CHARTA LEARNING · LESSON NOTES

Liquidity and Leverage

FRM Part II

1 A story: solvent, yet out of cash

- Kestrel Bank: assets exceed debts, so it is solvent
- A rumour: depositors and lenders pull their money
- Its loans cannot be turned into cash quickly
- It may fail for lack of cash, not lack of value
- Liquidity risk: unable to pay when payment is due

IN THE LESSON

- Picture Kestrel Bank on a Monday.
- Its assets are worth more than everything it owes, so it is solvent.
- Then a rumour spreads.
- Depositors withdraw, and lenders who fund the bank week to week decline to lend again.
- Kestrel owns plenty of loans, but a loan to a business cannot be turned into cash by Friday without a heavy discount.
- So Kestrel may fail, not because it lacks value, but because it lacks cash at the moment cash is due.
- That is liquidity risk: the risk of being unable to pay what you owe when it falls due, or of paying far too much to raise the cash.

2 Two origins of liquidity risk

Two origins that feed each other

Feature	Market (transaction)	Funding
Balance-sheet side	Assets	Liabilities
The risk	Cannot sell or hedge cheaply	Cannot meet obligations due
Shows up as	Wide spreads, price moves	Borrowing not rolled over

IN THE LESSON

- Liquidity risk has two origins, and they feed each other: market liquidity risk, also called transaction liquidity risk, and funding liquidity risk.
- Market liquidity risk sits on the asset side, and funding liquidity risk on the liability side.
- Market liquidity risk is the risk that a position cannot be sold or hedged quickly without a wide bid ask spread, the gap between the bid and ask prices, or without moving the price.
- Funding liquidity risk is the risk that a firm cannot meet cash or collateral obligations as they fall due.
- Market liquidity risk shows up as wide spreads and price moves.
- Funding liquidity risk shows up as borrowing that cannot be rolled over, meaning maturing loans that lenders will not replace.

3 Example 1: solvent but illiquid

Kestrel Bank: assets $1,000m, liabilities $920m. A 30-day stress begins

Equity	1,000 − 920	**$80m: solvent**
Cash and reserves	at the central bank	**$56m**
Government bonds, after haircut	150 × (1 − 4%)	**$144m**
Stressed net outflows, 30 days	deposits and wholesale	**$260m**
Shortfall	260 − (56 + 144)	**$60m: illiquid**

IN THE LESSON

- Kestrel Bank has assets of one thousand million dollars and liabilities of nine hundred and twenty million, and a thirty day stress begins.
- Equity is one thousand minus nine hundred and twenty, eighty million dollars, so Kestrel is solvent.
- It holds fifty six million of cash and reserves at the central bank.
- Government bonds worth one hundred and fifty million, after a four per cent haircut, a deduction from market value that lenders apply, give one hundred and forty four million.
- Under stress, net outflows of deposits and wholesale funding over thirty days are two hundred and sixty million.
- Two hundred and sixty, less the two hundred of cash and bonds, leaves a shortfall of sixty million.
- Solvent, yet illiquid.

4 Where the strain comes from

For the exam: find the origin first, then the institution's channel

Institution	Main source of strain
Banks	Deposit runs, rollover, credit lines
Insurers	Surrenders and large claims
Pension funds, derivative users	Variation margin calls
Hedge funds	Broker margins, redemptions
Open-end funds	Redemptions, illiquid holdings

IN THE LESSON

- For the exam, find the origin first, then the channel specific to the institution.
- Banks face deposit runs, the failure to roll over wholesale borrowing, and drawdowns of committed credit lines, when customers draw on lines the bank has promised.
- Insurers face surrenders, where policyholders cash in early, and large claims.
- Pension funds and other derivative users face variation margin calls, the cash payments that settle daily losses on derivatives.
- Hedge funds face higher margins from their prime brokers, the banks that finance them, and investor redemptions.
- Open end funds, which let investors withdraw at short notice, face redemptions against illiquid holdings.

5 A fractional reserve bank

- Lends long: loans that are illiquid
- Borrows short: demand deposits
- Keeps only a fraction as reserves
- Liquidity and maturity transformation
- Exposed if withdrawals exceed liquid assets

IN THE LESSON

- Why are banks so exposed?
- Kestrel is a fractional reserve bank.
- It lends long, making loans that run for years and are hard to sell.
- It borrows short, mostly through demand deposits that customers can withdraw at any time.
- And it keeps only a fraction of those deposits as reserves, in cash and balances at the central bank.
- Turning liquid deposits into illiquid loans is liquidity transformation, and turning short borrowing into long lending is maturity transformation.
- Savers get claims they can spend at any moment, and borrowers get credit for years.
- The price is fragility.
- If withdrawals exceed the bank's liquid assets, it cannot pay, however good its loans are.

6 Example 2: Kestrel's maturity ladder

Expected cash flows in normal times, $m: in, out, gap and cumulative gap

Up to 7 days	in 30, out 80	**−50; cum −50**
8 to 30 days	in 40, out 60	**−20; cum −70**
1 to 12 months	in 150, out 110	**+40; cum −30**
Over 1 year	in 600, out 450	**+150; cum +120**
Worst cumulative gap	against the 200 buffer	**−70: covered**

IN THE LESSON

- Here are Kestrel's expected cash flows in normal times, in millions, with the gap and the cumulative gap, or running total.
- This is a maturity ladder, the start of asset liability management, A L M.
- Up to seven days, thirty comes in and eighty goes out, a gap of minus fifty.
- From eight to thirty days, forty in and sixty out, minus twenty, so the cumulative gap is minus seventy.
- From one to twelve months, one hundred and fifty in and one hundred and ten out, plus forty, bringing the cumulative gap to minus thirty.
- Over one year, six hundred in and four hundred and fifty out, plus one hundred and fifty, so plus one hundred and twenty cumulatively.
- The worst cumulative gap, minus seventy, is covered by the two hundred million buffer of liquid assets.

7 The rest of the ALM toolkit

- Hold a buffer of high-quality liquid assets (HQLA)
- Diversify funding across sources and terms
- Funds transfer pricing: charge lines for term funding
- Stress tests check the run-off assumptions
- Backstops: deposit insurance, lender of last resort

IN THE LESSON

- Kestrel also holds a buffer of high quality liquid assets, H Q L A, which can be sold or pledged for cash even in stress.
- It diversifies its funding across sources and terms, so no single group of lenders can drain it.
- Funds transfer pricing, F T P, charges each business line for the term funding its assets need, so ten year loans pay for ten year money.
- Stress tests check the run off assumptions, meaning how much of each kind of funding is assumed to leave in a crisis.
- Behind these sit two backstops: deposit insurance, which keeps small depositors calm, and the central bank as lender of last resort.

8 Example 3: the face-value trap

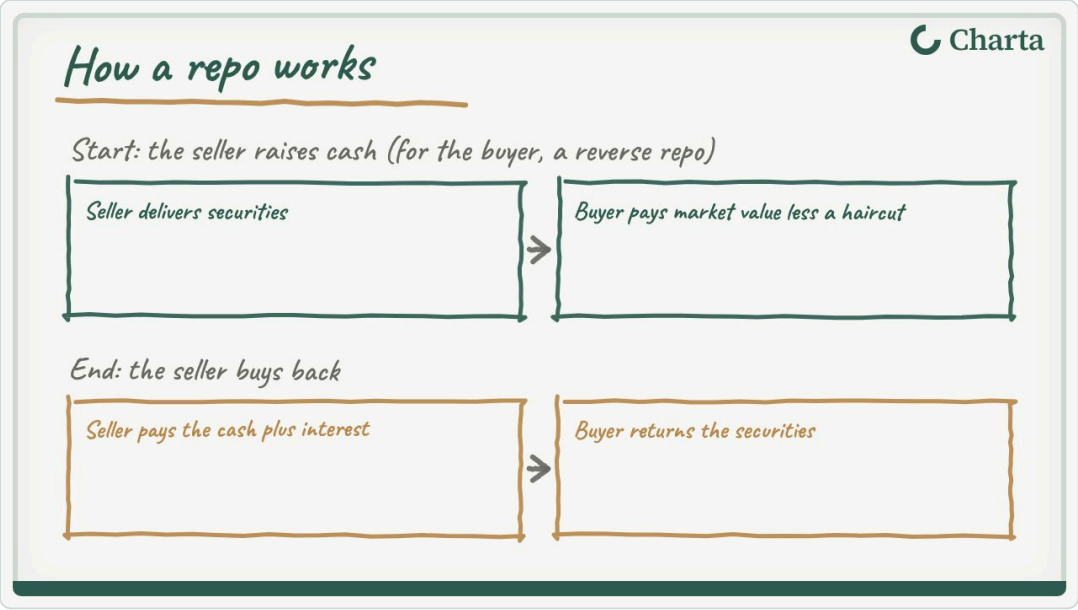
Cover the $60m shortfall by selling loans that fetch 80% of face; equity $80m

Shortfall from Example 1	260 − 200	**$60m**
Face value of loans to sell	60 ÷ 0.80	**$75m**
Loss on the sale	75 − 60	**$15m**
Equity after the sale	80 − 15	**$65m**
Wrong: loans at face value	60 of loans raises 60	**no loss seen**

IN THE LESSON

- Kestrel must cover its sixty million dollar shortfall by selling loans that fetch only eighty per cent of face value in a hurry, and it has eighty million of equity.
- The shortfall from example one is two hundred and sixty minus two hundred.
- At eighty cents in the dollar, Kestrel must sell sixty divided by point eight, seventy five million of loans at face value.
- The loss on the sale is seventy five minus sixty, fifteen million, so equity falls from eighty to sixty five million.
- An analyst who counts loans at face value thinks that sixty of loans raises sixty, and sees no loss at all.
- That is the common error: in a run, illiquid assets sell at a discount, and the discount eats capital.

9 How a repo works



IN THE LESSON

- Kestrel funds some bonds in collateral markets, where a party obtains cash or securities against collateral.
- The workhorse is the repo, short for repurchase agreement.
- At the start, the seller, here Kestrel, raises cash, and seen from the buyer's side the same deal is a reverse repo.
- The seller delivers securities to the buyer.
- The buyer pays cash equal to their market value less a haircut, the safety margin that protects the buyer if the securities fall in value.
- At the end, the seller buys back.
- It pays the cash plus interest, and the buyer returns the securities.
- In substance, a repo is a secured loan.

10 Example 4: Kestrel repos its bonds

Desk's bonds worth $125m; haircut 4%; repo rate 5%; 30 days; actual/360

Cash received	125 × (1 − 4%)	**$120m**
Interest	120 × 5% × 30 ÷ 360	**$0.5m**
Repurchase price	120 + 0.5	**$120.5m**
Kestrel's own money	125 − 120 = 4% of 125	**$5m**
Maximum leverage	1 ÷ 4% = 125 ÷ 5	**25 times**

IN THE LESSON

- Kestrel's trading desk holds corporate bonds worth one hundred and twenty five million dollars.
- It repos them for thirty days at a four per cent haircut and a five per cent repo rate, on the U S interest convention: cash times rate times days over three hundred and sixty.
- The cash received is one hundred and twenty five times ninety six per cent, one hundred and twenty million.
- Interest is one hundred and twenty times five per cent times thirty over three hundred and sixty, half a million.
- So the repurchase price is one hundred and twenty point five million.
- The desk funds the other five million, four per cent of the bonds, with its own money.
- That sets the maximum leverage on repo financing: one over the haircut, twenty five times.

11 Three collateral transactions

One idea, a different motive each

Transaction	Driven by	Paid for by
Repo	Need for cash	Interest at the repo rate
Securities lending	Need for a specific security	A lending fee
Total return swap	Wish for leveraged exposure	A financing payment

IN THE LESSON

- Collateral transactions share one idea, but each has a different motive.
- A repo is driven by the need for cash, and the cash borrower pays interest at the repo rate.
- Securities lending is driven by the borrower's need for a specific security, say to deliver on a short sale.
- The borrower pays a lending fee, and gives cash or other securities as collateral.
- A total return swap is driven by the wish for leveraged exposure.
- The receiver gets the asset's total return, its income plus any change in price, and pays a financing payment in exchange.
- It gains the exposure without buying the asset, so the leverage sits largely off its balance sheet.

12 Risks these transactions create

- Haircuts and margins rise with volatility
- Lenders may refuse to roll the funding
- The counterparty may fail
- Reinvested cash collateral can lose value
- Rehypothecation: 125 posted for 120, 5 left unsecured

IN THE LESSON

- Secured deals still create risks.
- First, haircuts and margins rise with volatility, so funding shrinks just as prices fall.
- Second, a lender may refuse to roll the funding when it matures.
- Third, the counterparty may fail.
- Fourth, a securities lender often reinvests cash collateral, and bears any losses on it.
- Fifth, rehypothecation, the re-use of posted collateral by the party that received it.
- Say Kestrel posts bonds worth one hundred and twenty five million against cash of one hundred and twenty, and the receiver re-pledges them and then fails.
- Kestrel still owes the one hundred and twenty, and for the five million of excess collateral it is only an unsecured creditor.

13 Leverage and the return on equity

Leverage = Assets ÷ Equity D/E = Leverage − 1
ROE = ROA + (ROA − Kd) × D/E

Leverage	Assets divided by equity
ROA	Return on assets, before interest
Kd	Cost of debt
D/E	Debt divided by equity

IN THE LESSON

- Now leverage.
- Balance sheet leverage is assets divided by equity.
- Debt to equity, written D over E, is that figure minus one, because assets are debt plus equity.
- Leverage magnifies the return on equity in both directions: this is the leverage effect.
- With R O A the return on assets, before interest, and K d the cost of debt, the return on equity, R O E, equals R O A, plus R O A minus K d, times debt to equity.
- Equity earns the asset return, plus the spread over the borrowing cost on every dollar of debt.
- So equity earns more than the assets only when the asset return beats the cost of debt.
- Keep R O A and K d on the same basis: both after tax, or both ignoring tax.

14 Example 5: leverage in a good and a bad year

Kestrel: assets $1,000m, debt $920m, equity $80m. Cost of debt 2.5%

Assets ÷ equity	1,000 ÷ 80	**12.5**
Debt ÷ equity	920 ÷ 80 = 12.5 − 1	**11.5**
ROE, ROA 3.0%	3.0% + (3.0% − 2.5%) × 11.5	**8.75%**
ROE, ROA 1.5%	1.5% + (1.5% − 2.5%) × 11.5	**−10.0%**
Check: ROA 3.0%	(30 − 23) ÷ 80	**8.75%**

IN THE LESSON

- Kestrel has assets of one thousand million, debt of nine hundred and twenty, equity of eighty, and pays two point five per cent on debt.
- Assets over equity is twelve point five.
- Debt over equity is nine hundred and twenty over eighty, eleven point five, or twelve point five minus one.
- In a good year, R O A is three per cent, so R O E is three per cent plus half a per cent times eleven point five, eight point seven five per cent.
- In a bad year, R O A is one point five per cent, below the cost of debt, so R O E is one point five per cent, less one per cent times eleven point five: a loss of ten per cent.
- Check the good year: thirty million earned, less twenty three of interest, leaves seven, and seven over eighty is eight point seven five per cent.

15 Example 6: the leverage effect on volatility

Asset volatility 0.8%; assets ÷ equity 12.5

A 1% fall in asset value	1% × 1,000 = $10m	**12.5% of equity**
Equity volatility	12.5 × 0.8%	**10.0%**
Wrong: using D/E	11.5 × 0.8%	**9.2%**

IN THE LESSON

- Kestrel's assets have a volatility of point eight per cent, and assets over equity is twelve point five.
- The leverage effect works on risk as well as return.
- A one per cent fall in asset value is ten million dollars, and against eighty million of equity that is a fall of twelve point five per cent.
- So equity volatility is roughly assets over equity times asset volatility: twelve point five times point eight per cent, which is ten per cent.
- A typical trap is to use debt to equity instead, eleven point five times point eight, which gives nine point two per cent and understates the risk.

16 Example 7: how transactions move leverage

Start: assets $1,000m, equity $80m. Each case is taken from the start

Start	1,000 ÷ 80	**12.5**
Buy $100m, all borrowed	1,100 ÷ 80	**13.75**
Instead: sell $200m, repay	800 ÷ 80	**10.0**
Ceiling, repo-financed	1 ÷ h; at 5%: 1 ÷ 0.05	**20 times**

IN THE LESSON

- Kestrel starts with assets of one thousand million and equity of eighty, and we take each case from that start, where leverage is twelve point five.
- Suppose Kestrel buys one hundred million of bonds, borrowing the whole price through a repo or a margin loan, and ignoring the haircut for now.
- Assets and liabilities grow together, so leverage rises to one thousand one hundred over eighty, thirteen point seven five.
- Suppose instead it sells two hundred million of assets and repays debt.
- Both sides shrink, and leverage falls to eight hundred over eighty, ten.
- For a position financed wholly by repo, haircuts set a ceiling.
- With a haircut h, each dollar of that position needs h of equity, so its leverage can reach at most one over h.
- At a five per cent haircut, that is twenty times.

17 Example 8: leverage off the balance sheet

Kestrel adds a total return swap: notional $50m, margin posted $5m

Balance sheet	margin replaces cash	**unchanged**
Balance-sheet leverage	1,000 ÷ 80	**still 12.5**
Swap's embedded leverage	notional ÷ margin = 50 ÷ 5	**10 times**
Exposure to asset prices	(1,000 + 50) ÷ 80	**13.1 times**

IN THE LESSON

- Kestrel adds a total return swap with a notional of fifty million dollars, and posts five million of margin.
- On the balance sheet, the margin simply replaces cash, so total assets are unchanged.
- Balance sheet leverage is still one thousand over eighty, twelve point five.
- Yet the swap has embedded leverage, meaning its notional relative to its margin, of fifty over five, ten times.
- Kestrel's exposure to asset prices is now one thousand and fifty million on eighty of equity, about thirteen point one times.
- So balance sheet leverage understates economic leverage.
- Ask what exposure a position gives, not only what it adds to the balance sheet.

18 Measuring funding liquidity risk

- Cash-flow and maturity ladders under stress
- Capacity: cash + liquid assets after haircuts
- Survival horizon: how long that capacity covers outflows
- Regulatory ratios: LCR and NSFR
- Manage: diverse, termed-out funding; buffers; limits; CFP

IN THE LESSON

- Funding liquidity risk is measured first with cash flow and maturity ladders, like Kestrel's, run under stress.
- Against the outflows, the bank sets its counterbalancing capacity, meaning cash plus liquid assets after haircuts.
- The survival horizon is how long that capacity covers cumulative net outflows.
- Then come the regulatory ratios, the L C R and the N S F R.
- To manage the risk, a bank diversifies its funding and terms it out, meaning it borrows for longer; it holds collateral buffers; it sets limits; and it keeps a contingency funding plan, C F P, a written plan of what to do, and who decides, when funding dries up.

19 Example 9: Kestrel's survival horizon

Capacity: cash $56m + bonds $150m after a 4% haircut. Stressed net outflows

Counterbalancing capacity	56 + 150 × 96%	**$200m**
Cumulative outflow, day 7	covered	**$60m**
By day 14	covered	**$120m**
By day 21	covered, 10 to spare	**$190m**
By day 30	260 − 200	**Short $60m**

Survival horizon: about 3 weeks

IN THE LESSON

- Kestrel's capacity is fifty six million of cash, plus government bonds of one hundred and fifty million after a four per cent haircut, set against its stressed net outflows.
- The bonds give one hundred and fifty times ninety six per cent, one hundred and forty four, so the counterbalancing capacity is two hundred million dollars.
- Now the cumulative net outflows.
- By day seven, sixty million has gone, which is covered.
- By day fourteen, one hundred and twenty, still covered.
- By day twenty one, one hundred and ninety, covered with ten to spare.
- By day thirty, two hundred and sixty, and Kestrel is sixty million short.
- So the survival horizon is about three weeks.
- The stress assumptions drive the answer, so read them before computing.

20 The two regulatory ratios

LCR = HQLA ÷ net outflows over 30 stressed days ≥ 100%
Net outflows = outflows − min(inflows, 75% × outflows)
NSFR = available ÷ required stable funding ≥ 100%

HQLA	High-quality liquid assets
Net outflows	Outflows less capped inflows
ASF	Available stable funding
RSF	Required stable funding

IN THE LESSON

- The liquidity coverage ratio, L C R, is high quality liquid assets divided by total net cash outflows over thirty stressed days, and it must be at least one hundred per cent.
- Net outflows are outflows minus inflows, but inflows count only up to seventy five per cent of outflows, so a bank must always hold liquid assets for at least a quarter of its outflows.
- The net stable funding ratio, N S F R, is available stable funding divided by required stable funding, also at least one hundred per cent.
- Available stable funding weights each source of funds by how reliable it is.
- Required stable funding weights each asset by how much stable funding it needs.

21 Example 10: Kestrel's LCR and NSFR

Outflows $340m, inflows $80m over 30 days. Stable funding factors assumed

HQLA, at full value	56 + 150	**$206m**
Net outflows	340 − 80 (cap 255)	**$260m**
LCR	206 ÷ 260	**79%: fails**
Available stable funding	200 + 90%×650 + 0×150	**$785m**
Required stable funding	85%×640 + 50%×154 + 5%×150	**$628.5m**

NSFR = 785 ÷ 628.5 = 125%: passes

IN THE LESSON

- Over thirty stressed days, Kestrel's outflows are three hundred and forty million and inflows eighty, and the stable funding factors are assumed.
- H Q L A, at full value, is fifty six of cash plus one hundred and fifty of bonds, two hundred and six.
- Net outflows are three hundred and forty minus eighty, two hundred and sixty, as inflows are under the cap of two hundred and fifty five.
- So the L C R is seventy nine per cent, a fail.
- Available stable funding counts two hundred of equity and long term debt in full, ninety per cent of six hundred and fifty of deposits, and nothing for one hundred and fifty of wholesale funding: seven hundred and eighty five.
- Required stable funding takes eighty five per cent of six hundred and forty of loans, half of one hundred and fifty four of other securities and five per cent of the bonds: six hundred and twenty eight point five.
- The N S F R is one hundred and twenty five per cent, a pass.

22 Measuring market liquidity risk

- Bid-ask spread: the cost of a round trip
- Market depth: size that trades near the price
- Price impact, e.g. the Amihud ratio
- Days to liquidate: position ÷ (participation × ADV)
- Manage: position limits, liquid assets, staged selling

IN THE LESSON

- Market liquidity risk, the risk of being unable to transact at fair prices, has its own measures.
- The bid ask spread is the cost of a round trip, buying and selling straight away.
- Market depth is how much can trade near the current price before the price moves.
- Price impact is how far the price moves per dollar traded, and a common measure is the Amihud ratio: the average, over days, of each day's absolute return divided by that day's traded value.
- Days to liquidate is the position divided by the participation rate, the share of daily volume you are willing to be, times average daily volume, A D V.
- To manage the risk, a firm sets position limits, diversifies across liquid assets and stages its selling.

23 Example 11: days to liquidate and Amihud

2,000,000 shares at $20. Daily volume 400,000; Kestrel trades at most 20%

Position value	2,000,000 × $20	**$40m**
Shares sold a day	20% × 400,000	**80,000**
Days to liquidate	2,000,000 ÷ 80,000	**25 days**
Each day's ratio, % per $m	2% ÷ 8, 1% ÷ 5, 3% ÷ 10	**0.25, 0.20, 0.30**
Amihud ratio	average of the three	**0.25% per $m**

IN THE LESSON

- Kestrel holds two million shares at twenty dollars.
- Daily volume is four hundred thousand shares, and Kestrel trades at most twenty per cent of it.
- The position is worth two million times twenty dollars, forty million.
- Twenty per cent of four hundred thousand is eighty thousand shares a day, so days to liquidate are two million over eighty thousand, twenty five days.
- Now price impact.
- On three recent days the absolute returns were two, one and three per cent, and the values traded were eight, five and ten million dollars.
- Each day's ratio is point two five, point two and point three per cent per million.
- The Amihud ratio is their average, point two five per cent per million dollars traded.
- Higher means less liquid.

24 Funding versus market liquidity

Different measures, different tools

Tool	Funding liquidity	Market liquidity
Measure	Stressed ladders, survival	Spread, depth, price impact
Headline numbers	LCR and NSFR	Amihud, days to liquidate
Manage: structure	Diverse, termed-out funding	Position limits
Manage: buffers	Collateral buffers, limits	Diversify into liquid assets
Manage: in stress	Contingency funding plan	Staged execution

IN THE LESSON

- Funding and market liquidity need different measures and tools.
- To measure funding liquidity, use stressed ladders and the survival horizon.
- For market liquidity, use the spread, depth and price impact.
- The headline numbers are the L C R and N S F R on the funding side, and the Amihud ratio and days to liquidate on the market side.
- To manage them, start with structure: diverse, termed out funding on one side, position limits on the other.
- Then buffers: collateral buffers and limits for funding, and diversifying into liquid assets for the market side.
- Finally, in stress: a contingency funding plan for funding, and staged execution for market liquidity.

25 The cost of the spread

Expected cost = ½ × value × mean s
Spread risk factor = mean s + z × σs
LVaR = VaR + ½ × value × (mean s + z × σs)

s	(ask − bid) ÷ mid
σs	Standard deviation of s
z	z for the confidence level
VaR	VaR before liquidity cost

IN THE LESSON

- Write s for the relative spread: the ask minus the bid, divided by the mid price.
- Selling costs about half the spread, because you trade from the mid price down to the bid.
- So the expected cost is half the position's value times the mean of s.
- Spreads widen in bad times, so to allow for that at a confidence level we add z standard deviations of s to its mean.
- That is the spread risk factor.
- In this simplified spread based adjustment, which ignores market impact, the liquidity cost is half the value times the spread risk factor.
- Liquidity adjusted V a R, or L V a R, adds that cost to ordinary V a R.

26 Example 12: liquidity-adjusted VaR

$40m position. Mean spread 0.8%, σs 0.6%, z = 2.33; daily volatility 2.5%

One-day VaR at 99%	2.33 × 2.5% × 40m	**$2,330,000**
Expected cost	½ × 40m × 0.8%	**$160,000**
Spread risk factor	0.8% + 2.33 × 0.6%	**2.198%**
Liquidity cost	½ × 40m × 2.198%	**$439,600**
LVaR	2,330,000 + 439,600	**$2,769,600**

Wrong: full spread, 40m × 2.198% = $879,200

IN THE LESSON

- Kestrel's shares are worth forty million dollars.
- The mean relative spread is point eight per cent, its standard deviation point six per cent, z at ninety nine per cent is two point three three, and daily volatility is two point five per cent.
- One day V a R is two point three three times two point five per cent times forty million, two million three hundred and thirty thousand dollars.
- The expected cost is half of forty million times point eight per cent, one hundred and sixty thousand.
- The spread risk factor is point eight plus two point three three times point six, two point one nine eight per cent.
- The liquidity cost is half of forty million times that: four hundred and thirty nine thousand six hundred.
- So L V a R is two million seven hundred and sixty nine thousand six hundred.
- The full spread would wrongly give eight hundred and seventy nine thousand two hundred.

27 Selling over several days

VaR(T) = one-day VaR × √[(T + 1)(2T + 1) ÷ 6T]
Not one-day VaR × √T

T	Days to sell, in equal parts
One-day VaR	VaR of the full position
Variance factor	(T + 1)(2T + 1) ÷ 6T

IN THE LESSON

- Suppose Kestrel sells a large position in T equal parts, one at the end of each of T days, and daily returns are independent.
- On the first day the whole position is at risk, on the second a smaller remainder, and so on until the last part goes.
- Adding the variance of each day's exposure gives a variance factor of T plus one, times two T plus one, divided by six T.
- Liquidation adjusted V a R scales one day V a R of the full position by the square root of that factor.
- The tempting mistake is to scale by the square root of T, as if the full position were held for all T days.
- Because the position shrinks, the true factor is smaller.

28 Example 13: a five-day sale

One-day VaR $2.33m; sold in 5 equal parts, one at the end of each day

Check: T = 1	2 × 3 ÷ 6	**factor 1**
Factor for T = 5	6 × 11 ÷ 30	**2.2**
Square root	√2.2	**1.483**
Liquidation VaR, 5 days	2.33m × 1.483	**$3.46m**
Wrong: √5 on the full position	2.33m × 2.236	**$5.21m**

Shares at risk, squared: $1^2 + 0.8^2 + 0.6^2 + 0.4^2 + 0.2^2 = 2.2$

IN THE LESSON

- Kestrel's one day V a R is two point three three million dollars, and it sells the shares in five equal parts, one at the end of each day.
- First a check: with T equal to one, the factor is two times three over six, which is one, giving plain one day V a R.
- For T equal to five, the factor is six times eleven over thirty, two point two.
- The square root is one point four eight three, so liquidation V a R is about three point four six million.
- Scaling by the square root of five, two point two three six, gives five point two one million, far too much.
- The factor of two point two comes from the shares left at risk each day, all, four fifths, three fifths, two fifths and one fifth, whose squares add up to it.

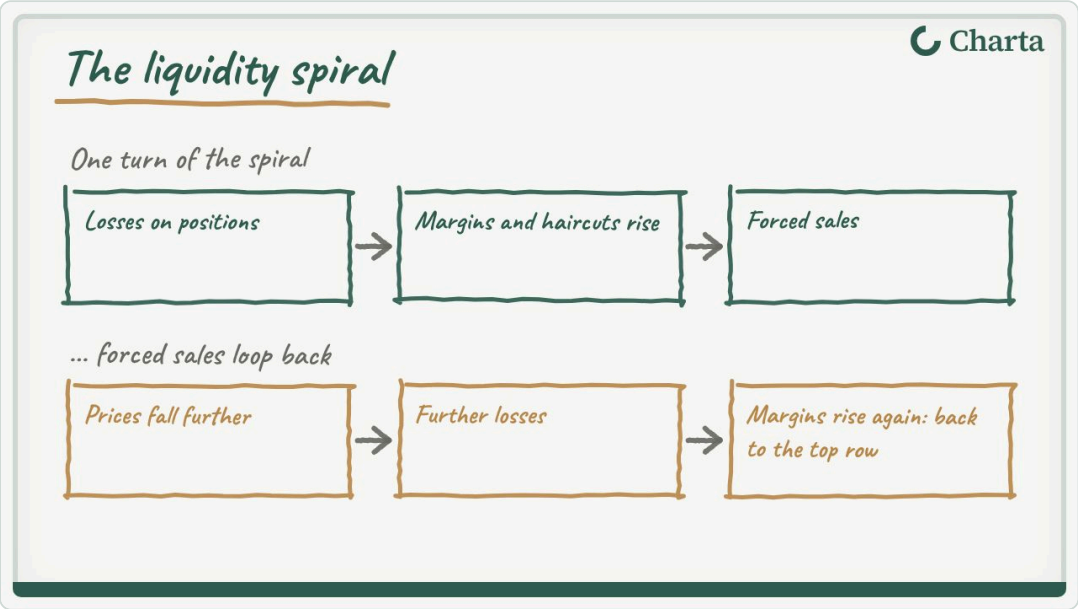
29 How the forms of liquidity risk feed each other

- Funding liquidity depends on market liquidity
- Haircuts and margins are set from prices and volatility
- Assets raise only the price the market will pay
- Market liquidity depends on funding liquidity
- Liquidity suppliers withdraw when funding tightens

IN THE LESSON

- Funding liquidity depends on market liquidity, in two ways.
- Lenders set haircuts and margins from the prices and volatility of the collateral, so when markets turn volatile, the same collateral raises less cash.
- And assets can be sold for cash only at the price the market will pay.
- Market liquidity, in turn, depends on funding liquidity.
- The dealers and leveraged funds that supply liquidity, by standing ready to buy, need funding to hold positions.
- When their own funding tightens, they withdraw, and markets thin out just when sellers appear.
- Each weakness feeds the other.

30 The liquidity spiral



IN THE LESSON

- Put the two together and you get a spiral.
- Look at the first row, one turn of the spiral.
- It starts with losses on positions.
- Lenders respond by raising margins and haircuts, so borrowers must post more or hold less.
- Those that cannot post more make forced sales.
- Now the second row, where the forced sales loop back.
- The selling pushes prices down further.
- Lower prices bring further losses for everyone holding the same assets.
- And with prices falling and volatility rising, margins rise again, back to the top row, which starts the next turn.

31 Example 14: a higher haircut forces a fire sale

Desk: equity $5m held fixed, bonds $125m. The haircut rises from 4% to 8%

Immediate funding gap	125 × (8% − 4%)	**$5m**
New ceiling on assets	5 ÷ 8%	**$62.5m**
Required sale	125 − 62.5	**$62.5m**
Sale versus funding gap	62.5 ÷ 5	**12.5 times**

Selling half the book depresses prices: the spiral turns

IN THE LESSON

- Kestrel's bond desk has five million of equity, held fixed, and one hundred and twenty five million of bonds financed by repo.
- Markets turn volatile and the haircut rises from four per cent to eight.
- The immediate funding gap, the extra cash lenders demand on the existing position, is one hundred and twenty five times the rise in the haircut, eight per cent minus four, which is five million.
- The new ceiling on assets is equity over the new haircut, five over eight per cent, sixty two point five million.
- So the required sale is one hundred and twenty five minus sixty two point five, sixty two point five million, half the book.
- The sale is twelve and a half times the funding gap.
- Selling half the book into a falling market depresses prices: that is how the spiral turns.

32 How a liquidity event becomes systemic

- Firms hold similar assets and similar short-term funding
- One fire sale lowers prices for every holder
- Contagion: through prices, margins and confidence
- Procyclicality: margins tighten in a downturn
- One firm's problem becomes the system's

IN THE LESSON

- Institutions hold similar assets and rely on similar short term funding.
- When Kestrel's desk sells sixty two point five million of bonds, prices fall for every holder.
- Contagion runs through three channels: through prices, as others mark down the same assets; through margins, as lenders raise haircuts for everyone; and through confidence, as lenders pull back from any firm that looks similar.
- The keyword is procyclicality.
- Haircuts and margins tighten in a downturn and loosen in a boom, so they amplify the cycle instead of damping it.
- That is how one firm's liquidity problem becomes a systemic event.

33 Mistakes to avoid

- Using D/E where assets ÷ equity is needed
- Forgetting ROE < ROA when ROA < cost of debt
- Charging the full spread instead of half
- Scaling the whole position by √T
- Counting illiquid assets at face value

IN THE LESSON

- Before the quiz, five mistakes to avoid.
- First, using debt to equity where assets over equity is needed, for example in the leverage effect on volatility.
- Second, forgetting that the return on equity falls below the return on assets whenever the asset return is below the cost of debt.
- Third, charging the full bid ask spread as the cost of selling, when you trade from the mid price and pay only half.
- Fourth, scaling the whole position by the square root of T when it is sold off in parts over T days.
- Fifth, counting illiquid assets at face value when, in a run, they would sell at a discount.

34 Try three quick questions

1. Bonds worth $50m are repoed at a 2% haircut. Cash received and maximum leverage?
   Answer: 50 × 98% = $49m. Maximum leverage 1 ÷ 2% = 50 times.
2. ROA 4%, cost of debt 3%, assets ÷ equity 10. Return on equity?
   Answer: D/E = 9; 4% + (4% − 3%) × 9 = 13%.
3. One-day VaR $1m; sold in 3 equal daily parts. Liquidation VaR?
   Answer: (4 × 7) ÷ 18 = 1.556; √1.556 = 1.247; about $1.25m.

IN THE LESSON

- Now try three quick questions.
- Pause, think, then listen to the answers.
- Question one.
- A firm repos bonds worth fifty million dollars at a two per cent haircut.
- How much cash does it receive, and what is its maximum leverage?
- It receives fifty times ninety eight per cent, forty nine million, and the maximum leverage is one over two per cent, fifty times.
- Question two.
- The return on assets is four per cent, the cost of debt three per cent, and assets over equity is ten.
- What is the return on equity?
- Debt to equity is nine, so four per cent plus one per cent times nine gives thirteen per cent.
- Question three.
- One day V a R is one million dollars, and the position is sold in three equal parts over three days.
- What is the liquidation V a R?
- The factor is four times seven over eighteen, about one point five five six.
- Its square root is one point two four seven, so about one point two five million dollars.