what is in the pool?

2 Pass-through or tranched, cash or synthetic

- Pass-through: pool cash shared pro rata
- Tranched: cash and losses split by seniority
- Cash deal: the vehicle owns the assets
- Synthetic: exposure through CDS, largely unfunded
- Classify by collateral, then assets or derivatives

IN THE LESSON

- Two more splits matter.
- First, how the pool's cash is shared.
- A pass-through simply passes the pool's cash flow to investors pro rata.
- A tranched product redistributes the cash and the losses by seniority, so some investors are protected at the expense of others.
- Not every securitisation is tranched.
- Second, how the exposure is obtained.
- In a cash structure, the vehicle actually owns the assets.
- In a synthetic structure it takes the credit exposure through credit default swaps, or C D S, so it is largely unfunded and holds no pool of cash collateral.
- For the exam, classify a product by its collateral, and by whether risk moves through assets or through derivatives.

3 How tranching spreads losses

Loss (% of pool) = max(0 , min(L, D) − A)
Loss (% of tranche) = that ÷ (D − A)

Pool loss, L (% of pool)	Loss on the whole pool
Attachment point, A	Pool loss where it starts to lose
Detachment point, D	Pool loss where it is wiped out
Thickness, D − A	Divide by this for tranche loss

IN THE LESSON

- Now to tranching.
- Picture a row of buckets under a tap: the bottom bucket fills first.
- Pool losses fill the junior tranches first, equity, then mezzanine, then senior.
- Each tranche is defined by two points.
- The attachment point, A, is the pool loss at which the tranche starts to lose.
- The detachment point, D, is the pool loss at which it is wiped out.
- Let L be the pool loss as a percentage of the pool.
- The tranche's loss, as a percentage of the pool, is the larger of zero and the smaller of L and D, minus A.
- Divide by the thickness, D minus A, to get the loss as a percentage of the tranche's own notional.
- The tranche losses always add up to the pool loss.

4 Example 1: losses climb the tranches

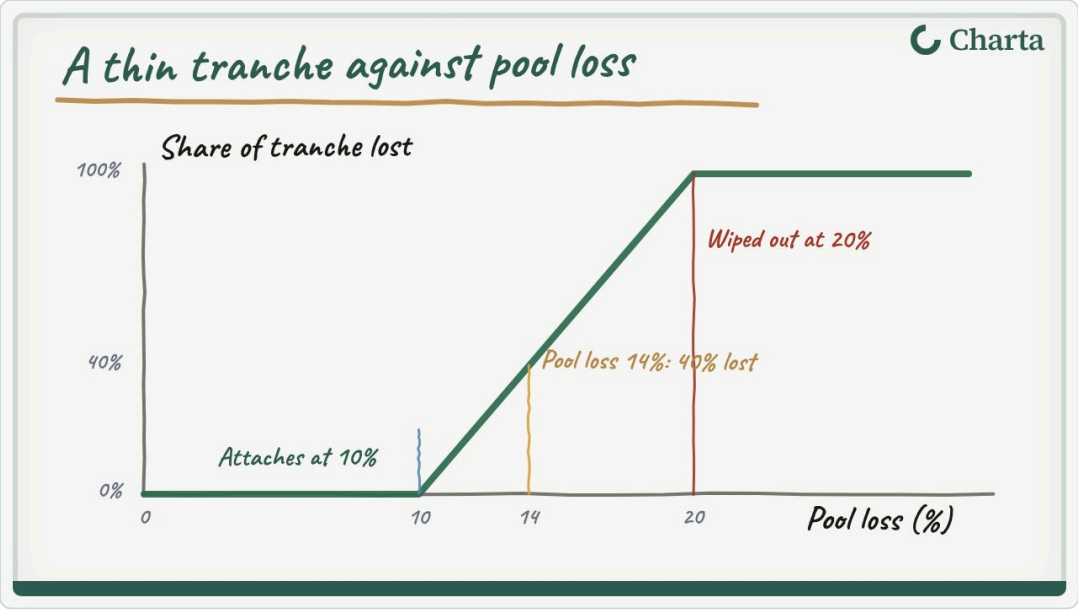
Pool $1,000m. Equity 0-10%, mezzanine 10-20%, senior 20-100%

Pool loss 14%: equity	10 ÷ 10	**100%**
Pool loss 14%: mezzanine	(14 − 10) ÷ 10	**40%**
Pool loss 14%: senior	14 is below 20	**0%**
Pool loss 25%: senior	(25 − 20) ÷ 80	**6.25%**
Pool loss 6%: equity	6 ÷ 10	**60%**

IN THE LESSON

- Harbourline pools a thousand million dollars of loans into three tranches.
- Equity attaches at zero and detaches at ten per cent.
- Mezzanine runs from ten to twenty per cent.
- Senior covers everything above twenty.
- Suppose the pool loses fourteen per cent.
- Equity loses ten points of the pool, over a thickness of ten: one hundred per cent, wiped out.
- Mezzanine loses fourteen minus ten, four points, over ten: forty per cent.
- Senior loses nothing.
- Ten plus four is fourteen, which matches the pool.
- Now the trap.
- If the pool loses only six per cent, equity loses sixty per cent, ten times the pool's rate.
- And at a pool loss of twenty five per cent, even senior loses six point two five per cent.

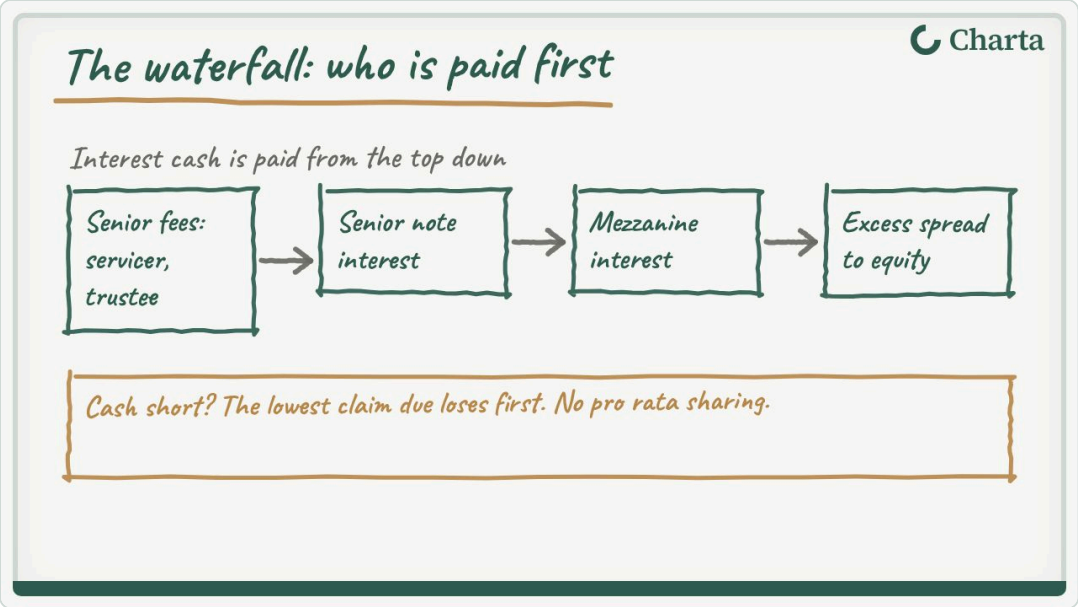
5 A thin tranche against pool loss



IN THE LESSON

- Here is the mezzanine tranche as a picture.
- Across is the pool loss, up the side the share of the mezzanine lost.
- Until the pool loses ten per cent, the line is flat at zero, because equity absorbs everything.
- That is the attachment point.
- Then the line climbs, one tenth of the tranche for each point of pool loss.
- At fourteen per cent it has lost forty per cent.
- At twenty per cent, the detachment point, the tranche is wiped out and the line stays at the top.
- The slope is steep because the tranche is thin.
- That is the exam point: a thin tranche can lose most of its notional while the pool loses only a small fraction.
- Always quote loss against the tranche, not the pool.

6 The waterfall: who is paid first



IN THE LESSON

- Next, the waterfall: the contractual order in which collected cash is paid out.
- Picture interest spilling down a staircase, one step at a time.
- First come the senior fees, to the servicer and the trustee.
- Then the senior note interest.
- Then the mezzanine interest.
- Whatever is left is excess spread, and it goes to the equity tranche.
- When cash is short, the shortfall lands on the lowest priority claim that is due, so equity first, then mezzanine.
- The rule to remember: always pay in priority order and stop when the cash runs out.
- The exam trap is to share a shortfall pro rata, which a waterfall does not do unless the deal says so.

7 Example 2: a weak year

Interest collected $45m. Fees $2m, senior due $40m, mezzanine due $7m

Cash collected	interest from the pool	**$45m**
Fees paid first	servicer and trustee	**$2m**
Senior interest	due 40, cash left 43	**$40m**
Mezzanine interest	due 7, cash left 3	**$3m**
Equity receives	nothing left; mezzanine short 4	**$0**

IN THE LESSON

- A weak year at Harbourline Trust one.
- The pool pays only forty five million dollars of interest.
- Fees are two million, senior interest due is forty million, and mezzanine interest due is seven million.
- Pay in order.
- Fees take two, leaving forty three.
- Senior takes its forty, leaving three.
- Mezzanine is due seven, but only three remains, so it receives three and is four million short.
- Equity receives nothing.
- The shortfall was not shared out: senior was paid in full, and it fell on the lowest priority claim that was due.

8 Principal: sequential or pro rata

Principal $90m. Senior notes $800m, mezzanine notes $100m

	Sequential	Pro rata
Rule	Senior first	By balance
Senior receives	$90m	$80m
Mezzanine receives	$0	$10m
Typical deal	After trigger	Before trigger

IN THE LESSON

- Principal collections are applied in one of two ways.
- Under the sequential rule, the senior tranche is repaid completely before the next note receives anything.
- Under the pro rata rule, every note is repaid in proportion to its balance.
- Take ninety million dollars of principal, with senior notes of eight hundred million and mezzanine notes of one hundred million.
- Sequentially, senior receives all ninety.
- Pro rata, the balances are in the ratio eight to one, so senior receives eighty and mezzanine receives ten.
- Many deals pay pro rata while the pool is healthy, then switch to sequential if a trigger, such as cumulative losses, is breached.
- After the switch, senior is paid first.

9 Who is in a securitization

The chief parties and what each does

Party	Role
Originator	Makes loans; SPV buys them, issues notes
Arranger or underwriter	Structures and sells the deal
Servicer	Collects payments, handles arrears
Trustee or custodian	Holds assets, runs the tests
Raters, investors, manager	Rate, buy, select collateral

IN THE LESSON

- Who takes part?
- Meet the cast.
- The originator, here Harbourline, makes or buys the loans and sells them to a special purpose vehicle, or S P V, which issues the notes.
- The arranger, or underwriter, structures the deal and sells it.
- The servicer collects the payments and handles borrowers who fall behind.
- The trustee, or custodian, holds the assets and runs the waterfall tests.
- Rating agencies rate the tranches, and investors buy them.
- In a C D O, a manager may also select the collateral.
- Each role is sensible on its own.
- The trouble is how they are paid.

10 Where interests clash

Each party's incentive that departs from the investor's

Party	Incentive
Originator	Sells the loans: weaker screening
Rating agency	Paid by the issuer
Manager holding equity	Prefers more risk
Servicer	Paid on balances
Arranger	Paid for completing the deal

IN THE LESSON

- Now the clash of interests.
- Many parties are paid for completing deals, not for how the loans perform years later.
- An originator that sells the loans has less reason to screen borrowers carefully.
- This is adverse selection: the loans sold may be the weaker ones.
- Rating agencies are paid by the issuer, who wants a high rating.
- A manager holding the equity prefers risk, because equity gains from the upside while investors in the notes want safety.
- A servicer is paid on balances, so it may be slow to write off or restructure loans that shrink the balance its fee is earned on.
- And an arranger is paid when the deal closes.
- The exam skill is to match each party with the incentive that departs from the investor's.

11 Interim cash flows: the recipe

Excess = collateral interest − senior coupon − mezzanine coupon
Loss = defaulted balance − recovery

Collateral interest	Rate × balance still performing
Coupon on a note	Rate × opening balance
Excess spread	What is left, before any tests
Loss on defaults	Written off against equity first

IN THE LESSON

- Time to run the numbers, period by period, for a three tier deal.
- Harbourline Trust one has a pool of one thousand million dollars paying eight per cent.
- The senior notes are eight hundred million at five per cent, mezzanine one hundred million at seven per cent, and equity one hundred million.
- Each period, first compute collateral interest, only on the balance still performing after defaults.
- Recoveries on defaulted assets are principal cash.
- Each coupon is the note's rate times its opening balance.
- Excess spread is collateral interest less the senior coupon and the mezzanine coupon.
- The loss is the defaulted balance less the recovery, and it is written off against equity first.

12 Example 3: year one

$50m of the pool defaults at the start; $20m is recovered

Interest on performing 950	8% × 950	**$76.0m**
Coupons: senior + mezzanine	40 + 7	**$47.0m**
Excess spread	76 − 40 − 7	**$29.0m**
Loss, written off to equity	50 − 20	**$30m**
Balances: senior/mezz/equity	recovery pays senior	**780 / 100 / 70**

IN THE LESSON

- Year one.
- Fifty million dollars of the pool defaults at the start of the year, and twenty million is recovered.
- The performing balance is nine hundred and fifty million, so collateral interest is eight per cent of that, seventy six million.
- The coupons are forty million on the senior notes and seven million on the mezzanine, so excess spread is twenty nine million.
- The loss is fifty less twenty, thirty million, and it is written off against equity, which falls from one hundred to seventy.
- The twenty million of recoveries is principal cash, applied to the senior notes first, so they fall from eight hundred to seven hundred and eighty.
- The usual error is charging interest on the defaulted assets.
- That would give eighty million, not seventy six.

13 Example 4: year two

Opening balances 780 / 100 / 70. $30m defaults, $15m recovered

Interest on performing 920	8% × 920	**$73.6m**
Coupons on opening balances	5% × 780 + 7% × 100	**$46.0m**
Excess spread	73.6 − 46.0	**$27.6m**
Loss, written off to equity	30 − 15	**$15m**
Balances: senior/mezz/equity	recovery pays senior	**765 / 100 / 55**

IN THE LESSON

- Year two uses the new balances, not last year's.
- Thirty million dollars defaults, with fifteen million recovered.
- The performing balance is nine hundred and twenty million, so interest is seventy three point six million.
- The senior coupon is five per cent of seven hundred and eighty million, thirty nine million, not forty.
- The mezzanine coupon is still seven, so the coupons total forty six million.
- Excess spread is twenty seven point six million.
- The loss is fifteen million, and equity falls from seventy to fifty five.
- The fifteen million recovered pays the senior notes down to seven hundred and sixty five.
- Check: seven sixty five, plus one hundred, plus fifty five, is nine hundred and twenty, the performing pool.

14 Excess spread and the OC account

- Excess spread = collateral interest − note interest − fees
- Equity could take it all, but much is diverted first
- Diverted cash sits in the OC account, a loss reserve
- Year-end test: collateral value + OC vs a required level
- Shortfall is paid in, never more than the excess spread

IN THE LESSON

- Where does excess spread go?
- Think of a household with a rainy day jar.
- Excess spread is the interest collected on the collateral minus the interest due on the notes and the fees.
- Rather than paying it all to equity, many structures first divert some into an overcollateralisation account, or O C account, a cash reserve that absorbs future losses.
- At each year end, a test compares the collateral value, plus the O C account, with a required level.
- If there is a shortfall, it is paid into the O C account out of excess spread, but never more than the excess spread available.

15 The overcollateralisation test

Collateral value = original balance − cumulative losses
Required = trigger × (senior + mezzanine notes)
Shortfall = Required − (collateral value + OC)

Collateral value	Not the original balance
Trigger	For example 110%, not 100%
Required level	Compared with value plus OC
OC balance	Carries forward each year

IN THE LESSON

- Here is the test.
- Collateral value is the original balance less cumulative losses.
- Add the O C balance.
- Compare that sum with the required level, which is the trigger times the senior plus mezzanine notes.
- The trigger is a percentage, for example one hundred and ten per cent, so value plus the O C account must exceed the debt by ten per cent.
- The shortfall is the required level less the sum, and it is paid into the O C account out of excess spread.
- The O C balance carries forward and counts towards the next test.
- The usual mistakes are diverting all the excess spread, ignoring the carried forward balance, and testing against one hundred per cent when the deal specifies a higher trigger.

16 Example 5: the OC account over three years

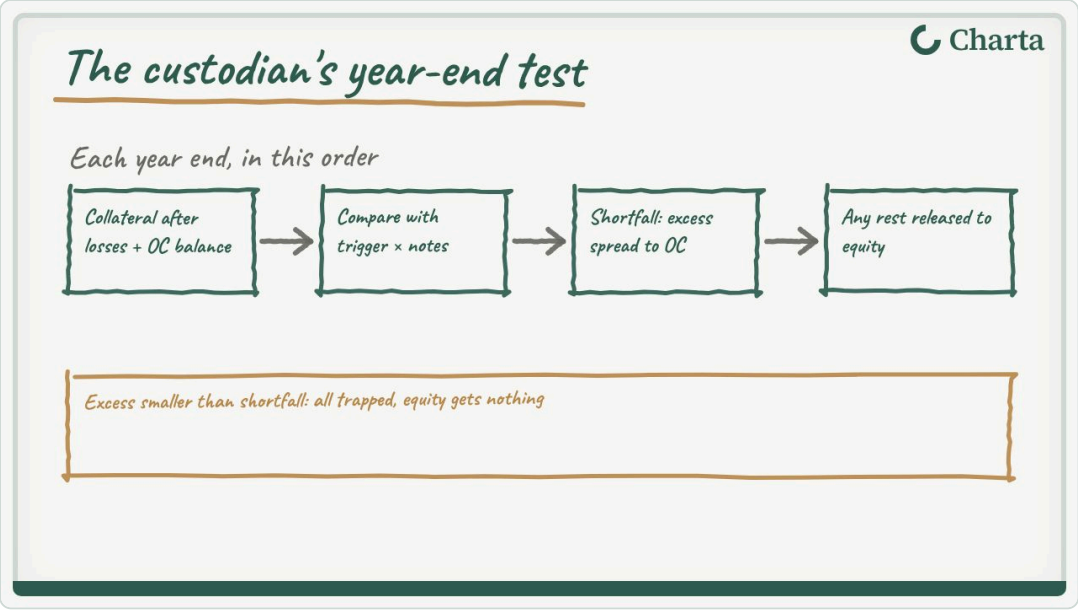
Trust 2: pool $1,000m, notes $900m, trigger 110%, required $990m. Millions

Year 1: loss 30, excess 40	970 + 0 vs 990: pay in 20	**OC = 20**
Year 2: loss 80, excess 25	920 + 20 vs 990: pay in 25	**OC = 45**
Year 3: loss 90, excess 60	910 + 45 vs 990: pay in 35	**OC = 80**
Released to equity, Y1 Y2 Y3	40 − 20, 25 − 25, 60 − 35	**20, 0, 25**
Test after year 3	910 + 80 vs 990	**met**

IN THE LESSON

- Harbourline Trust two has a pool of one thousand million dollars, senior notes of seven hundred million, mezzanine notes of two hundred million, and a trigger of one hundred and ten per cent.
- The required level is nine hundred and ninety million.
- At the end of year one, cumulative losses are thirty, so collateral value is nine seventy.
- Excess spread is forty.
- The shortfall is twenty, so twenty goes to the O C account, and twenty is released to equity.
- In year two: cumulative losses eighty, so collateral nine twenty, plus twenty is nine forty.
- The shortfall is fifty but excess spread is only twenty five, so all of it is paid in and the O C account holds forty five.
- In year three, losses are ninety, so collateral is nine ten, and excess spread is sixty.
- Nine ten plus forty five is nine fifty five.
- The shortfall is thirty five, so O C rises to eighty, twenty five is released, and nine ten plus eighty meets the requirement exactly.

17 The custodian's year-end test



IN THE LESSON

- Now the custodian's job at each year end.
- Step one: add the collateral value after cumulative losses to the O C account and compare the sum with the required amount, which is the trigger times the outstanding senior and mezzanine notes.
- If there is a shortfall, excess spread goes into the O C account, up to the shortfall.
- Step two: whatever excess spread remains is released to the equity noteholders.
- If the excess spread is smaller than the shortfall, all of it is trapped.
- Equity receives nothing, and part of the shortfall remains for next year.
- If the test is met, all excess spread flows to equity.
- The usual errors are paying equity before testing, and forgetting the O C balance.

18 Example 6: cash trapped, then released

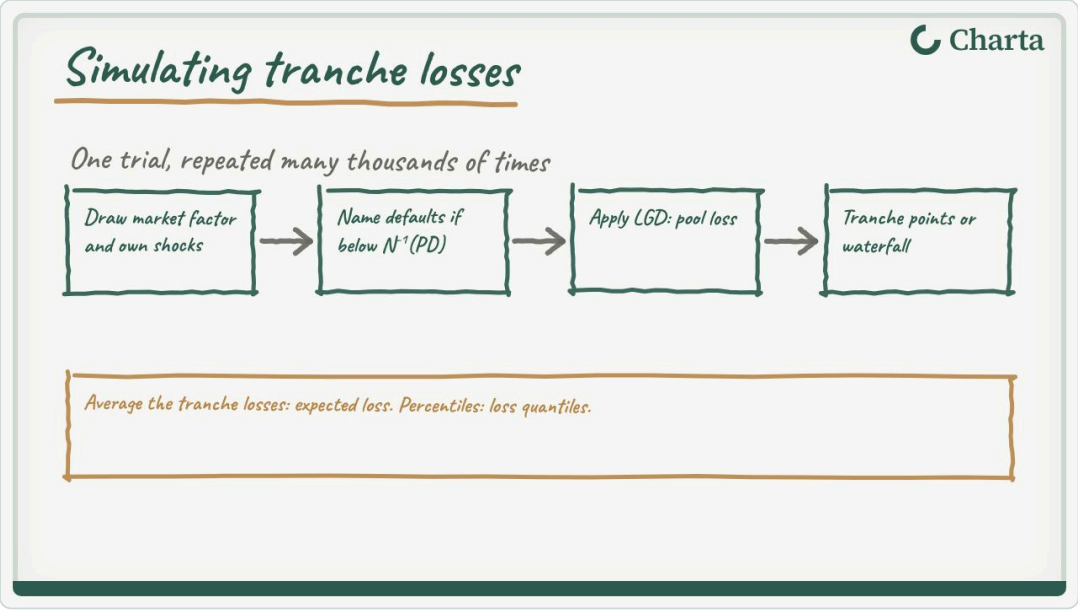
Trust 3: collateral $850m, notes $750m, trigger 120%, required $900m, OC starts at $30m

Year 1 test	790 + 30 = 820 vs 900	**Short 80**
Year 1 excess spread: 50	all 50 trapped in OC	**Equity 0**
Year 2 test	780 + 80 = 860 vs 900	**Short 40**
Year 2 excess spread: 90	40 to OC, rest released	**Equity 50**
OC balance, end of year 2	80 + 40	**120**

IN THE LESSON

- Harbourline Trust three has collateral of eight hundred and fifty million dollars, senior notes of six hundred million, mezzanine of one hundred and fifty million, and a trigger of one hundred and twenty per cent.
- Required is one hundred and twenty per cent of seven fifty, which is nine hundred.
- At the end of year one, cumulative losses are sixty, so collateral is seven ninety, and the O C account already holds thirty.
- Seven ninety plus thirty is eight twenty, short by eighty.
- Excess spread is only fifty, so all fifty is trapped, equity gets nothing, and the O C account rises to eighty.
- In year two cumulative losses are seventy: seven eighty plus eighty is eight sixty, short by forty.
- Excess spread is ninety.
- Forty goes to O C, and fifty is released to equity.

19 Simulating tranche losses



IN THE LESSON

- Pricing a tranche means knowing its losses, and the usual tool is Monte Carlo simulation.
- In each trial, draw correlated default events for the collateral.
- A Gaussian copula is typical.
- Every name has a latent variable built from a common market factor and its own shock, and defaults when that falls below the inverse normal of its default probability.
- Apply the loss given default to the defaulted names to get the pool loss.
- Run that through the tranche attachment and detachment points, or through the full waterfall, to get each tranche's loss.
- Repeat many thousands of times.
- The average over trials is each tranche's expected loss, and percentiles give loss quantiles.

20 Example 7: five trials

Tranches 0-10%, 10-20%, 20-100%. Pool loss in five trials: 2, 5, 9, 14, 26%

Equity: average of losses	20, 50, 90, 100, 100	**72%**
Mezzanine: average	0, 0, 0, 40, 100	**28%**
Senior: average	0, 0, 0, 0, 7.5	**1.5%**
Wrong: tranche the average 11.2%	equity / mezz / senior	**100 / 12 / 0%**
Check: pool loss	0.1×72 + 0.1×28 + 0.8×1.5	**11.2%**

IN THE LESSON

- Here are five trials, kept tiny for illustration.
- The pool losses are two, five, nine, fourteen and twenty six per cent.
- Equity loses twenty, fifty, ninety, one hundred and one hundred per cent, averaging seventy two.
- Mezzanine loses nothing in three trials, forty per cent in one and one hundred in another, averaging twenty eight.
- Senior loses only in the last trial, seven point five per cent, averaging one point five.
- The common error is to average the pool loss first, eleven point two per cent, and tranche that.
- It gives equity one hundred per cent, mezzanine twelve and senior zero: wrong in all three.
- And because senior losses occur in very few trials, senior needs a great many trials.

21 Default probability and correlation

Pjoint = p1 × p2 + ρ × √(p1(1 − p1) × p2(1 − p2))

Default probability, p1	Chance asset 1 defaults
Default probability, p2	Chance asset 2 defaults
Default correlation, rho	Zero gives independence
Both default	Both default in the period

IN THE LESSON

- Why does correlation matter?
- Expected pool loss depends only on default probabilities and loss given default, so correlation does not change it.
- Correlation changes how it is distributed.
- With low correlation, defaults are spread evenly, the pool loss stays close to its mean, equity absorbs it, and senior is nearly safe.
- With high correlation, outcomes are all or nothing: either few defaults occur, or many occur together.
- That lowers the expected loss of equity and raises that of senior, with mezzanine in between.
- For two assets with default probabilities p one and p two and correlation rho, the joint default probability is p one times p two, plus rho times the square root of p one times one minus p one times p two times one minus p two.

22 Example 8: two borrowers

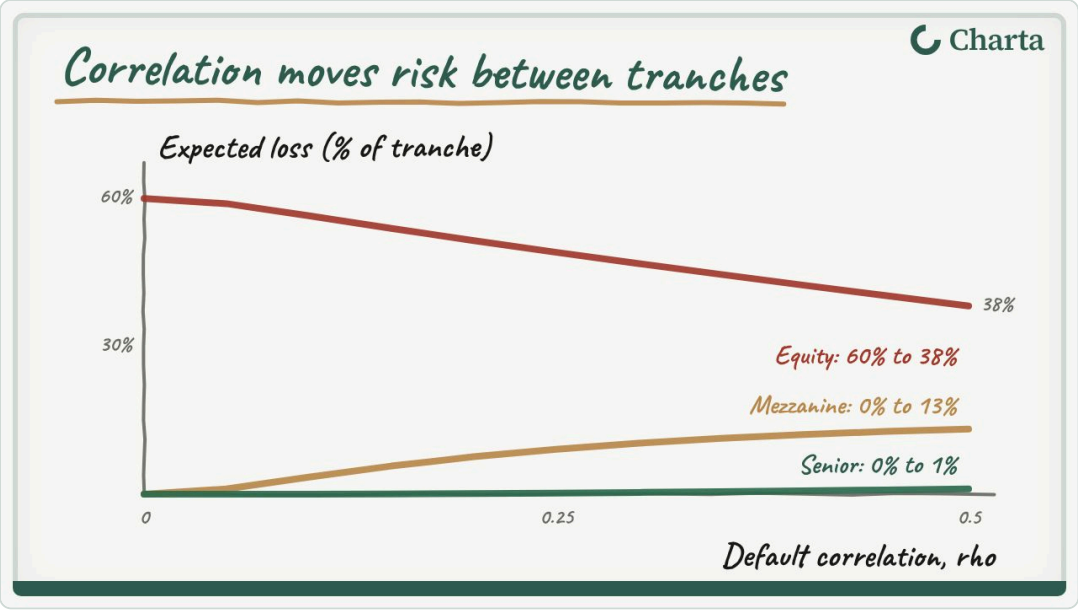
p1 = 3%, p2 = 5%, correlation ρ = 0.2

If independent	3% × 5%	**0.150%**
Correlation term	0.2 × √(0.03×0.97×0.05×0.95)	**0.744%**
Joint probability	0.150% + 0.744%	**0.894%**
Compared with independent	0.894 ÷ 0.150	**6.0 times**
Chance of 2nd if 1st defaults	0.894% ÷ 3%	**29.8%**

IN THE LESSON

- Take two Harbourline borrowers.
- One has a three per cent chance of default, the other five per cent, and their default correlation is point two.
- If they were independent, both would default with probability three per cent times five per cent, which is point one five per cent.
- The correlation term is point two times the square root of point zero three times point nine seven times point zero five times point nine five.
- The root is about point zero three seven two, so the term is about point seven four four per cent.
- The joint probability is about point eight nine four per cent, six times the independent figure.
- So if the first borrower defaults, the second follows with probability twenty nine point eight per cent, against five per cent normally.
- Correlation moves risk between tranches without changing the pool's expected loss.

23 Correlation moves risk between tranches



IN THE LESSON

- This chart shows the effect on a Harbourline pool with a ten per cent default probability and sixty per cent loss given default, using a one factor Gaussian copula.
- Along the bottom is correlation, from zero to point five.
- Up the side is each tranche's expected loss as a share of its own notional.
- Equity, the top line, falls from sixty per cent towards thirty eight.
- Mezzanine climbs from nothing to thirteen per cent.
- Senior hugs the floor, but rises from nothing to just over one per cent.
- These are small numbers, but they are the losses of the tranche everyone thought safe, and they arrive in extreme states of the world.
- Correlation, the least observable input, decides who bears the risk.

24 Tranche default sensitivities

Sensitivity = (value after bump − value before) ÷ size of bump
Same random numbers in both runs

The bump	A small change in one input
Change in value or loss	Tranche value or expected loss
Sensitivity	Per unit of tranche notional

IN THE LESSON

- How sensitive is a tranche to the collateral's credit risk?
- Sensitivity is measured by bump and revalue.
- First, change one input by a small amount: the default probability, the credit spread or the correlation.
- Then re-run the pricing or the simulation, using the same random numbers, so that the difference is the effect of the bump and not simulation noise.
- Then divide the change in tranche value or expected loss by the size of the change in the input.
- The second form is the loss caused by one more default.
- It depends on how close the pool already is to the tranche's attachment point, so sensitivity is not linear in the number of defaults.

25 Example 9: bumping the default probability

PD 10%→11%, LGD 60%, ρ = 0.3. Extra default: 1% × 60% = 0.6% of pool

Equity expected loss	46.79% → 50.14%	**+3.35 pts**
Mezzanine expected loss	10.34% → 12.21%	**+1.87 pts**
Senior expected loss	0.36% → 0.46%	**+0.10 pts**
One more default, pool at 5%	0.6 ÷ 10 of equity	**+6% of equity**
One more default, pool at 24%	0.6 ÷ 80 of senior	**+0.75% of senior**

IN THE LESSON

- Harbourline's pool again: default probability ten per cent, loss given default sixty per cent, correlation point three.
- Bump the default probability by one percentage point, to eleven per cent.
- Equity's expected loss rises from forty six point seven nine to fifty point one four per cent, up three point three five points.
- Mezzanine rises one point eight seven points.
- Senior rises only point one zero points, from point three six to point four six per cent.
- Per unit of notional, equity is the most sensitive, about thirty four times senior, so sensitivities also serve as hedge ratios against index positions.
- Now one more default, a one per cent name at sixty per cent loss, so point six per cent of the pool.
- If the pool has lost five per cent, it costs equity six per cent of its notional.
- If the pool has lost twenty four, it costs senior just point seven five.

26 The risk factors of structured products

Each factor and where it comes from

Factor	Source
Market	Interest rates and credit spreads
Prepayment	Mortgage and consumer pools
Liquidity	Wide bid-ask spreads in stress
Counterparty	Protection seller, swap provider
Model	Unobservable inputs, correlation

IN THE LESSON

- A structured product carries the credit risk of its collateral: default probability, recovery, and above all default correlation.
- But there are other risk factors, and each has its own source.
- Market risk comes from interest rates and credit spreads, which move tranche prices even when nothing defaults.
- Prepayment risk matters for mortgage and consumer pools, because borrowers repay early.
- Liquidity risk shows up as wide bid-ask spreads in stress.
- Counterparty risk arises in synthetic structures, through the protection seller or swap provider.
- Model risk comes from unobservable inputs, such as correlation.

27 Structure, leverage and systemic risk

- Credit core: default probability, recovery, correlation
- Structural: waterfall, triggers, servicer behaviour
- Leverage: loss = pool loss ÷ thickness, once attached
- Senior: pays in full except in extreme states (systemic)
- A rating is not a measure of all these risks

IN THE LESSON

- Three more factors.
- Structural risk comes from the waterfall, the triggers and the servicer's behaviour.
- Tranching also adds leverage.
- A tranche's loss rate, once attached, is the pool loss divided by its thickness, so a ten per cent thick equity tranche loses ten times the pool's rate: a pool loss of three per cent costs it thirty per cent.
- And senior tranches carry systemic risk.
- They pay in full except in extreme states of the world, so their losses arrive exactly when the whole economy is in trouble.
- Which brings the exam point: do not treat a rating as a measure of all of these risks.

28 Implied correlation

- Implied rho: correlation that matches the market price
- Root-finding: try, price, compare, adjust
- Compound: per tranche; mezzanine: two answers or none
- Base: each tranche = difference of two equity tranches
- Different rho across tranches (smile): model misspecified

IN THE LESSON

- Now the idea behind the correlation numbers quoted in the market.
- Implied correlation is the default correlation that, put into a pricing model such as the Gaussian copula, reproduces the observed market price or spread of a tranche, just like implied volatility.
- It is found by root finding: try a correlation, price the tranche, compare with the market, and adjust until they match.
- Compound correlation does this tranche by tranche.
- But a mezzanine tranche's price may not be monotonic in correlation, so it can yield two answers or none.
- Base correlation avoids that by treating each tranche as the difference between two equity tranches from zero, each with its own correlation.
- When different tranches imply different correlations, the correlation smile, the single correlation model is misspecified.

29 Example 10: finding an implied correlation

Market: equity (0-10%) expected loss 45%. Pool PD 10%, LGD 60%

Try ρ = 0.30	model gives 46.8%	**too high**
Try ρ = 0.40	model gives 42.4%	**too low**
Try ρ = 0.35	model gives 44.6%	**a little low**
Try ρ = 0.34	model gives 45.0%	**match**
Implied correlation	root-finding result	**about 0.34**

IN THE LESSON

- Let us find an implied correlation.
- The market price of Harbourline's equity tranche implies an expected loss of forty five per cent of its notional.
- The pool has a default probability of ten per cent and loss given default of sixty per cent, and the tranche runs from zero to ten per cent.
- Equity's expected loss falls as correlation rises.
- Try point three: the model gives forty six point eight per cent, too high, so raise the correlation.
- Try point four: forty two point four, too low.
- Try point three five: forty four point six, still a little low.
- Try point three four: forty five point zero, a match.
- The implied correlation is about point three four.
- Equity is monotonic in correlation, so this works; mezzanine might not be.

30 Base correlation

EL(A to D) = EL(0 to D) − EL(0 to A)
Example: 5.6% − 4.5% = 1.1% of pool; ÷ 10% = 11%

EL of tranche 0 to D	Equity tranche up to D
EL of tranche 0 to A	Equity tranche up to A
EL of tranche A to D	As a % of the pool

IN THE LESSON

- Base correlation builds each tranche from equity tranches.
- Let E L of the tranche from A to D, as a percentage of the pool, equal the expected loss of the zero to D tranche, minus the expected loss of the zero to A tranche.
- Each of those two equity tranches uses the base correlation implied for its own detachment point.
- Suppose the zero to twenty tranche has an expected loss of five point six per cent of the pool, and the zero to ten tranche four point five.
- The difference is one point one per cent of the pool.
- Divide by the thickness, ten per cent, and Harbourline's mezzanine has an expected loss of eleven per cent of its notional.

31 Why structured credit is created

Separate balance sheet motives from arbitrage motives

Motive	Idea
Balance sheet	Off balance sheet, free capital, risk transfer
Cheaper funding	Below own unsecured borrowing
Arbitrage	Pool yield above tranche cost; equity keeps it
Tailoring	Triple A senior; leveraged junior
Liquidity	Illiquid loans become tradable

IN THE LESSON

- Why do these products exist?
- Separate two families of motives.
- Balance sheet motives belong to originators.
- Harbourline can move loans off its balance sheet, free regulatory capital, transfer credit risk, and raise cheaper funding than its own unsecured borrowing would give.
- Arbitrage structures buy a pool of higher yielding assets and fund it with tranches whose combined cost is lower, with the equity holder keeping the spread.
- Tranching tailors risk and return to investors.
- A pool of lower rated loans can support a senior tranche rated triple A, for investors with rating or capital constraints, while the junior tranches serve those who want leveraged credit exposure.
- And securitisation creates liquidity from illiquid assets.

32 Example 11: the arbitrage in numbers

Buy $1,000m of loans at 8%. Senior $800m at 5%, mezzanine $100m at 7%, equity $100m

Pool income	8% × 1,000	**$80m**
Senior cost	5% × 800	**$40m**
Mezzanine cost	7% × 100	**$7m**
Left for equity	80 − 40 − 7	**$33m**
Equity return, before losses	33 ÷ 100	**33%**

Extra senior yield is not free: it pays for correlation, model and liquidity risk

IN THE LESSON

- Here is the arbitrage in numbers.
- Harbourline's vehicle buys a thousand million dollars of loans yielding eight per cent, so pool income is eighty million.
- It funds them with senior notes of eight hundred million at five per cent, costing forty million, and mezzanine notes of one hundred million at seven per cent, costing seven.
- What is left for the equity holder, who put in one hundred million, is thirty three million: a return of thirty three per cent, before any losses.
- And a related trap: treating the extra yield that a senior tranche pays, compared with an equally rated bond, as free.
- It compensates for correlation, model and liquidity risk.

33 Recap: the main ideas

- Classify by collateral; cash or synthetic; tranched or not
- Tranche loss = max(0, min(L, D) − A) ÷ thickness
- Waterfall: priority order, stop when cash runs out
- OC test: value + OC vs trigger × notes; shortfall trapped
- Correlation moves risk between tranches, not pool EL

IN THE LESSON

- Let us pull it together.
- Structured products are named by their collateral, and are cash or synthetic, pass-through or tranched.
- A tranche loses the larger of zero and the smaller of pool loss and detachment, minus attachment, divided by its thickness.
- The waterfall pays in priority order and stops when cash runs out.
- Interim cash flows charge interest on performing balances only, and write losses off against equity.
- The year-end test adds collateral value to the O C account, compares with the trigger times the notes, and traps excess spread before equity sees any.
- Simulation averages tranche losses.
- Correlation moves risk between tranches.
- Sensitivities come from bump and revalue, implied correlation from root finding, and the motives are balance sheet or arbitrage.

34 Try three quick questions

1. Tranche 5%-15%, pool loses 11%. Share of the tranche lost?
   Answer: (11 − 5) ÷ 10 = 60%.
2. Notes $400m, trigger 115%, collateral $500m − $50m losses, OC $5m, excess $12m. To equity?
   Answer: Required 460; 450 + 5 = 455; $5m to OC, $7m to equity.
3. p1 = 2%, p2 = 4%, ρ = 0.1. Joint default probability?
   Answer: 0.08% + 0.1 × 2.74% = about 0.35%.

IN THE LESSON

- Now try three quick questions.
- Pause the video, think, then listen to the answers.
- Question one.
- A tranche attaches at five per cent and detaches at fifteen per cent.
- The pool loses eleven per cent.
- What share of the tranche is lost?
- Eleven minus five is six, over a thickness of ten: sixty per cent.
- Question two.
- Notes are four hundred million dollars, the trigger is one hundred and fifteen per cent, collateral is five hundred million less fifty million of cumulative losses, the O C balance is five million, and excess spread is twelve million.
- What reaches equity?
- Required is four sixty, value plus O C is four fifty five, so five goes to O C and seven million to equity.
- Question three.
- Two assets have default probabilities of two per cent and four per cent, with correlation point one.
- Joint default probability?
- The root is about two point seven four per cent, a tenth is point two seven, and point zero eight plus point two seven is about point three five per cent.