CHARTA LEARNING · LESSON NOTES

Portfolio Risk: Analytical Methods

FRM Part II

1 The running example: the Corran Balanced Fund

- Corran Balanced Fund: $100m in two positions
- Equities $40m, volatility 16%
- Corporate bonds (credit) $60m, volatility 10%
- Correlation between them: 0.3
- VaR at 95% over one year, normal returns: z = 1.65

IN THE LESSON

- Here is the fund.
- Corran holds forty million dollars of equities, with a yearly volatility of sixteen per cent.
- Volatility means the standard deviation of returns.
- It also holds sixty million dollars of corporate bonds, which we will call credit, with a volatility of ten per cent.
- Their correlation is point three.
- The risk manager uses V a R, value at risk, the loss that will not be exceeded at a chosen confidence level over a chosen horizon.
- We assume normal returns and measure V a R at ninety five per cent over one year, so z is one point six five, the number of standard deviations that cuts off the worst five per cent.

2 Six VaR figures, six questions

Six figures describe one portfolio, and each answers its own question

VaR figure	Question it answers
Individual; undiversified	One position alone; sum of them all
Diversified	Whole fund, true correlations
Incremental	Add or drop a whole position
Marginal	One more dollar of a position
Component	Each position's share of the total

IN THE LESSON

- Six figures describe one portfolio, and each answers its own question.
- Individual V a R is one position's own risk, ignoring everything else held, and undiversified V a R is the sum of those individual figures.
- Diversified V a R is the risk of the whole fund using the true correlations.
- Incremental V a R is the change in fund V a R from adding or dropping a whole position.
- Marginal V a R is the change in fund V a R per extra dollar of one position.
- And component V a R is each position's share of the total, built so that the shares add up exactly to the diversified figure.
- The exam tests which figure answers which question, not only the arithmetic.

3 Individual and undiversified VaR

Individual VaR = z × σ × x
Undiversified VaR = sum of individual VaRs

z, the confidence multiplier	1.65 at the 95% level
Volatility of returns, σ	Volatility of the position
Position value, x	Dollars invested
Undiversified: every ρ = +1	Assumes every correlation is +1

IN THE LESSON

- Start with one position.
- Its individual V a R is z times its volatility times its value.
- Here z is the confidence multiplier, one point six five at ninety five per cent.
- Sigma is the volatility of the position's returns.
- And x is the dollars invested.
- Individual V a R ignores everything else the fund holds.
- Undiversified V a R adds the individual figures together.
- That is only right if every pair of positions moves in lockstep, which means every correlation is plus one.
- So undiversified V a R is what the fund's V a R would be if diversification gave no help at all.

4 Example 1: individual and undiversified VaR

Equities $40m, σ 16%; credit $60m, σ 10%; z = 1.65

Individual VaR, equities	1.65 × 16% × 40	**$10.56m**
Individual VaR, credit	1.65 × 10% × 60	**$9.90m**
Undiversified VaR	10.56 + 9.90	**$20.46m**

Undiversified VaR treats the correlation as +1

IN THE LESSON

- Equities are forty million at sixteen per cent, credit sixty million at ten per cent, and z is one point six five.
- For equities, one point six five times sixteen per cent times forty million gives ten point five six million.
- For credit, one point six five times ten per cent times sixty million gives nine point nine million.
- Credit is the bigger position in dollars, yet it has the smaller individual V a R, because its volatility is lower.
- Undiversified V a R is the sum, ten point five six plus nine point nine, which is twenty point four six million dollars.
- That figure treats the correlation between equities and credit as plus one.
- It is really only point three, so the true figure must be lower.

5 Diversified VaR

Diversified VaR = z × √(x^{T}Σx)
Two positions: x^{T}Σx = d1² + d2² + 2ρ × d1 × d2

Position value, x	Vector of position values
Covariance matrix, Σ	Variances and covariances
Dollar volatility, d	Each position's x × σ
Correlation, ρ	Correlation of the two returns

IN THE LESSON

- Diversified V a R uses the real correlations.
- Write x for the vector, or list, of position values, and capital sigma for the covariance matrix of returns, which holds every variance and covariance.
- Then x transpose, sigma, x is the variance of the fund's profit and loss in dollars, and diversified V a R is z times its square root.
- With two positions it is simpler than it looks.
- Write d one and d two for the dollar volatilities, each position's value times its volatility.
- The dollar variance is d one squared plus d two squared plus two times rho times d one times d two, where rho is the correlation.
- Under normal returns, diversified V a R is never above the undiversified figure.

6 Example 2: Corran's diversified VaR

d1 = 40 × 16% = 6.4; d2 = 60 × 10% = 6.0; ρ = 0.3. Amounts in $m

Variances of returns	0.16^2 and 0.10^2	**0.0256, 0.01**
Covariance of returns	0.3 × 0.16 × 0.10	**0.0048**
Dollar variance	$6.4^2 + 6^2 + 2\times0.3\times6.4\times6$	**100**
Fund volatility	√100, or 10% of $100m	**$10m**
Diversified VaR	1.65 × 10	**$16.5m**

Diversification benefit: 20.46 − 16.5 = $3.96m

IN THE LESSON

- The dollar volatilities are forty times sixteen per cent, six point four million, and sixty times ten per cent, six million, and rho is point three.
- The variances of returns are point one six squared, point zero two five six, and point one squared, point zero one.
- The covariance is point three times point one six times point one, point zero zero four eight.
- Now the dollar variance: six point four squared is forty point nine six, six squared is thirty six, and the cross term, two times point three times six point four times six, is twenty three point zero four.
- They sum to one hundred, so fund volatility is ten million dollars, ten per cent of the fund.
- Diversified V a R is one point six five times ten, sixteen point five million.
- The benefit is twenty point four six minus sixteen point five, three point nine six million.

7 Incremental VaR

- Change in fund VaR on adding or dropping a position
- Exact: recompute the fund's VaR with and without it
- Answers: should we hold this position at all?
- Depends on how it moves with what is already held
- Trap: incremental VaRs do not add up to the total

IN THE LESSON

- Incremental V a R is the change in diversified V a R when a whole position is added or dropped.
- It is computed exactly, by working out the fund's V a R with the position and again without it, and taking the difference.
- It answers a yes or no question: should we hold this position at all?
- Two positions with the same individual V a R can change fund risk by very different amounts, because it depends on how each moves with what is already held.
- That leads to the usual trap.
- Each incremental figure is measured against a different fund, so the incremental V a Rs generally do not add up to the total.

8 Example 3: incremental VaR

Fund VaR $16.5m. Drop each position in turn and recompute

Drop credit: equities alone	fund VaR becomes	**$10.56m**
Incremental VaR, credit	16.5 − 10.56	**$5.94m**
Drop equities: credit alone	fund VaR becomes	**$9.90m**
Incremental VaR, equities	16.5 − 9.90	**$6.60m**
Sum of incrementals	5.94 + 6.60	**$12.54m**

12.54 is not 16.5: incremental VaRs do not add up

IN THE LESSON

- Corran's fund V a R is sixteen point five million.
- Drop credit, and the fund is equities alone, with a V a R of ten point five six million.
- So the incremental V a R of credit is sixteen point five minus ten point five six, five point nine four million.
- Drop equities instead, and credit alone has a V a R of nine point nine million, so the incremental V a R of equities is sixteen point five minus nine point nine, six point six million.
- The two incremental figures sum to five point nine four plus six point six, twelve point five four million, well short of sixteen point five.

9 Marginal VaR

Marginal VaR_i = z × cov(R_i, R_p) ÷ σ_p
Marginal VaR_i = (VaR ÷ W) × β_i

Covariance with the fund	How the position moves with fund
Fund volatility, σp	Fund's volatility of returns
Fund value, W	VaR ÷ W: VaR per fund dollar
Beta to the fund, β	cov(R_i, R_p) ÷ σ_p²

IN THE LESSON

- Marginal V a R measures how much fund V a R moves for each extra dollar of one position.
- For position i it is z times the covariance of that position's return with the fund's return, divided by sigma p, the fund's volatility of returns.
- Covariance is used because what matters is how the position moves with the fund, not on its own.
- There is a neater form.
- Write W for the fund's value, so V a R divided by W is V a R per dollar of fund.
- Multiply that by beta, the position's covariance with the fund divided by the fund's variance.
- A beta above one means more than average risk per dollar.

10 Example 4: each position's beta

Weights: equities 40%, credit 60%. Fund volatility 10%, so σp² = 0.01

Cov(equities, fund)	0.4×0.0256 + 0.6×0.0048	**0.01312**
Beta, equities	0.01312 ÷ 0.01	**1.312**
Cov(credit, fund)	0.6×0.01 + 0.4×0.0048	**0.00792**
Beta, credit	0.00792 ÷ 0.01	**0.792**
Check: weighted betas	0.4×1.312 + 0.6×0.792	**1.000**

IN THE LESSON

- First the betas.
- Equities are forty per cent of the fund and credit sixty per cent, and the fund's variance is ten per cent squared, point zero one.
- The covariance of equities with the fund is point four times point zero two five six plus point six times point zero zero four eight, which is point zero one three one two.
- Divide by point zero one, and the beta of equities is one point three one two.
- For credit, point six times point zero one plus point four times point zero zero four eight is point zero zero seven nine two, so its beta is point seven nine two.
- Check: point four times one point three one two plus point six times point seven nine two is exactly one, as a fund's beta to itself must be.

11 Example 5: Corran's marginal VaRs

Fund VaR $16.5m on a fund of $100m

Fund VaR per dollar	16.5 ÷ 100	**0.165**
Marginal VaR, equities	0.165 × 1.312	**0.2165**
Same, by the first formula	1.65 × 0.01312 ÷ 0.10	**0.2165**
Marginal VaR, credit	0.165 × 0.792	**0.1307**
Reading it	one more $1m of equities	**VaR + $0.22m**

IN THE LESSON

- Fund V a R per dollar is sixteen point five over one hundred, point one six five.
- For equities, multiply by the beta: point one six five times one point three one two is point two one six five.
- The first formula agrees: one point six five times point zero one three one two, divided by point one, is also point two one six five.
- For credit, point one six five times point seven nine two is point one three zero seven.
- Each extra million dollars of equities adds about two hundred and sixteen thousand dollars of V a R, roughly point two two million.

12 Component VaR

Component VaR_i = Marginal VaR_i × x_i
Sum of components = diversified VaR

Marginal VaR	Change in VaR per extra dollar
Position value, x	Size of the position in dollars
Component VaR	Adds up exactly to the total
A hedge	Negative component: cuts risk

IN THE LESSON

- Component V a R turns marginal V a R into shares of the total.
- The component V a R of position i is its marginal V a R times its size in dollars.
- The components add up exactly to diversified V a R, with nothing left over and nothing counted twice.
- Scale every position up in the same proportion and V a R scales up in that proportion too, and that property makes the sizes times the marginal figures sum to the whole.
- A component can also be negative.
- A hedge has a negative marginal V a R, so its component is negative: it takes risk away from the total.

13 Example 6: Corran's component VaRs

Marginal VaRs 0.2165 and 0.1307 per dollar; positions $40m and $60m

Component, equities	0.2165 × 40	**$8.66m**
Component, credit	0.1307 × 60	**$7.84m**
Sum of components	8.66 + 7.84	**$16.50m**
Share of risk, equities	8.66 ÷ 16.5	**52.5%**
Share of risk, credit	7.84 ÷ 16.5	**47.5%**

Equities: 40% of the money, 52.5% of the risk (0.4 × 1.312)

IN THE LESSON

- For equities, point two one six five times forty million is eight point six six million.
- For credit, point one three zero seven times sixty million is seven point eight four million.
- Their sum is sixteen point five million, exactly the diversified V a R.
- So equities carry eight point six six over sixteen point five, about fifty two point five per cent of the risk, and credit carries the other forty seven point five per cent.
- Equities hold only forty per cent of the money but more than half of the risk.
- In fact each risk share is the weight times the beta, and point four times one point three one two is about fifty two point five per cent.

14 The six figures side by side

Corran's VaR figures in $m; marginal VaR is per dollar

Figure	Equities	Credit	Total
Individual	10.56	9.90	20.46: undiversified
Incremental	6.60	5.94	12.54: not the total
Marginal, per $	0.2165	0.1307	none
Component	8.66	7.84	16.50: diversified

IN THE LESSON

- Here are Corran's figures in millions, with marginal V a R per dollar.
- The individual figures, ten point five six and nine point nine, add to twenty point four six, the undiversified V a R.
- The incremental figures, six point six and five point nine four, add to only twelve point five four, which is not the total.
- The marginal V a Rs, point two one six five and point one three zero seven, are per dollar, so adding them would mean nothing, and no total is shown.
- The components, eight point six six and seven point eight four, add to sixteen point five, the diversified V a R.

15 Correlation and two-position VaR

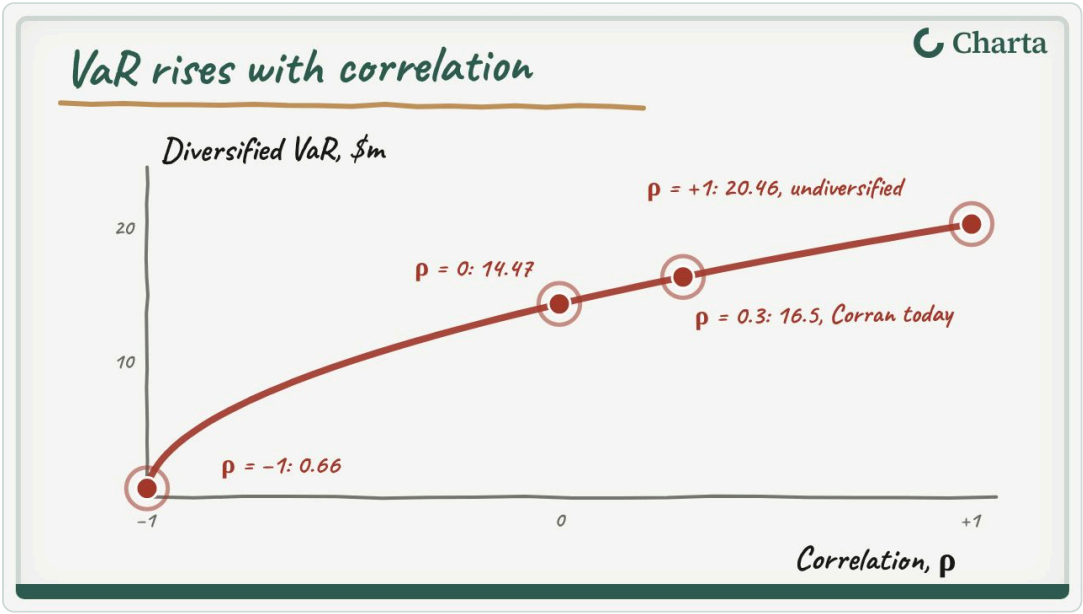
VaR_d = √(V1² + V2² + 2ρ × V1 × V2)
ρ = +1: V1 + V2 ρ = 0: √(V1² + V2²) ρ = −1: abs(V1 − V2)

Individual VaRs V1, V2	Same horizon and confidence level
Correlation, ρ	Correlation of the two returns
Diversification benefit	Undiversified − diversified

IN THE LESSON

- Suppose returns are normal, or more generally elliptical, and both V a Rs use the same horizon and confidence level.
- Then the diversified V a R of two positions follows from their individual V a Rs, V one and V two, and their correlation rho.
- It is the square root of V one squared plus V two squared plus two times rho times V one times V two.
- At rho of plus one this is V one plus V two, the undiversified V a R.
- At rho of zero it is the square root of V one squared plus V two squared.
- At rho of minus one it is the absolute difference between V one and V two, so equal and opposite positions cancel completely.
- The diversification benefit is undiversified minus diversified V a R.

16 VaR rises with correlation



IN THE LESSON

- This chart shows Corran's diversified V a R as the correlation between equities and credit moves from minus one to plus one, with the individual V a Rs fixed at ten point five six and nine point nine million.
- At plus one, on the right, it reaches twenty point four six, the undiversified figure.
- At Corran's actual correlation of point three it is sixteen point five.
- At zero it is the square root of ten point five six squared plus nine point nine squared, about fourteen point four seven.
- At minus one it falls to the difference, just point six six million.
- Holding positions fixed, a higher correlation always raises portfolio V a R, and the benefit shrinks.

17 Example 7: a floor under diversification

Equal weights; each stock σ = 20%; every pair ρ = 0.25

Rule for N equal weights	σ × √(1/N + (1 − 1/N) × ρ)	**volatility**
N = 1	20% × √1	**20.0%**
N = 4	20% × √(0.25 + 0.75×0.25)	**13.2%**
N = 25	20% × √(0.04 + 0.96×0.25)	**10.6%**
N very large: the floor	20% × √0.25	**10.0%**

IN THE LESSON

- Correlation also sets a floor.
- Take a separate illustration, not Corran's own numbers.
- A manager is choosing how many stocks to hold, in equal weights.
- Each has a volatility of twenty per cent, and every pair a correlation of point two five.
- With N stocks, volatility is sigma times the square root of one over N plus one minus one over N, times rho.
- With one stock, twenty per cent.
- With four, twenty per cent times the root of point two five plus point seven five times point two five, about thirteen point two per cent.
- With twenty five, twenty per cent times the root of point zero four plus point nine six times point two five, about ten point six per cent.
- As N grows, volatility falls towards sigma times the square root of rho, twenty per cent times point five, ten per cent: common risk that no number of stocks removes.

18 Example 8: correlations jump in a crisis

In a crisis, the equity-credit correlation jumps from 0.3 to 0.9

VaR at ρ = 0.3, today	from Example 2	**$16.5m**
Cross term at ρ = 0.9	2 × 0.9 × 10.56 × 9.90	**188.18**
VaR at ρ = 0.9	√(10.56² + 9.90² + 188.18)	**$19.94m**
Benefit today	20.46 − 16.5	**$3.96m**
Benefit in the crisis	20.46 − 19.94	**$0.52m**

IN THE LESSON

- In a crisis, assets that normally move loosely together tend to fall together, and correlations jump towards one.
- Suppose Corran's correlation between equities and credit jumps from point three to point nine.
- Today's V a R, at point three, is sixteen point five million.
- At point nine the cross term is two times point nine times ten point five six times nine point nine, about one hundred and eighty eight point one eight.
- Add the squares of ten point five six and nine point nine, take the root, and V a R is about nineteen point nine four million.
- The diversification benefit shrinks from three point nine six million to just point five two million.
- V a R built on calm period correlations understates losses exactly when it matters.

19 Marginal VaR for decisions

- Change in VaR ≈ marginal VaR × amount traded
- Rank by marginal VaR, not individual or component VaR
- Highest marginal VaR: most risk cut per dollar sold
- Negative marginal VaR: a hedge; adding to it lowers VaR
- A big position with a low marginal VaR: a poor place to cut

IN THE LESSON

- A manager wants to know how the fund's V a R will respond to a small change in one position, and marginal V a R answers that directly.
- The estimated change in V a R from a trade is marginal V a R times the amount traded.
- So rank positions by marginal V a R, not by individual or component V a R.
- The position with the highest marginal V a R cuts the most risk per dollar sold.
- A negative marginal V a R marks a hedge: adding to it lowers V a R.
- And a big position with a low marginal V a R can be a poor place to cut.

20 Example 9: where to cut

Corran wants to cut risk by selling $5m of one position

Marginal VaR, equities	per dollar	**0.2165**
Marginal VaR, credit	per dollar	**0.1307**
Sell $5m of equities	5 × 0.2165	**VaR − $1.08m**
Sell $5m of credit	5 × 0.1307	**VaR − $0.65m**
Biggest position is credit	$60m, but low marginal VaR	**cut equities**

IN THE LESSON

- Corran wants to cut risk by selling five million dollars of one position.
- Equities have a marginal V a R of point two one six five per dollar, and credit point one three zero seven.
- Selling five million of equities is estimated to cut V a R by five times point two one six five, about one point zero eight million.
- Selling five million of credit cuts it by five times point one three zero seven, only about point six five million.
- Notice the trap.
- Credit is the biggest position, at sixty million dollars, but it has the low marginal V a R.
- A manager who cuts the biggest holding by habit gets less risk reduction per dollar, so the answer is to cut equities.

21 Example 10: small trades and big trades

Linear estimate versus exact recomputation. Amounts in $m

Sell $5m equities: estimate	5 × 0.2165	**− $1.08m**
Exact: equities now $35m	$1.65\times\sqrt{(5.6^2+6^2+2\times0.3\times5.6\times6)}$	**$15.44m**
Exact change	16.5 − 15.44	**− $1.06m**
Sell all $40m: estimate	40 × 0.2165	**− $8.66m**
Exact: credit alone	16.5 − 9.90	**− $6.60m**

Use marginal VaR for direction and rough size; recompute big trades

IN THE LESSON

- Marginal V a R is a linear approximation, the slope at today's mix.
- For a five million dollar sale of equities, the estimate is a cut of one point zero eight million.
- Exactly, equities fall to thirty five million, a dollar volatility of five point six million, and V a R is one point six five times the root of five point six squared plus six squared plus two times point three times five point six times six, fifteen point four four million.
- The exact cut is sixteen point five minus fifteen point four four, one point zero six million, very close.
- Now sell all forty million of equities.
- The estimate is a cut of eight point six six million, but the exact fall, to credit's nine point nine, is only six point six million.
- Use marginal V a R for direction and rough size, and recompute big trades.

22 Example 11: a hedge has a negative marginal VaR

Corran sells $20m of equity futures that track its equities

Net equity exposure	40 − 20	**$20m**
Fund dollar volatility	√(3.2² + 6² + 2×0.3×3.2×6)	**$7.6m**
Fund VaR	1.65 × 7.6	**$12.54m**
Marginal VaR of the hedge	−1.65 × 0.8 ÷ 7.6	**−0.1737**
Component of the hedge	20 × −0.1737	**−$3.47m**

0.8 = 20 × 0.0256 + 60 × 0.0048, sign reversed. Per $ sold, so size = $20m sold

IN THE LESSON

- Now a hedge.
- Corran sells twenty million dollars of equity index futures, which we assume move exactly with its equities.
- Net equity exposure falls to forty minus twenty, twenty million, a dollar volatility of three point two million.
- With credit's six million and rho of point three, fund dollar volatility is exactly seven point six million, and V a R is one point six five times seven point six, twelve point five four million.
- The hedge's marginal V a R is minus one point six five times point eight over seven point six, minus point one seven three seven per dollar sold, so its size is the twenty million sold, and its component is twenty times that, minus three point four seven million: it takes risk away.
- The note shows where point eight comes from: twenty times point zero two five six plus sixty times point zero zero four eight, with the sign reversed because the hedge is short.

23 Two special positions

- Minimum risk: fully invested mix with the lowest volatility
- Condition: every position has the same marginal VaR
- Best Sharpe ratio: most excess return per unit of risk
- Condition: excess return ÷ marginal VaR equal for all
- Equivalently: excess return = β × fund excess return

IN THE LESSON

- Two special portfolios come from the same marginal V a R machinery.
- The first is the minimum risk position, the fully invested mix with the lowest volatility.
- There, every position has the same marginal V a R.
- If one were higher than another, moving a dollar from it to the other would cut risk, so the mix could not be the minimum.
- The second is the best ratio of expected excess return to risk, the highest Sharpe ratio, where excess return means return above the risk free rate.
- Its condition is that each position's expected excess return divided by its marginal V a R is the same for all.
- Equivalently, each position's excess return equals its beta times the fund's excess return.

24 The minimum-risk mix of two assets

$w1 = (s2^2 - \rho \times s1 \times s2) \div (s1^2 + s2^2 - 2\rho \times s1 \times s2)$
$w2 = 1 - w1$

Weight in asset 1	Fully invested: w1 + w2 = 1
Volatilities s1, s2	Volatilities of assets 1 and 2
Correlation, ρ	Correlation of the two returns

IN THE LESSON

- For two assets the minimum risk weights have a formula.
- Let w one be the weight in asset one, s one and s two the volatilities, and rho the correlation.
- The weight in asset one is s two squared minus rho times s one times s two, divided by s one squared plus s two squared minus two times rho times s one times s two.
- The weight in asset two is one minus w one, because the fund is fully invested.
- If s two squared is smaller than rho times s one times s two, the weight in asset one turns negative, a short position.
- And these weights depend only on risk, never on expected returns.

25 Example 12: Corran's minimum-risk mix

Equities s1 = 16%, credit s2 = 10%, ρ = 0.3; covariance 0.0048

Numerator	0.01 − 0.0048	**0.0052**
Denominator	0.0256 + 0.01 − 2×0.0048	**0.0260**
Weight in equities	0.0052 ÷ 0.0260	**20%**
Check: cov with fund, equities	0.2×0.0256 + 0.8×0.0048	**0.00896**
Check: cov with fund, credit	0.2×0.0048 + 0.8×0.01	**0.00896**

Volatility √0.00896 = 9.466%; VaR 1.65 × 9.466 = $15.62m, not $16.5m

IN THE LESSON

- Corran has s one of sixteen per cent, s two of ten per cent and rho of point three.
- The numerator is credit's variance, point zero one, minus the covariance, point zero zero four eight, which is point zero zero five two.
- The denominator is point zero two five six plus point zero one minus two times point zero zero four eight, point zero two six.
- The weight in equities is point zero zero five two over point zero two six, exactly twenty per cent, so eighty per cent goes to credit.
- Now the check.
- The covariance of equities with the fund is point two times point zero two five six plus point eight times point zero zero four eight, point zero zero eight nine six.
- For credit, the second check row gives the same, point zero zero eight nine six.
- So the marginal V a Rs are equal.
- Volatility is nine point four seven per cent, so V a R is fifteen point six two million, against sixteen point five today.

26 Example 13: is Corran at the best Sharpe ratio?

Forecast excess returns: equities 6.56%, credit 3.96%

Fund excess return	0.4×6.56% + 0.6×3.96%	**5.00%**
Equities: return ÷ marginal VaR	6.56% ÷ 0.2165	**0.303**
Credit: return ÷ marginal VaR	3.96% ÷ 0.1307	**0.303**
Beta check, equities	1.312 × 5.00%	**6.56%**
Sharpe: today vs minimum risk	5÷10 vs 4.48÷9.47	**0.50 vs 0.47**

4.48 = 0.2×6.56 + 0.8×3.96. Never mix the conditions or use individual VaR

IN THE LESSON

- Corran forecasts excess returns of six point five six per cent for equities and three point nine six for credit.
- The fund's excess return is point four times six point five six plus point six times three point nine six, exactly five.
- Divide by marginal V a R: six point five six over point two one six five is about point three zero three for equities, and three point nine six over point one three zero seven is point three zero three for credit too.
- The ratios are equal, so today's mix has the best Sharpe ratio.
- The beta check agrees: one point three one two times five is six point five six.
- Compare Sharpe ratios: five over ten is point five today.
- The minimum risk mix earns point two times six point five six plus point eight times three point nine six, four point four eight, over nine point four seven, about point four seven.
- Never mix up the two conditions, and never divide by individual V a R.

27 Risk management and portfolio management

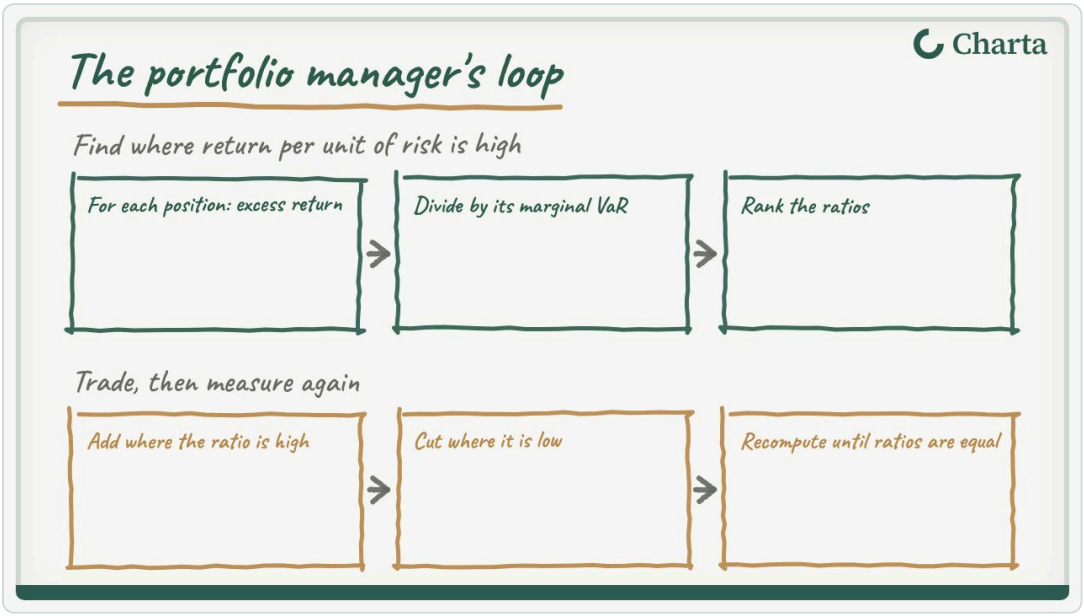
Two jobs that overlap but are not the same

	Risk management	Portfolio management
Aim	Measure, report, limit risk	Best return for the risk
Main tool	VaR and its limits	Risk plus return forecasts
Question	How much could we lose?	What should we hold?
Marginal VaR	Sizes a cut under a limit	Moves money to return per risk

IN THE LESSON

- Risk management and portfolio management overlap, but they are not the same job.
- Risk management aims to measure, report and limit the risk being taken, while portfolio management aims for the best expected return for the risk.
- The risk manager's main tool is V a R and its limits, while the portfolio manager combines risk figures with return forecasts.
- The risk manager asks how much the fund could lose, and the portfolio manager asks what it should hold.
- Marginal V a R links the two.
- It sizes the cut that brings V a R back under a limit, and it moves money towards positions with a high return per unit of risk.

28 The portfolio manager's loop



IN THE LESSON

- Here is the portfolio manager's loop.
- The first row is to find where return per unit of risk is high.
- For each position, take the expected excess return.
- Divide it by that position's marginal V a R.
- Then rank the ratios.
- The second row is to trade, then measure again.
- Add to positions where the ratio is high, and cut those where it is low.
- Recompute until the ratios are equal, because marginal V a Rs move as the mix changes.
- At that point no shift of money can raise the Sharpe ratio any further.
- The loop needs both sets of inputs: the risk figures from the risk manager and the return forecasts from the portfolio manager.

29 Example 14: back under a VaR limit

VaR limit $15m; fund VaR $16.5m. Cut equities, the highest marginal VaR

Excess VaR	16.5 − 15	**$1.5m**
Cut = excess ÷ marginal VaR	1.5 ÷ 0.2165	**$6.93m**
Exact VaR after the sale	equities 40 − 6.93 = $33.07m	**$15.036m**
Second pass: new MVaR 0.2054	0.036 ÷ 0.2054	**$0.18m more**
Total sale; VaR	6.93 + 0.18 = 7.11	**VaR $15.00m**

IN THE LESSON

- Corran's V a R limit is fifteen million dollars and V a R is sixteen point five, so the excess is one point five million.
- Cut equities, the position with the highest marginal V a R.
- The required sale is the excess divided by marginal V a R, one point five over point two one six five, about six point nine three million.
- With equities at thirty three point zero seven million, the exact V a R is fifteen point zero three six million, a touch over.
- Recomputed at the new mix, the marginal V a R of equities is point two zero five four, so a second pass sells point zero three six over point two zero five four, about point one eight million more.
- In total about seven point one one million is sold, and V a R is fifteen million.

30 Example 15: sizing a hedge

Size an equity-futures hedge so the hedge's marginal VaR is zero

Hedge marginal VaR = 0 when	net × 0.0256 + 60 × 0.0048 = 0	**condition**
Net equity exposure	−0.288 ÷ 0.0256	**− $11.25m**
Futures to sell	40 + 11.25	**$51.25m**
Fund VaR	$1.65\times\sqrt{(1.8^2 + 6^2 - 2\times0.3\times1.8\times6)}$	**$9.44m**

1.8 = 11.25 × 16%. The short also offsets some credit risk

IN THE LESSON

- A hedge is correctly sized when the hedge instrument's marginal V a R is zero.
- Then V a R is at its lowest.
- For Corran's equity futures, that happens when the net equity exposure times point zero two five six, plus sixty times point zero zero four eight, equals zero.
- Sixty times point zero zero four eight is point two eight eight, so the net equity exposure is minus point two eight eight over point zero two five six, minus eleven point two five million.
- Corran sells forty plus eleven point two five, fifty one point two five million of futures.
- At sixteen per cent, the net short has a dollar volatility of one point eight million, and the formula on the board gives a V a R of about nine point four four million.
- The short also offsets some credit risk.

31 Example 16: splitting a VaR limit across desks

A firm-wide VaR limit of $15m, split between two desks

Equities' share of risk	8.66 ÷ 16.5	**52.48%**
Credit's share of risk	7.84 ÷ 16.5	**47.52%**
Equity desk limit	52.48% × 15	**$7.87m**
Credit desk limit	47.52% × 15	**$7.13m**
Check: limits add up	7.87 + 7.13	**$15.00m**

Components add up exactly; individual VaRs do not

IN THE LESSON

- Last, component V a R allocates a total limit.
- Corran's firm wide V a R limit of fifteen million must be split between the equity desk and the credit desk.
- Because components add up exactly, the split can follow them.
- Equities carry eight point six six over sixteen point five, fifty two point four eight per cent of the risk, and credit seven point eight four over sixteen point five, forty seven point five two per cent.
- The equity desk limit is fifty two point four eight per cent of fifteen, seven point eight seven million, and the credit desk limit is forty seven point five two per cent of fifteen, seven point one three million.
- They add to exactly fifteen.
- Components add up exactly, and individual V a Rs do not.

32 Try three quick questions

1. Individual VaRs $12m and $5m, correlation 0. Diversified VaR and benefit?
   Answer: √(144 + 25) = $13m; benefit 17 − 13 = $4m.
2. Fund $200m, VaR $20m; a position's beta is 1.5. Marginal VaR; sell $4m?
   Answer: 0.1 × 1.5 = 0.15 per $; VaR falls about $0.6m.
3. Volatilities 30% and 10%, correlation 0. Minimum-risk weight in the first?
   Answer: 0.01 ÷ (0.09 + 0.01) = 10%.

IN THE LESSON

- Now try three quick questions.
- Pause, think, then listen.
- Question one.
- Two positions have individual V a Rs of twelve million and five million dollars, with zero correlation.
- What are the diversified V a R and the benefit?
- The root of one hundred and forty four plus twenty five is thirteen million, and the benefit is seventeen minus thirteen, four million dollars.
- Question two.
- A two hundred million dollar fund has a V a R of twenty million, and a position has a beta of one point five.
- What is its marginal V a R, and what does selling four million do?
- Point one times one point five is point one five per dollar, so V a R falls by about point six million.
- Question three.
- Two assets have volatilities of thirty and ten per cent and zero correlation.
- What is the minimum risk weight in the first?
- Point zero one over the sum of point zero nine and point zero one, which is ten per cent.