CHARTA LEARNING · LESSON NOTES

Asset-Liability Management and Duration

FRM Part II

1 A story: two hits from one rate rise

- Harbourside lends long at fixed rates, funds short
- Rates rise one point
- Hit 1: deposits reprice first, so income falls
- Hit 2: fixed-rate loans lose market value
- Both come from mismatched rates and maturities

IN THE LESSON

- Imagine Harbourside Bank.
- It lends long at fixed rates, making loans that run for years, and it funds them short, with deposits that reprice within months.
- One morning, market interest rates rise by one percentage point.
- The first hit: within months Harbourside must pay more on its deposits, while most of its loans still earn the old rate, so its net interest income falls.
- Net interest income is the interest earned on assets minus the interest paid on liabilities.
- The second hit: its fixed rate loans are now worth less, because new loans pay more, so the market value of its equity, its net worth, falls too.
- Both hits come from one source: assets and liabilities whose rates and maturities do not match.

2 Harbourside's balance sheet

Harbourside Bank, market values in $m. We use these figures all lesson

Assets	loans and securities	**$1,000m**
Liabilities	deposits and borrowing	**$920m**
Equity	1,000 − 920	**$80m**
Leverage ratio L ÷ A	920 ÷ 1,000	**0.92**
Equity share of assets	80 ÷ 1,000	**8%**

IN THE LESSON

- Here is Harbourside's balance sheet at market values, in millions of dollars, and we will use these figures for the whole lesson.
- Assets, its loans and securities, are worth one thousand million dollars.
- Liabilities, its deposits and borrowing, are worth nine hundred and twenty million.
- Equity is what is left: one thousand minus nine hundred and twenty, which is eighty million dollars.
- The leverage ratio, liabilities divided by assets, written L over ay, is nine hundred and twenty over one thousand, which is point nine two.
- And the equity share of assets is eighty over one thousand, only eight per cent.
- Keep that thin slice of equity in mind.
- A small move in asset values is a big move for equity.

3 Asset-liability management in three layers

- Targets: net interest income and market value of equity
- Layer 1, structural: reshape the balance sheet
- Layer 2, measurement-based: gap and duration gap
- Layer 3: derivatives and securitisation
- Exam: which tool fits which exposure

IN THE LESSON

- Asset liability management, or A L M, is how a bank manages the mix of its assets and liabilities to control this risk.
- It has two targets: net interest income, shown in the accounts each year, and the market value of equity, the economic value of the bank.
- A L M attacks the risk in layers.
- Layer one is structural: reshape the balance sheet itself.
- Layer two is measurement based: measure the exposure, then set a policy on it, using the gap and the duration gap.
- Gap management targets net interest income, and duration gap management targets the market value of equity.
- Layer three uses derivatives and securitisation: futures, swaps, caps, floors and loan sales.
- The exam often asks which tool fits which exposure.

4 Layer 1: structural choices

- Match repricing dates and maturities
- Shorten fixed-rate assets
- Lengthen or fix the rate on liabilities
- Make loans floating-rate
- But floating rates push risk to borrowers: credit risk

IN THE LESSON

- The simplest layer is structural.
- First, match repricing dates and maturities, so that assets and liabilities reset at about the same time.
- Second, shorten fixed rate assets, for example by making three year loans rather than ten year loans.
- Third, lengthen liabilities or fix the rate on them, for example by issuing five year term deposits or bonds rather than relying on overnight funding.
- Fourth, make loans floating rate, so that the rate the borrower pays resets with the market.
- That last choice has a catch.
- It does not remove the rate risk.
- Floating rates push the risk onto borrowers.
- When rates rise, the borrower's payments rise, and a borrower who cannot afford them may default.
- So floating rate lending trades interest rate risk for credit risk.

5 Example 1: a floating-rate loan moves the risk

Floating-rate loan $10m. Borrower's cash flow $0.9m a year. Rate rises from 5% to 8%

Interest at 5%	10m × 5%	**$0.5m**
Coverage at 5%	0.9 ÷ 0.5	**1.8 times**
Interest at 8%	10m × 8%	**$0.8m**
Coverage at 8%	0.9 ÷ 0.8	**1.125 times**

Harbourside's income is protected; the borrower's cushion is not

IN THE LESSON

- Harbourside lends ten million dollars at a floating rate to Kettle Joinery, a small manufacturer whose cash flow is nine hundred thousand dollars a year, and the rate then rises from five to eight per cent.
- At five per cent, interest is ten million times five per cent, five hundred thousand dollars.
- Coverage, cash flow divided by interest, is point nine over point five, one point eight times.
- At eight per cent, interest becomes ten million times eight per cent, eight hundred thousand dollars, and coverage falls to point nine over point eight, one point one two five times.
- Harbourside's interest income rose with the market, so Harbourside's income is protected.
- But the borrower's cushion is not: it has nearly gone, and the chance of default has risen.
- The rate risk has turned into credit risk.

6 Layer 2: measure, then set a policy

- Gap management: repricing within a window
- Targets net interest income over that window
- Duration gap management: timing of all cash flows
- Targets the market value of equity
- One protects income, the other net worth

IN THE LESSON

- The second layer starts by measuring the exposure, then sets a policy on it.
- Gap management, also called interest sensitive gap management, looks at repricing within a window: it sorts assets and liabilities by whether they reprice within a chosen period, such as the next twelve months.
- It targets net interest income over that window, which is what the accounts will show.
- Duration gap management looks instead at the timing of all cash flows, however far away, through a measure called duration.
- It targets the market value of equity, the bank's net worth.
- So one protects income, the other net worth, and they can give different answers for the same bank.
- We study each in depth shortly.
- First, the third layer.

7 Layer 3: derivatives

Mirror tools for the two exposures

Tool	Hurt by rising rates	Hurt by falling rates
Financial futures	Sell futures	Buy futures
Interest rate swap	Pay fixed, receive floating	Receive fixed, pay floating
Options	Buy a cap	Buy a floor

IN THE LESSON

- The third layer uses derivatives, and each tool comes in a mirror pair for the two exposures.
- Read the table row by row.
- With financial futures, a bank hurt by rising rates sells futures.
- Futures prices fall when rates rise, so the short position gains just as the balance sheet loses.
- A bank hurt by falling rates buys futures.
- With an interest rate swap, a bank hurt by rising rates pays fixed and receives floating.
- Its floating receipts offset rising deposit costs.
- A bank hurt by falling rates receives fixed and pays floating.
- With options, a bank hurt by rising rates buys a cap, which pays when rates rise above a strike rate.
- A bank hurt by falling rates buys a floor, which pays when rates fall below a strike rate.

8 Example 2: a cap, its cost and its upside

Cap on $200m at a 6% strike, premium $1m. One annual reset

Rate rises to 8%: payoff	200 × (8% − 6%)	**$4m**
Net gain after premium	4 − 1	**$3m**
Rate falls to 4%: payoff	4% is below the strike	**$0**
Cost if rates fall	premium only	**$1m**

Hedging costs money, but the bank keeps the gain from cheaper funding

IN THE LESSON

- Harbourside buys a cap on two hundred million dollars of floating rate funding, with a strike rate of six per cent and a premium of one million dollars.
- To keep it simple, the cap has one annual reset.
- Suppose the rate rises to eight per cent.
- The cap pays two hundred million times the excess over the strike, eight minus six, which is two per cent, so the payoff is four million dollars.
- After the premium, four minus one, the net gain is three million.
- Now suppose the rate falls to four per cent instead.
- Four per cent is below the strike, so the cap pays nothing, and the cost if rates fall is the premium only, one million.
- Hedging costs money, but with a cap the bank keeps the gain from cheaper funding.

9 Choosing a tool, and what hedging costs

- Sell or securitise fixed-rate assets
- Futures and swaps lock in a rate: lose the upside too
- Caps and floors keep the upside, for a premium
- Every hedge costs money or gives up some gain
- Exam: name the exposure first, then the tool

IN THE LESSON

- Two more points complete the toolkit.
- Loan sales and securitisation move fixed rate assets off the balance sheet altogether.
- Securitisation means pooling loans and selling securities backed by them to investors.
- Now compare the derivatives.
- Futures and swaps lock in a rate: they protect against the bad move, but you lose the upside too, the gain if rates move the bank's way.
- Caps and floors keep the upside, for a premium paid up front.
- Either way, every hedge costs money or gives up some gain, so a bank does not hedge everything.
- For the exam, name the exposure first.
- Is the bank hurt by rising rates or by falling ones, and is the target income or net worth?
- Then choose the tool.

10 Interest-sensitive gap: the idea

- Rate-sensitive: reprices within the chosen window
- Windows: e.g. 0–3 months, 3–12 months, 1–5 years
- RSA: rate-sensitive assets. RSL: rate-sensitive liabilities
- Gap = RSA − RSL in that window
- Fixed rate beyond the window: not rate-sensitive

IN THE LESSON

- Now the mechanics of gap management.
- An asset or liability is rate sensitive if it reprices within the chosen window.
- Repricing means its interest rate resets, either because it matures and is replaced, or because it is a floating rate item.
- The bank picks its windows, for example nought to three months, three to twelve months, and one to five years.
- Rate sensitive assets are written R S A, and rate sensitive liabilities are written R S L.
- The gap for a window is R S A minus R S L in that window.
- Anything whose rate stays fixed beyond the window, such as a ten year fixed rate mortgage when the window is one year, is not rate sensitive for that window.
- It still carries rate risk, but the gap does not see it.

11 Harbourside's repricing schedule

$m. The one-year gap adds the first two rows

Window	RSA	RSL	Gap	Running gap
0–3 months	250	400	−150	−150
3–12 months	150	150	0	−150
1–5 years	300	270	+30	−120
Over 5 years	300	100	+200	+80
Total	1,000	920	+80	= equity

IN THE LESSON

- Here is Harbourside's repricing schedule in millions, and the one year gap adds the first two rows.
- In the first window, nought to three months, two hundred and fifty of assets and four hundred of liabilities reprice, a gap of minus one hundred and fifty.
- From three to twelve months, one hundred and fifty reprice on each side, a gap of zero, so the running gap stays at minus one hundred and fifty.
- That is the one year gap.
- From one to five years, three hundred against two hundred and seventy, a gap of plus thirty, and the running gap is minus one hundred and twenty.
- Over five years, three hundred against one hundred, plus two hundred, and the running gap is plus eighty.
- In total, one thousand of assets against nine hundred and twenty of liabilities: the final running gap of eighty equals equity.

12 Gap, and what it does to income

Gap = RSA − RSL
ΔNII = Gap × Δr

Rate-sensitive assets	Assets repricing in window
Rate-sensitive liabilities	Liabilities repricing in window
Repricing gap	Positive: asset-sensitive
Change in the rate Δr	Same rate change on both sides

IN THE LESSON

- Here are the two formulas.
- The gap equals rate sensitive assets minus rate sensitive liabilities.
- Write N I I for net interest income, and delta for a change.
- If rates on both sides move by the same amount, delta r, the change in net interest income equals the gap times delta r.
- Now the terms.
- Rate sensitive assets are the assets repricing in the window, and rate sensitive liabilities are the liabilities repricing in the window.
- A positive gap means the bank is asset sensitive: more assets than liabilities reprice, so rising rates raise income and falling rates cut it.
- A negative gap is liability sensitive and does the reverse.
- And delta r must be the same rate change on both sides, or the formula fails.

13 Example 3: Harbourside's one-year gap

One-year window: RSA $400m, RSL $550m, total assets $1,000m

One-year gap	400 − 550	**−$150m**
Relative gap	−150 ÷ 1,000	**−15%**
Sensitivity ratio RSA ÷ RSL	400 ÷ 550	**0.73**
Rates rise 1 point: ΔNII	−150 × 1%	**−$1.5m**
Rates fall 1 point: ΔNII	−150 × (−1%)	**+$1.5m**

IN THE LESSON

- In Harbourside's one year window, rate sensitive assets are four hundred million dollars, rate sensitive liabilities are five hundred and fifty million, and total assets are one thousand million.
- The one year gap is four hundred minus five hundred and fifty, minus one hundred and fifty million.
- The relative gap is the gap divided by total assets, minus fifteen per cent.
- The interest sensitivity ratio is R S A divided by R S L, four hundred over five hundred and fifty, about point seven three.
- It is below one, as for any negative gap.
- If rates rise one point, net interest income changes by minus one hundred and fifty million times one per cent, a fall of one and a half million dollars a year.
- If rates fall one point, income rises by one and a half million.

14 Which gap wins in which forecast

Effect on net interest income over the window. NIM = NII ÷ earning assets

Gap	Rates rise	Rates fall
Positive (RSA > RSL)	Income rises	Income falls
Negative (RSA < RSL)	Income falls	Income rises
Zero (RSA = RSL)	No change	No change
Strategy	Go positive	Go negative

IN THE LESSON

- This table turns the formula into a strategy, showing the effect on net interest income over the window.
- Net interest margin, written N I M, is net interest income divided by earning assets, so more income on the same assets means a higher margin.
- With a positive gap, where R S A is greater than R S L, income rises when rates rise and falls when rates fall.
- With a negative gap, where R S A is less than R S L, income falls when rates rise and rises when rates fall.
- With a zero gap, where they are equal, the formula gives no change either way: income is hedged, at least on paper.
- So the strategy for the highest net interest margin follows.
- If you forecast rising rates, go positive.
- If you forecast falling rates, go negative.

15 Example 4: positioning for a rate rise

Base income $30m on $1,000m: margin 3%. Harbourside forecasts a 1-point rise

Keep gap at −150: income	30 − 150 × 1%	**$28.5m**
Margin with gap −150	28.5 ÷ 1,000	**2.85%**
Reprice: RSA 600, RSL 450	600 − 450	**gap +$150m**
Gap +150: income	30 + 150 × 1%	**$31.5m**
Margin with gap +150	31.5 ÷ 1,000	**3.15%**

If rates fall instead, the +150 gap cuts the margin to 2.85%

IN THE LESSON

- Harbourside's base income is thirty million dollars on one thousand million of earning assets, a margin of three per cent, and it forecasts a one point rise.
- If it keeps its gap at minus one hundred and fifty, income becomes thirty minus one and a half, twenty eight and a half million, a margin of two point eight five per cent.
- Instead, it moves two hundred million of fixed rate loans to floating rates, so R S A becomes six hundred, and it replaces one hundred million of short term deposits with two year term deposits, so R S L falls to four hundred and fifty.
- The gap is now plus one hundred and fifty.
- Income becomes thirty plus one and a half, thirty one and a half million, a margin of three point one five per cent.
- But this is a bet on the forecast.
- If rates fall instead, the positive gap cuts the margin to two point eight five per cent.

16 Example 5: when the two sides move differently

RSA $400m, RSL $550m. Asset rates rise 1.0 point, deposit rates only 0.6

Gap rule, 1-point move	−150 × 1%	**−$1.5m**
Asset income change	400 × 1.0%	**+$4.0m**
Liability cost change	550 × 0.6%	**+$3.3m**
Actual ΔNII	4.0 − 3.3	**+$0.7m**

Unequal moves: ΔNII = RSA × ΔrA − RSL × ΔrL. The gap rule's sign was wrong

IN THE LESSON

- Take Harbourside's original one year window, with R S A of four hundred million and R S L of five hundred and fifty.
- Asset rates rise one full point, but deposit rates rise only point six, because deposit rates often lag.
- The gap rule, using a one point move, predicts minus one hundred and fifty times one per cent, a fall of one and a half million.
- The asset income change is four hundred times one per cent, plus four million.
- The liability cost change is five hundred and fifty times point six per cent, plus three point three million.
- The actual change in net interest income is four minus three point three, a gain of seven hundred thousand dollars.
- With unequal moves, the change is R S A times the asset rate change, minus R S L times the liability rate change.
- Here the gap rule got even the sign wrong.

17 Gap management: exam traps

- Sign: a negative gap loses when rates rise
- Gap × Δr needs equal moves on both sides
- Unequal moves: RSA × ΔrA − RSL × ΔrL
- Relative gap uses total assets; ratio above 1 = positive gap
- A gap position is a bet on the rate forecast

IN THE LESSON

- Before moving on, here are the traps for gap management.
- First, the sign.
- A negative gap loses income when rates rise, and a positive gap loses when they fall.
- Many wrong answers simply flip this.
- Second, the gap times delta r formula needs equal moves on both sides.
- Third, when the moves are unequal, use R S A times the asset rate change, minus R S L times the liability rate change.
- Fourth, the relative gap divides by total assets, and an interest sensitivity ratio above one means a positive gap.
- Last, a gap position is a bet on the rate forecast.
- It raises the margin only if the forecast is right, and a wrong forecast cuts the margin by just as much.

18 Duration: a measure of rate sensitivity

- Duration: weighted average time to the cash flows
- Longer duration: bigger value change when rates move
- %ΔP ≈ −D × Δy ÷ (1 + y)
- A zero-coupon bond's duration = its maturity
- Equity = assets − liabilities, so its value moves too

IN THE LESSON

- Duration gap management needs one idea: duration.
- Duration is the weighted average time to the cash flows of a bond, with each time weighted by the present value of the payment made then.
- It also measures sensitivity to rates.
- The longer the duration, the bigger the value change when rates move.
- The percentage change in price is approximately minus duration times the change in yield, divided by one plus the yield.
- A zero coupon bond, which pays a single amount at maturity, has a duration equal to its maturity.
- Now apply this to a bank.
- Assets and liabilities each have a duration.
- Equity is assets minus liabilities, so when rates move, its value moves too, by the difference between the two changes.

19 The duration gap

DG = DA − (L ÷ A) × DL
ΔE ≈ −DG × A × Δy ÷ (1 + y)

Asset duration DA	Duration of the assets
Leverage L ÷ A	Liabilities ÷ assets
Liab. duration DL	Duration of the liabilities
Yield change Δy	Change in yield; y is the yield

IN THE LESSON

- Here is the formula.
- The duration gap, D G, equals the duration of assets, D A, minus the leverage ratio, L over ay, times the duration of liabilities, D L.
- Then the change in the market value of equity, delta E, is approximately minus the duration gap, times total assets, times the change in yield, divided by one plus the yield.
- Next, the terms.
- D A is the duration of the assets.
- The leverage ratio is liabilities divided by assets.
- It is there because liabilities fund only part of the assets, so their duration counts for less.
- D L is the duration of the liabilities.
- And delta y is the change in yield, while y itself is the starting yield.

20 Example 6: Harbourside's duration gap

A $1,000m, L $920m, DA 4 years, DL 1.5 years, yield 5%. Rates rise 1 point

Duration gap	4 − 0.92 × 1.5	**2.62 years**
Asset value change	−4 × 1,000 × 1% ÷ 1.05	**−$38.10m**
Liability value change	−1.5 × 920 × 1% ÷ 1.05	**−$13.14m**
Equity change	−2.62 × 1,000 × 1% ÷ 1.05	**−$24.95m**
Share of equity lost	24.95 ÷ 80	**31.2%**

IN THE LESSON

- Harbourside has assets of one thousand million dollars and liabilities of nine hundred and twenty million, durations of four years and one and a half years, a yield of five per cent, and rates rise one point.
- The duration gap is four minus point nine two times one point five, two point six two years.
- The asset value change is minus four times one thousand times one per cent, over one point zero five, a loss of thirty eight point one zero million.
- Liabilities fall by thirteen point one four million, the same way, which helps the bank.
- The equity change is minus two point six two times one thousand times one per cent, over one point zero five, a loss of twenty four point nine five million: the asset loss less the liability fall, up to rounding.
- The share of equity lost is twenty four point nine five over eighty, thirty one point two per cent.

21 Example 7: the trap of matching DA and DL

Same bank, but liabilities lengthened so DL = DA = 4 years. Rates rise 1 point

Duration gap	4 − 0.92 × 4	**0.32 years**
Equity change	−0.32 × 1,000 × 1% ÷ 1.05	**−$3.05m**
Why not zero?	L is only 92% of A	**L ≠ A**

Matching DA and DL hedges only if liabilities equal assets

IN THE LESSON

- Suppose Harbourside lengthens its liabilities so that D L equals D A, four years, and rates again rise one point.
- This is a common trap: many people think the bank is now hedged.
- The duration gap is four minus point nine two times four, which is point three two years, not zero.
- The equity change is minus point three two times one thousand times one per cent, over one point zero five, a loss of about three point zero five million dollars.
- Why not zero?
- Because L is only ninety two per cent of ay.
- The leverage ratio is below one, so the liability side cannot fully offset the asset side, and L is not equal to ay.
- Matching D A and D L hedges only if liabilities equal assets, and for a bank with any equity they never do.

22 Example 8: setting the duration gap to zero

Immunise: make DG = 0, so DA = (L ÷ A) × DL

Option 1: lengthen liabilities	DL = 4 ÷ 0.92	**4.35 years**
Option 2: shorten assets	DA = 0.92 × 1.5	**1.38 years**
Option 3: derivatives	sell futures or pay fixed	**DG → 0**
Check option 1	4 − 0.92 × 4.35	**≈ 0**

Restructuring is slow; derivatives are faster but cost money

IN THE LESSON

- To immunise means to make the duration gap zero, so that D A equals the leverage ratio times D L, and equity value no longer moves with small rate changes.
- Harbourside has three options.
- Option one is to lengthen liabilities: D L must become four divided by point nine two, about four point three five years.
- Option two is to shorten assets: D A must fall to point nine two times one and a half, one point three eight years.
- Option three uses derivatives.
- With a positive gap, the bank sells futures or pays fixed in a swap, until the duration gap goes to zero.
- Check option one: four minus point nine two times four point three five is approximately zero.
- Restructuring is slow and can cost business; derivatives are faster but cost money.

23 Duration gap: exam points

- Positive DG: equity falls when rates rise
- Apply the leverage adjustment L ÷ A to DL
- Divide by 1 + y
- Thin slice of equity: small asset moves, big equity moves
- DG targets net worth, not this year's income

IN THE LESSON

- Here are the points the exam rewards.
- First, the sign: a positive duration gap means equity falls when rates rise, and a negative gap means it rises.
- Second, apply the leverage adjustment: multiply the liability duration by L over ay before subtracting.
- Third, divide by one plus the yield, which turns duration into price sensitivity.
- Fourth, equity is a thin slice of assets.
- For Harbourside, a move of under four per cent in asset values became a loss of nearly a third of equity, so small asset moves mean big equity moves.
- Last, the duration gap targets net worth, not this year's income.

24 Drawbacks of gap management

- Ignores timing inside a bucket
- Assumes equal repricing: ignores basis risk
- Overlooks the market value of equity
- Treats balances as fixed: prepayments, runoff, options

IN THE LESSON

- Both methods are approximations, and each has blind spots.
- Start with gap management.
- First, it ignores timing inside a bucket.
- Everything in the three to twelve month window counts the same, whether it reprices in month four or month eleven.
- Second, it assumes equal repricing, that assets and liabilities reprice by the same amount.
- That ignores basis risk, the risk that related rates, such as a loan rate and a deposit rate, move by different amounts, as when deposit rates lagged in our earlier example.
- Third, it overlooks the market value of equity, because it looks only at income within the window.
- Fourth, it treats balances as fixed.
- In reality borrowers prepay loans when rates fall, deposits run off when rates rise, and embedded options, such as a borrower's right to repay early, change the cash flows.

25 Example 9: a zero gap that still loses

3–12 month bucket: $150m RSL reprice in month 4, $150m RSA in month 11. Rates +1

Gap in the bucket	150 − 150	**0**
Extra cost, months 5–12	150 × 1% × 8 ÷ 12	**+$1.0m**
Extra income, month 12	150 × 1% × 1 ÷ 12	**+$125k**
Change in income this year	0.125 − 1.0	**−$875k**

Narrower buckets shrink this blind spot, but never remove it

IN THE LESSON

- Take Harbourside's three to twelve month bucket, where one hundred and fifty million of liabilities reprice in month four, one hundred and fifty million of assets reprice in month eleven, and rates rise one point at the start of the year.
- The gap in the bucket is zero, so gap management sees no risk.
- But the liabilities cost one per cent more for the last eight months of the year, months five to twelve: one hundred and fifty times one per cent times eight twelfths, one million dollars.
- The assets earn one per cent more only in month twelve: one hundred and fifty times one per cent times one twelfth, one hundred and twenty five thousand.
- The change in income this year is one hundred and twenty five thousand minus one million: a loss of eight hundred and seventy five thousand dollars.
- Narrower buckets shrink this blind spot, but never remove it.

26 Drawbacks of duration gap management

- Assumes small, parallel shifts of the yield curve
- Linear: ignores convexity, so big moves are misestimated
- Durations drift as time passes and rates move: rebalance
- Prepayable loans and deposits: durations are estimates
- Zero DG stabilises equity value, not net interest income

IN THE LESSON

- Duration gap management has its own blind spots.
- First, it assumes small, parallel shifts of the yield curve, meaning every rate moves by the same amount.
- If short rates rise while long rates fall, the measure can mislead.
- Second, it is linear: it ignores convexity, the curvature in the link between value and yield, so big moves are misestimated.
- Third, durations drift as time passes and as rates move.
- A four year zero coupon bond is a three year bond a year later.
- So a hedge that is right today needs rebalancing.
- Fourth, the durations of prepayable loans and of deposits with no fixed term, such as current accounts, are estimates built on assumptions about customer behaviour.
- Last, a zero duration gap stabilises equity value, not net interest income.

27 Example 10: small and large moves

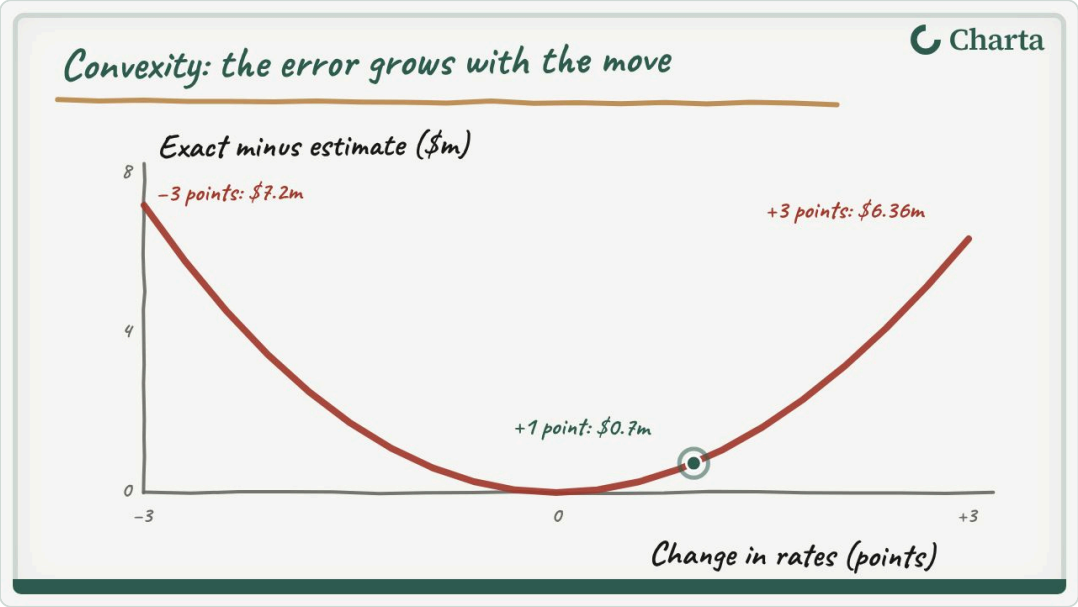
Assets: one 4-year zero, $1,000m. Liabilities: one 1.5-year zero, $920m. Yield 5%

+1 point: duration estimate	−2.62 × 1,000 × 1% ÷ 1.05	**−$24.95m**
+1 point: exact repricing	962.79 − 907.01 − 80	**−$24.22m**
+3 points: duration estimate	−2.62 × 1,000 × 3% ÷ 1.05	**−$74.86m**
+3 points: exact repricing	893.43 − 881.93 − 80	**−$68.50m**
Error at +3 points	74.86 − 68.50	**$6.36m**

IN THE LESSON

- To see the problem in numbers, simplify Harbourside: its assets are one four year zero coupon bond worth one thousand million dollars, its liabilities one zero coupon bond of one and a half years worth nine hundred and twenty million, and the yield is five per cent.
- For a one point rise, the duration estimate is a loss of twenty four point nine five million.
- Repricing both at six per cent takes assets to nine hundred and sixty two point seven nine million and liabilities to nine hundred and seven point zero one.
- Less the starting equity of eighty, that is a loss of twenty four point two two.
- Close.
- For three points, the estimate triples, a loss of seventy four point eight six million.
- Repricing at eight per cent gives assets of eight hundred and ninety three point four three and liabilities of eight hundred and eighty one point nine three.
- Less the eighty, a loss of sixty eight point five zero.
- The error at three points is six point three six million.

28 Convexity: the error grows with the move



IN THE LESSON

- This chart plots the error of the duration estimate for the simplified Harbourside, meaning the exact value of equity minus the estimate.
- Along the bottom is the change in rates, from minus three to plus three percentage points.
- Up the side is the error, from zero to eight million dollars.
- The red curve is the error.
- At no change, it is zero.
- For a one point rise, it is twenty four point nine five minus twenty four point two two, only about point seven million, so for small moves the estimate works.
- For a three point rise, it is the six point three six million we found.
- For a three point fall, repricing both at two per cent gives an exact gain of eighty two point zero five million, against an estimate of seventy four point eight six, an error of about seven point two.
- The error is always positive here: the estimate overstates losses and understates gains.

29 Which blind spot belongs to which method

Know which limitation each method suffers from

Method	Blind spots
Gap	Bucket timing; basis risk
Gap	Ignores equity value; fixed balances
Duration gap	Small parallel shifts; no convexity
Duration gap	Drift, rebalancing, estimated durations
Zero duration gap	Equity value steady, income not

IN THE LESSON

- For the exam, know which limitation each method suffers from.
- Gap management misses timing inside a bucket, and it misses basis risk between benchmarks.
- It also ignores equity value, and it treats balances as fixed.
- Duration gap management assumes small parallel shifts and has no convexity term.
- Its durations drift, so the hedge needs rebalancing, and some durations are only estimates.
- Last, a zero duration gap keeps equity value steady, but not income.
- That is why a bank watches both measures together.

30 Try three quick questions

1. RSA $300m, RSL $200m. Rates fall 2 points on both sides. ΔNII?
   Answer: Gap +100. 100 × (−2%) = −$2m.
2. A $500m, L $450m, DA 5, DL 2, y 4%. Rates rise 0.5 point. ΔE?
   Answer: DG = 5 − 0.9 × 2 = 3.2. −3.2 × 500 × 0.5% ÷ 1.04 = −$7.69m.
3. A bank loses when rates fall. Which swap and which option?
   Answer: Receive fixed, pay floating; buy a floor.

IN THE LESSON

- Pause the video, think, then listen to the answers.
- Question one.
- A bank has rate sensitive assets of three hundred million dollars and rate sensitive liabilities of two hundred million.
- Rates fall two points on both sides.
- What is the change in net interest income?
- The gap is plus one hundred million, times minus two per cent: a fall of two million dollars.
- Question two.
- Assets are five hundred million, liabilities four hundred and fifty million, asset duration five years, liability duration two years, and the yield four per cent.
- Rates rise half a point.
- What is the change in equity?
- The duration gap is five minus point nine times two, three point two.
- Minus three point two, times five hundred million, times half of one per cent, over one point zero four, is a loss of about seven point six nine million.
- Question three.
- A bank loses when rates fall.
- Which swap and which option hedge it?
- Receive fixed and pay floating, and buy a floor.