CHARTA LEARNING · LESSON NOTES

VaR and Risk Budgeting

FRM Part II

1 Meet Redwater Pension Plan

- Redwater: a defined-benefit plan, $1,000m of assets
- Policy mix: 70% equities, 30% bonds
- Managers A and B run equities; Manager C runs bonds
- A trustee asks: is 70/30 a balanced split of risk?

IN THE LESSON

- Meet Redwater, a made up defined benefit pension plan.
- Defined benefit means the plan has promised its members a pension, so it has liabilities to meet.
- Redwater has one thousand million dollars of assets.
- Its policy mix, the long term split of its money, is seventy per cent equities and thirty per cent bonds.
- The money is run by outside managers.
- Manager A and Manager B run equities, and Manager C runs bonds.
- At a meeting, a trustee asks whether seventy thirty is a balanced split of risk.
- It is not, and seeing why is where this lesson starts.
- Money and risk are different things.

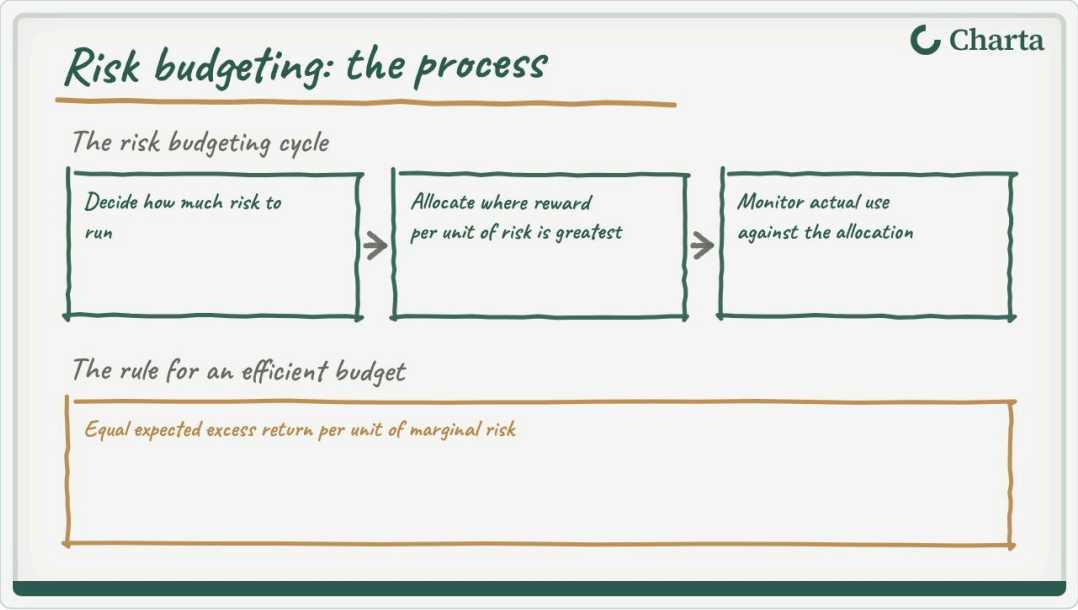
2 What a risk budget is

- Risk budget: the total risk an investor will bear
- Stated in risk units: VaR or tracking error
- Divided among asset classes, managers or strategies
- Built from component VaRs, which sum to the total
- Stand-alone VaRs do not add if correlations are below 1

IN THE LESSON

- So what is a risk budget?
- It is the total amount of risk an investor is willing to bear.
- The budget is stated in risk units, such as V a R, value at risk, the loss that should be exceeded only with a small probability over a given horizon.
- Or it is stated as tracking error, the volatility of returns relative to a benchmark.
- That total is then divided among asset classes, managers or strategies.
- Budgets are built from component V a Rs, the pieces each part contributes, which add up exactly to the total.
- Stand-alone V a Rs, each part measured on its own, do not add up when correlations are below one.

3 Risk budgeting: the process



IN THE LESSON

- Risk budgeting is the process around the budget.
- The risk budgeting cycle has three steps.
- First, decide how much risk to run in total.
- For a pension plan, that comes from its liabilities and how much risk the sponsor can bear.
- Second, allocate that risk where the expected reward per unit of risk is greatest.
- Third, monitor the actual use of risk against the allocation.
- Underneath sits the rule for an efficient budget.
- It tends to equalise the expected excess return per unit of marginal risk, the extra risk the fund takes on from a small extra dose of a position.
- If one manager earns more per unit of marginal risk than another, moving budget toward it improves the fund.

4 Example 1: capital share is not risk share

Equities: 70% of capital, volatility 16%. Bonds: 30%, volatility 5%

Equities: weight × volatility	70% × 16%	**11.2 points**
Bonds: weight × volatility	30% × 5%	**1.5 points**
Equities' share, rough	11.2 ÷ 12.7	**88%**
Equities' share, exact	with correlation: at the end	**95.8%**

Capital 70/30 is risk about 96/4: budgets are shares of risk

IN THE LESSON

- Let us test the trustee's question.
- Equities are seventy per cent of capital, with a volatility of sixteen per cent a year.
- Bonds are thirty per cent, with a volatility of five per cent.
- A rough look multiplies each weight by its volatility.
- Equities give seventy per cent times sixteen per cent, eleven point two percentage points.
- Bonds give thirty per cent times five per cent, one point five points.
- On this rough view, equities carry eleven point two out of twelve point seven, about eighty eight per cent of the risk.
- Handling correlation properly, as we do at the end, the equity share is ninety five point eight per cent.
- So seventy thirty in capital is about ninety six four in risk.
- Budgets are shares of risk, not shares of capital.

5 Holding period, turnover and leverage

- Holding period: months or years, so longer horizons
- Turnover: VaR from today's holdings describes today only
- High turnover: monitor often; long-horizon VaR says little
- Leverage: VaR against equity grows with assets ÷ equity
- Exam: match the horizon to the time to change positions

IN THE LESSON

- Three features of an investment operation decide how V a R should be used.
- The first is the holding period.
- A pension fund holds positions for months or years and cannot unwind quickly, so it uses weekly, monthly or longer horizons.
- The second is turnover, how fast the portfolio changes.
- V a R is computed from today's holdings, so it describes today's portfolio only.
- For a manager with high turnover, it says little about risk over a long horizon, so monitoring must be frequent.
- The third is leverage, meaning borrowing to hold more assets.
- Measured against equity capital, V a R grows with assets divided by equity.
- The exam point ties these together: match the horizon to the time needed to change the position.

6 Example 2: stretching the horizon

Redwater's one-day 95% VaR is $12.1m

One-day VaR	reported by the risk system	**$12.1m**
Square-root-of-time rule	VaR(T) = VaR(1) × √T	**if positions fixed**
25 days, about five weeks	12.1 × √25 = 12.1 × 5	**$60.5m**
100 days to rebalance	12.1 × √100 = 12.1 × 10	**$121m**

IN THE LESSON

- Redwater's risk system reports a one day V a R, at ninety five per cent confidence, of twelve point one million dollars.
- But Redwater cannot change its portfolio in a day.
- The square root of time rule stretches the horizon: V a R over T days is the one day V a R times the square root of T.
- It holds only if positions stay fixed and daily returns are independent of one another.
- Over twenty five trading days, about five weeks, multiply by the square root of twenty five, which is five, giving sixty point five million.
- If a large rebalance takes about one hundred trading days, multiply by ten, giving one hundred and twenty one million dollars.

7 Leverage: VaR measured against equity

Equity VaR = Asset VaR × A ÷ E
Not × D ÷ E: a common error

Asset VaR	VaR as a share of total assets
A	Total assets
E	Equity capital
D	Debt: not the factor

IN THE LESSON

- Now leverage.
- When a fund borrows, its dollar V a R scales with its total assets.
- But the owners care about losses measured against their equity capital, the money they put in.
- So the equity V a R, as a share of equity, is the asset V a R, as a share of assets, multiplied by the leverage factor, total assets, ay, divided by equity, E.
- The common error, on the second line, is to multiply by debt over equity instead.
- In the terms, asset V a R is measured as a share of total assets.
- Total assets are ay.
- Equity capital is E.
- Debt is D, and it is not the factor.
- Debt over equity is always one less than assets over equity, because assets equal debt plus equity, so that mistake understates the owners' risk.

8 Example 3: a leveraged hedge fund

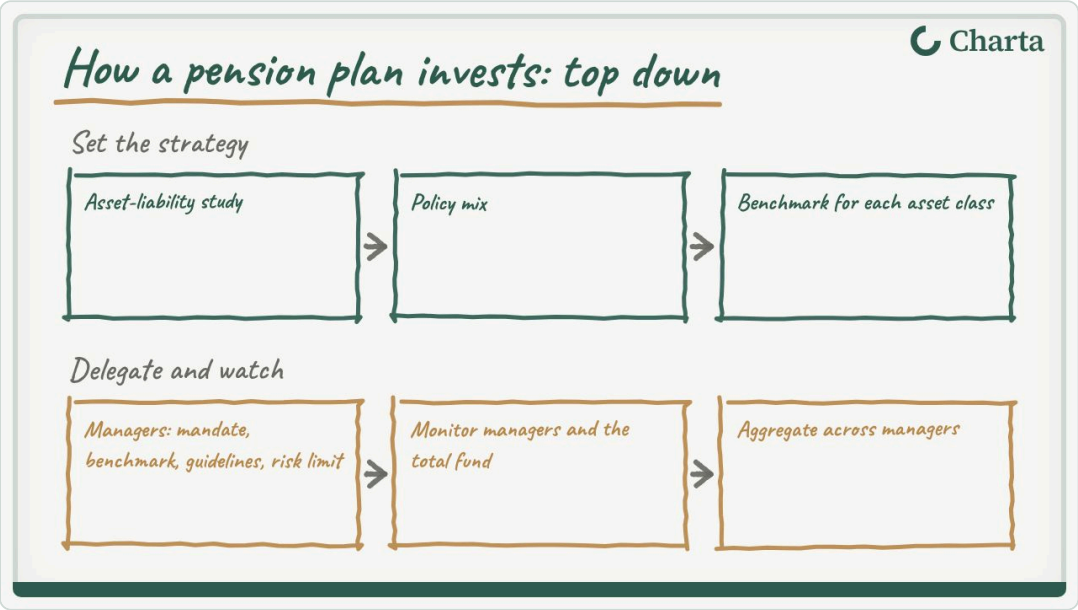
Fenwick fund: assets $500m, equity $100m, debt $400m. Asset VaR 5%

Asset VaR in dollars	5% × 500	**$25m**
Leverage factor	A ÷ E = 500 ÷ 100	**5**
Equity VaR	5% × 5	**25% of equity**
Check in dollars	25 ÷ 100	**25%**
Wrong: D ÷ E	5% × 400 ÷ 100 = 5% × 4	**20% of equity**

IN THE LESSON

- Redwater is looking at Fenwick, a made up hedge fund that uses leverage.
- It has assets of five hundred million dollars, financed by one hundred million of equity and four hundred million of debt.
- Its V a R is five per cent of assets.
- In dollars, that is five per cent of five hundred million, twenty five million.
- The leverage factor is assets over equity, five hundred over one hundred, which is five.
- So the equity V a R is five per cent times five, twenty five per cent of equity.
- Check it in dollars: twenty five million lost out of one hundred million of equity is twenty five per cent.
- Using debt over equity, which is four, would give only twenty per cent, and understate the owners' risk.

9 How a pension plan invests: top down



IN THE LESSON

- Large investors such as pension funds invest from the top down.
- They first set the strategy.
- It starts with an asset liability study, which examines the liabilities, the sponsor's finances and its tolerance for risk.
- The sponsor is the employer that must fund the plan.
- The study sets the policy mix, the long term allocation across asset classes.
- Next, a benchmark is chosen for each asset class, such as a broad equity index.
- Then they delegate and watch.
- The money goes to several outside managers.
- Each gets a mandate, a benchmark, guidelines and a risk limit.
- Some are passive, tracking the index, some active.
- Then the plan monitors the managers and the total fund against benchmarks and budgets.
- Finally, the plan aggregates risk across the managers.

10 Example 4: Redwater's structure

Policy mix: 70% equities, 30% bonds, on $1,000m of assets

Manager A: active equities	40% of the fund	**$400m**
Manager B: active equities	30% of the fund	**$300m**
Equities in total	400 + 300	**$700m = 70%**
Manager C: passive bonds	30% of the fund	**$300m**
Tracking-error budgets	A 5%, B 4%, C 0.5%	**in each mandate**

IN THE LESSON

- Here is Redwater's version.
- Its policy mix is seventy per cent equities and thirty per cent bonds, on one thousand million dollars.
- Manager A is an active equity manager running forty per cent of the fund, four hundred million dollars.
- Manager B is also an active equity manager, with thirty per cent, three hundred million.
- Together that is seven hundred million, the seventy per cent equity weight in the policy mix.
- Manager C runs a passive bond fund with thirty per cent, three hundred million.
- Each mandate carries a tracking error budget: five per cent for A, four per cent for B, and half of one per cent for C, since a passive fund should stay close to its index.

11 Who carries the total risk

- The policy mix drives most of total fund risk
- Active management usually adds a small share of variance
- Managers are watched one by one, so the plan aggregates
- Bets can offset: A buys bank shares, B sells them
- Or overlap: both make the same bet, and risk doubles up

IN THE LESSON

- Where does Redwater's risk come from?
- The policy mix drives most of the total fund risk.
- Active management, the managers' bets against their benchmarks, usually adds only a small share of the variance.
- But the managers are watched one by one, each against its own benchmark, and none sees the whole fund.
- So the plan itself must aggregate across them.
- Bets can offset.
- If Manager A buys extra bank shares and Manager B sells them, the plan pays two active fees for a position that nets to roughly nothing.
- Or bets can overlap.
- If both make the same bet, the risk doubles up, even though each looks fine alone.

12 Hedge funds: why risk is harder to see

- Limited transparency: positions and leverage unseen
- So no position-based VaR: work from reported returns
- Short histories, fat tails, option-like payoffs
- So a normal VaR from history is unreliable
- Illiquid holdings: smoothed returns understate volatility

IN THE LESSON

- Hedge funds make risk management harder.
- First, transparency is limited.
- The investor usually cannot see the fund's positions or its leverage, so it cannot compute a V a R from positions, and must work from the returns the fund reports.
- Second, those return histories are short and not normal.
- Many strategies have option like payoffs, steady small gains with occasional large losses, and fat tails, meaning extreme results are more common than a normal curve suggests.
- So a normal V a R fitted to that history is unreliable.
- Third, illiquid holdings are priced infrequently, so their reported returns are smoothed, and look calmer than the assets really are.

13 Smoothed returns understate volatility

Multiplier = (1 + ρ) ÷ (1 − ρ)
True variance = naive variance × multiplier

Serial correlation, rho	Serial correlation of returns
Naive variance	Variance if returns independent
Multiplier	Scales up to the long horizon

IN THE LESSON

- Smoothing leaves a fingerprint: each month's reported return carries over part of the previous month's.
- That is positive serial correlation, written rho.
- Suppose the pattern is first order autoregressive, meaning each return depends on the one just before.
- Then the multiplier is one plus rho, divided by one minus rho.
- The true long horizon variance is the naive variance times that multiplier.
- Here rho is the serial correlation of returns.
- The naive variance is the one you would compute assuming returns are independent.
- And the multiplier scales it up to the long horizon.
- With rho of zero, the multiplier is one and nothing changes.
- As rho rises, the multiplier grows fast, so the naive figure understates the true risk.

14 Example 5: unsmoothing Fenwick

Fenwick's reported volatility: 5% a year, naive. Serial correlation ρ = 0.6

Variance multiplier	(1 + 0.6) ÷ (1 − 0.6)	**4**
Volatility multiplier	√4	**2**
Adjusted volatility	5% × 2	**10%**
95% VaR, $50m stake, reported	1.645 × 5% × 50	**$4.1m**
95% VaR, adjusted	1.645 × 10% × 50	**$8.2m**

IN THE LESSON

- Fenwick reports a volatility of five per cent a year.
- That is the naive figure, monthly volatility scaled by the square root of time as if returns were independent.
- Its monthly returns have a serial correlation of point six.
- The variance multiplier is one point six divided by point four, which is four.
- Volatility is the square root of variance, so the volatility multiplier is the square root of four, which is two.
- The adjusted volatility is five per cent times two, ten per cent.
- Now take a fifty million dollar stake and a ninety five per cent V a R, with normal quantile one point six four five.
- On the reported figure, the V a R is one point six four five times five per cent times fifty million, about four point one million dollars.
- On the adjusted figure it is about eight point two million, twice as much.

15 More hedge fund challenges

- Lockups and notice periods: exposure cannot be cut fast
- So the VaR horizon should be longer
- Correlations with other assets rise in stress
- Operational and style-drift risk: due diligence
- Exam: historical hedge fund VaR usually understates risk

IN THE LESSON

- There are more challenges.
- Lockups are periods during which the investor may not withdraw, and redemption notice periods make it wait further after asking for its money.
- Both mean the exposure cannot be cut quickly, so the V a R horizon should be longer.
- Next, correlations between hedge funds and other assets tend to rise in stress, just when diversification is needed most.
- Finally there are operational risks, such as weak controls or pricing, and style drift risk, where a fund moves away from the strategy it was hired for.
- Both need due diligence, a careful investigation of the fund before and after investing.
- The exam point: historical hedge fund V a R usually understates true risk.

16 Six risks, part one: the asset side

Measured on the assets alone, or against a benchmark

Risk	What it measures
Absolute risk	Total volatility or VaR of the assets
Relative risk	Tracking error against a benchmark
Active management risk	Against the policy benchmark

IN THE LESSON

- Six kinds of risk are distinguished for a plan like Redwater, and the first three are measured on the assets alone, or against a benchmark.
- Absolute risk is the total volatility or V a R of the assets, with no reference to any benchmark.
- Relative risk is the volatility of returns relative to a benchmark, which is the tracking error.
- It applies to any portfolio and its benchmark, for example Manager A against its equity index.
- Active management risk is the tracking error of the actual portfolio against the policy benchmark.
- It is the risk that the managers' bets, taken together, add to the plan.

17 Six risks, part two: against the liabilities

A defined-benefit plan must pay the pensions it has promised

Risk	What it measures
Funding risk	Shortfall risk: surplus volatility
Policy-mix risk	Surplus risk from the strategic allocation
Sponsor risk	Sponsor earnings move with the surplus

IN THE LESSON

- A defined benefit plan must pay the pensions it has promised, so the other three risks bring in the liabilities.
- The surplus is assets minus liabilities.
- Funding risk is the risk that assets fall short of liabilities, measured as the volatility of the surplus.
- Policy mix risk is the surplus risk generated by the strategic allocation alone, that is, by the policy benchmark measured against the liabilities.
- Sponsor risk is the risk that the sponsor's own business weakens just when the plan needs contributions.
- It depends on how the sponsor's earnings move with the surplus.
- If one downturn cuts both share prices and the sponsor's profits, the deficit and the weakness arrive together.

18 Funding risk from its two parts

σ^2(funding) = σ^2(policy mix) + σ^2(active)
Uncorrelated: variances add, SDs do not

Funding risk	Volatility of the surplus
Policy-mix risk	Policy benchmark vs liabilities
Active risk	Actual vs policy benchmark

IN THE LESSON

- How do these fit together?
- The surplus moves for two reasons: the policy mix moves against the liabilities, and the managers move away from the policy mix.
- If active risk and policy mix risk are uncorrelated, the variance of funding risk is the sum of the two variances, policy mix variance plus active variance.
- Variance is the square of volatility, written sigma squared.
- The second line is the exam point.
- Variances add, but standard deviations do not, so you square, add, and then take the square root.
- In the terms, funding risk is the volatility of the surplus.
- Policy mix risk is the policy benchmark against the liabilities.
- And active risk is the actual portfolio against the policy benchmark.

19 Example 6: Redwater's funding risk

Policy-mix risk 10%. Active risk 2.337%, built at the end. Uncorrelated

Policy-mix variance	10^2	**100**
Active variance	2.337^2	**5.46**
Funding variance	100 + 5.46	**105.46**
Funding risk	√105.46	**10.27%**
Wrong: add the volatilities	10 + 2.337	**12.34%**

Active risk is only 5.46 ÷ 105.46 = 5.2% of the variance

IN THE LESSON

- Redwater's asset liability study puts its policy mix risk at ten per cent.
- Its active risk, built from its managers at the end of the lesson, is two point three three seven per cent.
- The two are uncorrelated, and we work in percentage points squared.
- The policy mix variance is ten squared, one hundred.
- The active variance is two point three three seven squared, about five point four six.
- The funding variance is one hundred plus five point four six, one hundred and five point four six.
- The funding risk is its square root, ten point two seven per cent.
- Adding the volatilities would wrongly give twelve point three four per cent.
- And active risk is only five point two per cent of the variance: the policy mix drives most of the risk.

20 Watching risk with VaR

- On a schedule: ex-ante VaR or tracking error
- For each manager, each asset class and the total fund
- Compare with the risk budget and limits
- Component or marginal VaR targets reductions
- Aggregate using correlations, not sums

IN THE LESSON

- Now to monitoring.
- V a R is used to watch risk by computing, on a regular schedule, the ex-ante V a R or tracking error of the current positions.
- Ex-ante means forward looking: the risk of the positions held now.
- This is done for each manager, for each asset class and for the total fund.
- The results are compared with the risk budget and the limits.
- When a figure is too high, component or marginal V a R shows which positions contribute most, so reductions are targeted at those, rather than cutting everything a little.
- And when figures are combined across managers or asset classes, aggregation must use correlations, not simple sums, because bets partly offset.

21 Ex-ante versus ex-post

- Ex-ante: the risk of the positions held now
- Ex-post: tracking error of past returns
- Ex-post lags changes in strategy
- Low realised risk can hide high forward risk
- Stand-alone VaR ignores correlation and can mislead

IN THE LESSON

- Compare two ways of measuring a manager's risk.
- Ex-ante measures reflect the positions held now.
- Ex-post tracking error looks backward: it is the volatility of the manager's past returns against the benchmark.
- Because it uses history, ex-post tracking error lags any change in strategy.
- So a manager whose realised tracking error is low may already run a much higher forward risk.
- One more caution: stand-alone V a R, a position measured on its own, ignores correlation and can mislead.
- A large position may hedge the rest of the fund, and a small one may add to its biggest bet.

22 Example 7: a breach the past would miss

Manager B runs $300m with a tracking-error budget of 4%

Realised tracking error	past three years	**3.5%**
Ex-ante tracking error	today's positions	**6.0%**
Over budget	6.0 − 4.0	**2.0 points**
In dollars	6% and 4% of $300m	**$18m vs $12m**
Action	cut the top contributors	**targeted**

IN THE LESSON

- Manager B runs three hundred million dollars with a tracking error budget of four per cent.
- Its realised tracking error over the past three years is three point five per cent, inside the budget, so a backward look raises no flag.
- But the risk team computes the ex-ante tracking error from today's positions, and finds six per cent.
- Six minus four is two percentage points over budget.
- In dollars, six per cent of three hundred million is eighteen million, against a budget of twelve million.
- The action is targeted.
- The team uses the decomposition to find the positions that contribute most, and asks the manager to cut those.

23 VaR needs support

- Delta-normal VaR misstates option risks
- Back-testing: losses beyond VaR as often as expected?
- Stress tests: extreme scenarios that VaR may not see
- Exam: monitor ex-ante, by contribution, against limits

IN THE LESSON

- V a R also needs supplementing.
- Delta normal V a R treats every position as a straight line response to market moves, with normally distributed returns.
- Options have curved payoffs, so this method misstates their risks, often understating them.
- Back-testing checks the model.
- It compares actual results with the V a R, and asks whether losses beyond the V a R happen about as often as the confidence level implies.
- For a ninety five per cent V a R, that is about one period in twenty.
- Stress tests look at extreme scenarios that a V a R built on normal times may not see.
- The exam point for monitoring: monitor ex-ante, by contribution, against limits.

24 Better guidelines: limit risk, not size

- Old: notional caps, e.g. derivatives at most 10% of assets
- New: a VaR or ex-ante tracking-error limit
- State the confidence level and the horizon
- Captures leverage, volatility and correlation
- Allows risk-reducing derivatives; caps risk-raising ones

IN THE LESSON

- V a R can also improve the investment guidelines, the written rules in each mandate.
- Old style guidelines use notional or instrument restrictions, for example a cap on derivatives at ten per cent of assets.
- But a notional cap says little about risk: a large hedge can make a portfolio safer.
- The new style replaces these with limits stated in risk: a V a R limit, or an ex-ante tracking error limit.
- Such a limit must state its confidence level and its horizon.
- It captures leverage, volatility and correlation all at once.
- And it allows derivatives that reduce risk, while capping those that raise it.

25 Example 8: Manager A's new guideline

Manager A: $400m mandate. One-year relative VaR limit at 99%: $46.5m

Normal quantile at 99%	z	**2.326**
Limit as a share of the mandate	46.5 ÷ 400	**11.6%**
Tracking-error limit	11.6% ÷ 2.326	**5.0%**
Rule	TE limit = VaR limit ÷ z	**matches A's budget**

IN THE LESSON

- Manager A runs a four hundred million dollar mandate.
- Its new guideline says its one year relative V a R, at ninety nine per cent, must not exceed forty six point five million dollars.
- Relative V a R is the V a R of the return measured against the benchmark.
- Under a normal distribution, the quantile at ninety nine per cent, written z, is two point three two six.
- As a share of the mandate, the limit is forty six point five over four hundred, about eleven point six per cent.
- Dividing by z gives the tracking error limit: eleven point six per cent over two point three two six, which is five per cent.
- The rule: tracking error limit equals V a R limit over z.
- It matches A's budget.

26 Example 9: removing an unrewarded bet

Manager A: expected active return 2%, tracking error 5%

Information ratio = return ÷ TE	2% ÷ 5%	**0.40**
Stock picks: variance	4^2	**16**
Unintended tilt: variance	3^2; earns nothing	**9**
Total tracking error	√(16 + 9)	**5%**
Tilt removed	2% ÷ √16 = 2% ÷ 4%	**IR 0.50**

IN THE LESSON

- Manager A expects an active return of two per cent with a tracking error of five per cent.
- The information ratio is the expected active return divided by the tracking error, the reward per unit of active risk, so A's is two over five, point four.
- Decompose A's risk.
- Its stock picks have a variance of four squared, sixteen.
- An unintended tilt toward small companies adds a variance of three squared, nine, and is expected to earn nothing.
- Uncorrelated, the total is the square root of sixteen plus nine, five per cent.
- Remove the tilt: tracking error falls to four per cent with no loss of expected return, and the information ratio rises to point five.
- Constrain risk rather than size, and remove risk that is not compensated.

27 Building the risk budget: the formulas

VaR = z × σp × fund value
βi = cov(Ri, Rp) ÷ var(Rp)
Risk share = wi × βi

z	Normal quantile: 1.645 at 95%
sigma p	Fund volatility
Beta of class i	Beta of class i to the fund
Weight, w i	Weight of class i in the fund

IN THE LESSON

- Now the risk budget.
- Total risk is the fund's V a R at the chosen confidence level: z, times the fund volatility sigma p, times the fund value.
- For each asset class i, we need its beta, measured against the fund itself.
- Beta is the covariance of the asset class return with the fund return, divided by the variance of the fund return.
- Asset class i's share of total fund risk is its weight, w i, times that beta.
- In the terms, z is the normal quantile, one point six four five at ninety five per cent.
- Sigma p is the fund volatility.
- Beta is each class's beta to the fund, and w is its weight.

28 Example 10: Redwater's volatility and VaR

Equities 70%, vol 16%; bonds 30%, vol 5%; correlation 0.2. Fund $1,000m

Equity term	0.7^2 × 0.16^2	**0.012544**
Bond term	0.3^2 × 0.05^2	**0.000225**
Cross term	2×0.7×0.3×0.2×0.16×0.05	**0.000672**
Fund variance; volatility	sum 0.013441; √	**11.59%**
95% VaR, one year	1.645 × 0.1159 × 1,000	**$190.7m**

Per day: 190.7 ÷ √250 = $12.06m, the $12.1m used earlier

IN THE LESSON

- First, the fund's volatility.
- Equities are seventy per cent at sixteen per cent volatility, bonds thirty per cent at five, with correlation point two.
- The equity term is point seven squared times point one six squared, point zero one two five four four.
- The bond term is point three squared times point zero five squared, point zero zero zero two two five.
- The cross term, two times both weights, the correlation and both volatilities, is point zero zero zero six seven two.
- The fund variance is the sum, point zero one three four four one, and its square root is eleven point five nine per cent.
- The one year V a R at ninety five per cent is one point six four five times that, times one thousand million, one hundred and ninety point seven million dollars.
- Divided by the square root of two hundred and fifty trading days, that is twelve point zero six million a day, the twelve point one used earlier.

29 Example 11: betas and risk shares

Variances 0.16^2 = 0.0256, 0.05^2 = 0.0025. Equity-bond cov 0.0016. Fund var 0.013441

Equities: cov with fund	0.7×0.0256 + 0.3×0.0016	**0.0184**
Equities: beta; share	0.0184 ÷ 0.013441 = 1.369	**× 70% = 95.8%**
Bonds: cov with fund	0.3×0.0025 + 0.7×0.0016	**0.00187**
Bonds: beta; share	0.00187 ÷ 0.013441 = 0.139	**× 30% = 4.2%**
Shares sum	95.8% + 4.2%	**100%**

Exam: covariance with the fund first, then times the weight

IN THE LESSON

- Next, the betas.
- The fund variance is point zero one three four four one.
- The variances are sixteen per cent squared, point zero two five six, and five per cent squared, point zero zero two five.
- The covariance between equities and bonds is point two times point one six times point zero five, point zero zero one six.
- A class's covariance with the fund adds its covariance with each class, times that class's weight.
- For equities: point seven times point zero two five six, plus point three times point zero zero one six, is point zero one eight four.
- Divided by the fund variance, beta is one point three six nine, and times seventy per cent the share is ninety five point eight per cent.
- For bonds, the covariance is point zero zero one eight seven, beta point one three nine, and times thirty per cent the share is four point two per cent.
- The shares sum to one hundred per cent.
- The exam point: covariance first, then times the weight.

30 Example 12: component versus stand-alone VaR

Fund VaR $190.7m. Stand-alone VaR = 1.645 × σi × wi × fund value

Equities: component VaR	95.8% × 190.7	**$182.7m**
Bonds: component VaR	4.2% × 190.7	**$8.0m**
Components sum	182.7 + 8.0	**$190.7m**
Stand-alone: equities; bonds	1.645×16%×700; ×5%×300	**$184.2m; $24.7m**
Stand-alone sum	184.2 + 24.7	**$208.9m**

Diversification benefit: 208.9 − 190.7 = $18.2m. Budget on components

IN THE LESSON

- Now the budget in dollars.
- The fund's V a R is one hundred and ninety point seven million.
- A stand-alone V a R is one point six four five, times the class's volatility, times its weight, times the fund value.
- Equities' component V a R is ninety five point eight per cent of one hundred and ninety point seven, which is one hundred and eighty two point seven million.
- Bonds' component is four point two per cent of it, eight million.
- The components sum to one hundred and ninety point seven million, exactly the total.
- Stand-alone, equities have one point six four five times sixteen per cent times seven hundred million, one hundred and eighty four point two million, and bonds have twenty four point seven million.
- Their sum, two hundred and eight point nine million, overstates the total by eighteen point two million, the diversification benefit.
- So budgets are built on components.

31 Active risk: the managers' budget

TE = √[Σ (wi × TEi)^2]
Share of i = (wi × TEi)^2 ÷ Σ (wi × TEi)^2

Weight, w i	Manager's weight in the fund
Manager tracking error, TE i	Manager's own tracking error
Fund tracking error, TE	The fund's active risk

IN THE LESSON

- The same idea works for active managers.
- Suppose their active returns are uncorrelated.
- Then the fund's tracking error is the square root of the sum of the squares of each manager's weight times its own tracking error.
- Manager i's share of active risk is its squared term divided by that sum.
- In the terms, w i is the manager's weight in the total fund.
- T E i is the manager's own tracking error against its benchmark.
- And T E on its own is the fund's tracking error, its active risk.
- Notice that the weight matters as much as the tracking error: a manager with a high tracking error but a small slice of the fund may use little of the budget.

32 Example 13: Redwater's managers

A: 40%, TE 5%. B: 30%, TE 4%. C: 30%, TE 0.5%. Uncorrelated

Weighted TEs: A, B, C	40%×5, 30%×4, 30%×0.5	**2.0, 1.2, 0.15**
Squared and summed	4 + 1.44 + 0.0225	**5.4625**
Fund tracking error	√5.4625	**2.337%**
Shares: A, B, C	each square ÷ 5.4625	**73.2, 26.4, 0.4%**
Wrong: sum, no square root	2.0 + 1.2 + 0.15	**3.35%**

Also wrong: weights as risk shares; summing stand-alone figures

IN THE LESSON

- Now Redwater's managers.
- A has forty per cent of the fund and a tracking error of five per cent, B thirty per cent and four per cent, C thirty per cent and half of one per cent.
- The weighted tracking errors are two, one point two, and point one five.
- Squared and summed: four, plus one point four four, plus point zero two two five, gives five point four six two five.
- The fund's tracking error is its square root, two point three three seven per cent, the active risk we used for funding risk.
- Each square divided by five point four six two five gives the shares: seventy three point two per cent for A, twenty six point four for B, and point four for C.
- Summing without the square root gives three point three five per cent, which is wrong.
- Also wrong: using weights as risk shares, or summing stand-alone figures.

33 Try three quick questions

1. Assets $800m, equity $200m, VaR 4% of assets. VaR as a share of equity?
   Answer: Factor A ÷ E = 4. 4% × 4 = 16% of equity.
2. Reported volatility 6% a year, serial correlation 0.5. Adjusted volatility?
   Answer: Multiplier 1.5 ÷ 0.5 = 3. 6% × √3 ≈ 10.4%.
3. Two uncorrelated managers, 50% each, TEs 4% and 2%. Fund TE, shares?
   Answer: Terms 2 and 1. √5 ≈ 2.24%. Shares 80% and 20%.

IN THE LESSON

- Now try three quick questions.
- Pause, think, then listen.
- Question one.
- A fund has assets of eight hundred million dollars and equity of two hundred million.
- Its V a R is four per cent of assets.
- What is the V a R as a share of equity?
- Assets over equity is four, so four per cent times four is sixteen per cent of equity.
- Question two.
- A hedge fund reports volatility of six per cent a year, with serial correlation of point five.
- What is the adjusted volatility?
- The variance multiplier is one point five over point five, which is three, so volatility is six per cent times root three, about ten point four per cent.
- Question three.
- Two uncorrelated managers each hold half of a fund, with tracking errors of four and two per cent.
- What is the fund's tracking error, and the shares?
- The weighted terms are two and one, so tracking error is root five, about two point two four per cent, and the shares are eighty and twenty per cent.