CHARTA LEARNING · LESSON NOTES

Data sufficiency: enough to decide?

Data Insights

1 A story: is the clue enough?

- A question, then two clues called statements
- You decide: do the clues pin down one answer?
- You do not need the answer itself
- Think like a detective, not a calculator

IN THE LESSON

- Picture a detective with a missing person.
- A neighbour says, the man wore a red coat.
- Is that enough to name him?
- No, plenty of people wear red coats.
- A second neighbour says, he lives at number nine.
- Alone, also not enough.
- But the two clues together might point to exactly one person.
- Notice what the detective never does: the detective does not need to find the man to know whether the clues are enough.
- She only asks, do these clues leave room for two different people?
- That is data sufficiency.
- You get a question and two statements, which are the clues.
- Your whole job is to decide whether the clues pin down one answer.
- Calculating the answer itself is usually a waste of time.

2 Two kinds of question

- Value question: sufficient only if one value fits
- Yes or no question: always yes, or always no
- A definite no IS sufficient
- Not sufficient = the answer could change

IN THE LESSON

- Before you touch the statements, decide what kind of question you are facing.
- There are only two.
- The first is a value question, such as, what is the value of x.
- A statement is sufficient only if it forces exactly one value.
- If it allows two values, even two close ones, it is not sufficient.
- The second is a yes or no question, such as, is x greater than four.
- Here a statement is sufficient if it forces the answer to be always yes, or always no.
- Read that again.
- A definite no is just as good as a definite yes.
- What fails is a statement that lets the answer be yes in one case and no in another.
- Sufficient means one answer.
- Not sufficient means more than one.

3 What a yes or no statement can do

- Sufficient: always no
- Not sufficient: could be yes or no
- Sufficient: always yes

IN THE LESSON

- Here is a yes or no question on a line.
- On the far left, the statement forces the answer to be no every time.
- That is sufficient.
- On the far right, the statement forces yes every time.
- Also sufficient.
- In the middle sits the dangerous case, a statement that lets some numbers give yes and others give no.
- That one is not sufficient.
- Most of the marks are lost by students who treat the left end as a failure.
- They find that the answer is always no, think I have not found yes, so it must be insufficient, and choose wrongly.
- A clean, certain no is an answer.
- Learn to like it.

4 The routine, in the same order every time

Decide, then test

Kind: value or yes or no → Statement 1 alone → Statement 2 alone → Both together

IN THE LESSON

- Here is the routine, and you will use it on every question.
- Step one, decide the kind of question: value, or yes or no.
- Step two, look at statement one alone, and forget statement two exists.
- Ask whether it gives one answer.
- Step three, clear your mind and look at statement two alone, pretending you never read statement one.
- Only if neither works by itself do you go to step four, and combine them.
- Why this order?
- Because combining is the expensive step, and many questions are settled earlier.
- Also, each statement is guaranteed to be true and never contradicts the other, so when you do combine them, you may use every fact from both together.

5 The five answers never change

- A: only statement 1 works alone
- B: only statement 2 works alone
- C: neither alone, both together
- D: each statement works alone
- E: not enough, even together

IN THE LESSON

- The five answer choices on this section are the same every single time, so learn them as a table and you will never need to read them again.
- If statement one alone works and statement two alone does not, choose A.
- If statement two alone works and one does not, choose B.
- If neither works alone but they work together, choose C.
- If each works by itself, choose D.
- And if even together they are not enough, choose E.
- The table is really your two tests laid side by side.
- One test result for each statement, then one more only if you need it.
- A quick tip: after testing a statement alone, say its verdict aloud in your head, sufficient or not sufficient, before moving on.
- It stops the two statements blurring together.

6 Example 1: a value question

What is n? (1) $4n - 9 = 15$ (2) $n^2 - 5n = 6$

Kind of question	needs one value	**value**
(1) alone	$4n = 24$	**$n = 6$**
(2) alone	$(n - 6)(n + 1) = 0$	**6 or −1**
Verdict on (2)	two values	**not enough**
Answer	(1) alone works	**A**

IN THE LESSON

- Example one.
- What is the value of n?
- Statement one: four n minus nine equals fifteen.
- Statement two: n squared minus five n equals six.
- First, the kind: a value question, so we need exactly one value.
- Statement one alone.
- Add nine to get four n equals twenty four, so n is six.
- One value, sufficient.
- Now forget that.
- Statement two alone.
- Move everything to one side: n squared minus five n minus six equals zero, which factorises into n minus six, times n plus one.
- So n is six or minus one.
- Two values, so it is not sufficient.
- Statement one works and two does not, which is choice A.
- Notice that statement two even contains the right answer, six, and was still not enough.

7 Example 2: a yes or no question

Is $k < 10$? (1) $k^2 < 100$ (2) $2k + 3 < 30$

Kind of question	yes or no	**yes/no**
(1) alone	$-10 < k < 10$	**always yes**
(2) alone	$k < 13.5$	**could vary**
Test cases for (2)	$k = 0$ and $k = 12$	**yes; no**
Answer	(1) alone works	**A**

IN THE LESSON

- Example two.
- Is k less than ten?
- Statement one: k squared is less than one hundred.
- Statement two: two k plus three is less than thirty.
- The kind: yes or no.
- Statement one alone.
- If the square of k is under one hundred, then k sits between minus ten and ten.
- Every such k is below ten, so the answer is always yes.
- Sufficient.
- Statement two alone.
- Take away three and halve it: k is below thirteen and a half.
- Try k equal to zero.
- The answer is yes.
- Try k equal to twelve.
- Then two k plus three is twenty seven, under thirty, but twelve is not below ten, so the answer is no.
- Yes in one case and no in another, not sufficient.
- The answer is A.

8 Traps in the method

- Never carry statement 1 into statement 2
- Not told integer or positive? Allow fractions, zero, negatives
- Squares have two roots: $x^2 = 9$ gives 3 and −3
- Same fact twice is still one fact

IN THE LESSON

- Four traps catch almost everyone at first.
- Trap one: carrying a fact from statement one into your test of statement two.
- Test each alone, with a clean head.
- Trap two: assuming a number is a whole number or is positive when the question does not say so.
- A number can be a fraction, zero, or negative unless told otherwise.
- Trap three: forgetting that a square has two roots.
- If x squared is nine, x can be three or minus three.
- Trap four: thinking that two statements that say the same thing in different words give more information than one.
- If statement two just repeats statement one, they cannot rescue each other, and the answer is D or E, never C.
- Check for these before you answer.

9 Number properties: test cases, do not solve

- Pick numbers that obey the statement
- Aim for one yes case and one no case
- Both found: not sufficient. Done
- Always the same answer? Find the reason

IN THE LESSON

- Now the first family: number properties.
- These are questions about odd and even, divisibility, remainders, primes and factors.
- Imagine a locksmith testing a key.
- The locksmith does not study the lock.
- They just try the key in a few well chosen doors.
- Number properties work like that.
- You rarely solve an equation.
- Instead you try numbers that obey the statement, and you aim deliberately for two results: one number that gives yes, and one that gives no.
- If you can find both, the statement is not sufficient, and you are done.
- If every number you try gives the same answer, stop and ask why.
- There is usually a reason, such as three numbers in a row always containing a multiple of three.
- That reason is your proof.
- Always read the stem first for words like integer and positive.

10 Remainders as a pattern

Number = d × k + r (0 ≤ r < d)

Divisor (d)	What you divide by
Whole number (k)	How many whole times it fits
Remainder (r)	Left over, less than d

IN THE LESSON

- A remainder is what is left over after dividing as many whole times as you can.
- Twenty three divided by six goes three times with five left over.
- The rule is that a number equals the divisor times some whole number, plus the remainder.
- Let us read the terms.
- The divisor, d, is what we divide by.
- The letter k is a whole number of times it fits, and it can be anything from zero upwards.
- And r is the remainder, which is always smaller than the divisor.
- So a number that leaves remainder four on division by six is six k plus four, which gives four, ten, sixteen, twenty two, and so on.
- Writing the pattern like this lets you list numbers quickly and combine two patterns without guessing.

11 Example 3: remainders, two patterns together

Remainder when n ÷ 12? (1) n ≡ 3 (mod 4) (2) n ≡ 2 (mod 3)

(1) alone	3, 7, 11, 15 …	**3 or 7**
(2) alone	2, 5, 8, 11 …	**2 or 5**
On both lists	11, 23, 35 …	**every 12th**
Remainder on ÷ 12	11 each time	**11**
Answer	together only	**C**

IN THE LESSON

- Example three.
- A positive integer n is divided by twelve.
- What is the remainder?
- Statement one: n leaves remainder three when divided by four.
- Statement two: n leaves remainder two when divided by three.
- A value question.
- Statement one alone gives three, seven, eleven, fifteen, and so on.
- Three has remainder three on division by twelve, but seven has remainder seven.
- Not sufficient.
- Statement two alone gives two, five, eight, eleven.
- Two and five give different remainders.
- Not sufficient.
- Together, we need numbers on both lists.
- Three is not on the second list, seven is not, but eleven is on both.
- The next one is twenty three, and twenty three divided by twelve leaves eleven.
- The pattern repeats every twelve.
- Always eleven.
- Sufficient together, choice C.

12 Example 4: a definite no is an answer

Is positive integer n prime? (1) n is a multiple of 15 (2) n is odd

Kind of question	yes or no	**yes/no**
(1) alone	15, 30, 45 … all divisible by 3	**always no**
(2) alone	n = 3 yes, n = 9 no	**could vary**
Lesson	always no still counts	**sufficient**
Answer	(1) alone works	**A**

IN THE LESSON

- Example four.
- Is the positive integer n prime?
- Statement one: n is a multiple of fifteen.
- Statement two: n is odd.
- A yes or no question.
- Statement one alone.
- The multiples of fifteen are fifteen, thirty, forty five, and so on.
- Each is divisible by three and by five, and each is bigger than five, so none is prime.
- The answer is always no.
- Many students feel uneasy here because the statement never says what n is.
- But always no is a definite answer, so it is sufficient.
- Statement two alone.
- Three is odd and prime, so the answer is yes.
- Nine is odd and not prime, so the answer is no.
- Yes and no, not sufficient.
- Answer A, and the key was spotting a statement that rules the property out.

13 Example 5: consecutive integers

Is n(n+1)(n+2) divisible by 12? (1) n even (2) n multiple of 3

(1) test n = 2	2 × 3 × 4	**24: yes**
(1) test n = 4	4 × 5 × 6	**120: yes**
(1) why	two evens: 8; three in a row: 3	**always yes**
(2) test n = 3 and 9	60 and 990	**yes; no**
Answer	(1) alone works	**A**

IN THE LESSON

- Example five, a trap one.
- For a positive integer n, is n times n plus one times n plus two divisible by twelve?
- Statement one: n is even.
- Statement two: n is a multiple of three.
- Statement one first.
- Try n equal to two: two times three times four is twenty four, divisible by twelve.
- Try n equal to four: four times five times six is one hundred and twenty, divisible by twelve again.
- Is it a coincidence?
- No.
- If n is even, then n and n plus two are two even numbers, giving a factor of eight, and any three numbers in a row include a multiple of three.
- So it is always divisible by twenty four, so always yes.
- Statement two: n equal to three gives sixty, yes, but nine gives nine hundred and ninety, which is not.
- Answer A.

14 Mistakes to avoid, so far

- 1. One example proves nothing: hunt for the opposite
- 2. Always no is still sufficient
- 3. Read the stem: integer? positive?
- 4. Try zero, negatives, fractions when allowed
- 5. Test each alone before combining

IN THE LESSON

- Time to stop and collect the mistakes that cost marks.
- One.
- Stopping at one example.
- A single number that gives yes proves nothing.
- You need to look for a number that gives no.
- Two.
- Treating a certain no as a failure.
- Always no is sufficient.
- Three.
- Forgetting what the stem allows.
- If it does not say integer, then one half is allowed, and remainders may not even make sense.
- If it does not say positive, zero and negatives are allowed.
- Four.
- Testing only friendly numbers.
- Try zero, one, a negative, a fraction and a large number when the stem permits.
- Five.
- Combining statements too early.
- Test each alone first, because the answer is often settled before you need to combine them.

15 Algebra: translate, then count

- Translate each statement into an equation
- Two different equations, two unknowns: usually enough
- Watch for equations that are the same in disguise
- Ask: what combination does the question want?

IN THE LESSON

- Now the second family: algebra and word problems.
- Think of translating from a foreign language.
- Each statement is a sentence in plain English, and your job is to turn it into an equation or an inequality.
- Then you count what you know.
- As a rough guide, two different equations in two unknowns usually fix both unknowns.
- One equation in two unknowns usually does not.
- But rough guides break, and the exam knows it.
- Two equations might secretly be the same equation, such as x plus y equals five and two x plus two y equals ten, which tell you nothing new.
- And the question might not ask for each unknown.
- It might ask for a combination, and one equation can fix a combination while leaving the pieces unknown.
- So count carefully, and read what is being asked.

16 The square of a sum

$(x + y)^2 = x^2 + y^2 + 2xy$

Sum squared	The sum, squared
Sum of squares	x^2 plus y^2
Twice the product	Two times x times y

IN THE LESSON

- One rule you must know cold for this family is the square of a sum.
- The sum of two numbers, squared, equals the first number squared, plus the second number squared, plus twice their product.
- Let us read the terms.
- The left side is the sum squared, written x plus y all squared.
- Then comes the sum of squares, x squared plus y squared.
- And last, twice the product, two times x times y.
- Why does it matter?
- Because it links three things the exam likes to give you in pieces.
- If a statement gives the sum of squares, and another gives the product, you can find the sum squared.
- But the sum itself is then a square root, which has two signs, and that is where the trap hides.

17 Example 6: the question asks for a combination

What is 2x + y? (1) 6x + 3y = 24 (2) x − y = 1

(1) alone	divide by 3	**2x + y = 8**
(2) alone	x = y + 1, y is free	**not fixed**
Two cases for (2)	(1, 0) and (2, 1)	**2 and 5**
Counting trap	one equation, two unknowns	**still enough**
Answer	(1) alone works	**A**

IN THE LESSON

- Example six.
- What is the value of two x plus y?
- Statement one: six x plus three y equals twenty four.
- Statement two: x minus y equals one.
- A value question, and the target is the combination two x plus y, not x or y separately.
- Statement one alone.
- Every term on the left is divisible by three, so divide the whole equation by three.
- We get two x plus y equals eight.
- That is exactly the target, so statement one is sufficient, even though x and y themselves are still unknown.
- This is the combination idea from the last slide.
- Statement two alone.
- It says x is y plus one, with y free.
- Try y equal to zero, so x is one, and the target is two.
- Try y equal to one, so x is two, and the target is five.
- Not sufficient.
- Answer A.

18 Example 7: squares hide a sign

x, y real. What is x + y? (1) $x^2 + y^2 = 10$ (2) $xy = 3$

(1) alone	(1,3): 4; (3,−1): 2	**not enough**
(2) alone	(1,3): 4; (−1,−3): −4	**not enough**
Together	$(x + y)^2 = 10 + 2 \times 3$	**16**
So x + y	square root of 16	**4 or −4**
Answer	the sign stays open	**E**

IN THE LESSON

- Example seven.
- Here is the classic trap.
- x and y are real numbers.
- What is x plus y?
- Statement one: x squared plus y squared equals ten.
- Statement two: x times y equals three.
- Statement one alone.
- One and three work in magnitude, but so does three and minus one.
- The sums are four and two.
- Not sufficient.
- Statement two alone.
- One and three give a sum of four, while minus one and minus three give minus four.
- Not sufficient.
- Now together.
- The sum squared equals ten, plus twice three, which is sixteen.
- So x plus y is the square root of sixteen, which is four or minus four.
- Both signs are possible: one and three, or minus one and minus three, each fit both statements.
- Two values, so even together it is not sufficient.
- Answer E.

19 Word problems: use every fact in the story

- Write the equation the story gives
- List the whole-number solutions that fit
- Check which fits each statement keeps
- Exactly one left? Sufficient. Stop

IN THE LESSON

- Word problems add a new weapon: the story itself.
- Imagine buying pens and notebooks with exact money.
- You cannot buy half a pen, and you cannot buy minus two notebooks.
- Items are whole numbers, never negative.
- That fact is in the story even if no statement repeats it, and it can cut an equation with many solutions down to just a few.
- So read the stem for every constraint: a fixed total, at least one of each, whole people, whole items.
- First write the equation.
- Then list the whole number solutions that fit.
- Then see which solutions each statement keeps.
- If one statement leaves exactly one solution, it is sufficient.
- This is just the detective again, using the story to rule people out.
- And once you see exactly one solution, stop and move on.

20 Example 8: the whole-number fits

Pens $3, notebooks $5, total exactly $41

3p + 5n = 41	Items
12 pens, 1 notebook	13
7 pens, 4 notebooks	11
2 pens, 7 notebooks	9

IN THE LESSON

- Example eight.
- A shop sold only pens at three dollars and notebooks at five dollars, and took exactly forty one dollars.
- How many notebooks were sold?
- We have three p plus five n equals forty one.
- If n is one, then three p is thirty six, so p is twelve.
- If n is four, then three p is twenty one, so p is seven.
- If n is seven, then three p is six, so p is two.
- Try n equal to ten and the notebooks alone cost fifty, which is too much.
- And n of two, three, five, six all leave a total that three does not divide.
- So there are exactly three fits.
- The table shows them, and the number of items for each is thirteen, eleven and nine.
- The next slide tests the statements against these three rows.

21 Example 8: testing the statements

How many notebooks? (1) fewer than 10 items (2) more pens than notebooks

(1) alone	only 9 items is under 10	**2 pens, 7 books**
(2) alone	12 > 1 and 7 > 4	**1 or 4**
Why it works	3p + 5n = 41 has only 3 fits	**whole numbers**
Answer	(1) alone works	**A**

IN THE LESSON

- Now the question: how many notebooks were sold?
- Statement one: fewer than ten items were sold in all.
- Look at the item counts: thirteen, eleven and nine.
- Only nine is fewer than ten.
- That leaves two pens and seven notebooks, one fit, so the answer is seven.
- Sufficient.
- Statement two: more pens than notebooks were sold.
- Twelve pens beats one notebook, and seven pens beats four notebooks.
- Two fits survive, so the number of notebooks could be one or four.
- Not sufficient.
- Answer A.
- Notice something nice.
- We never needed algebra beyond the first equation.
- The real work was listing a handful of whole number fits and asking which statement trims the list to one.
- That is a method you can run in under two minutes.

22 Logic questions: puzzles, not sums

- Order, overlap, extremes, what must be true
- Build a yes case and a no case
- Cannot build the no case? Say why
- Skip arithmetic the question never asked for

IN THE LESSON

- Now the third family: questions that turn on logic rather than calculation.
- They are about order, overlap, extremes, or what must be true.
- Treat them as puzzles.
- The golden move is the same one we have used all along: try to build a counterexample.
- Build one arrangement that obeys the statement and gives yes, and another that obeys it and gives no.
- If you can build both, the statement is not sufficient.
- If you cannot build the second one, ask why it is impossible, and the reason is your proof.
- Another golden rule: resist doing arithmetic the question does not ask for.
- If it asks whether the average is above ten, you do not need the average.
- You only need to know whether every possible case lands above ten.

23 Example 9: order by chaining

Together: a chain

Ana is older than Cara →

Cara is older than Ben →

So Ana is older than Ben

Alone: each leaves a gap

(1) alone: where is Ben? →

(2) alone: where is Ana?

IN THE LESSON

- Example nine, an ordering question.
- Is Ana older than Ben?
- Statement one: Ana is older than Cara.
- Statement two: Cara is older than Ben.
- Statement one alone.
- Ana is older than Cara, but where is Ben?
- Ben could be older than both, so Ana is not older than him, or younger than both, so she is.
- The answer could be yes or no.
- Not sufficient.
- Statement two alone fails the same way about Ana.
- Now put them together on one line.
- Ana is older than Cara, and Cara is older than Ben, so Ana must be older than Ben.
- Chain the two statements and the answer is always yes.
- Sufficient together, choice C.
- Sketching the order on a line takes ten seconds and removes all doubt.

24 Example 10: overlapping groups

60 students. Do at least 12 study both? (1) 40 French, 32 Spanish (2) 5 study neither

Add the two groups	40 + 32	**72**
Compare with the whole	72 − 60	**12 extra**
Extras must be shared	overlap at least	**12**
(2) alone	no group sizes given	**not enough**
Answer	(1) alone works	**A**

IN THE LESSON

- Example ten.
- Sixty students each study French, Spanish, both or neither.
- Do at least twelve study both languages?
- Statement one: forty study French and thirty two study Spanish.
- Statement two: five study neither.
- Picture the sixty students as sixty seats.
- Forty seats are taken by French students and thirty two by Spanish students.
- Add them: seventy two.
- But there are only sixty seats.
- So at least twelve students sat in two seats at once, meaning they study both.
- If anyone studies neither, even more are squeezed together.
- So the overlap is at least twelve every time, sufficient.
- Statement two alone says nothing about the language numbers, so it cannot say.
- The rule: if two groups add up to more than the whole, they must share members.
- Answer A.

25 Example 11: extremes of an average (set-up)

Five positive integers, mean 20, so sum 100. Largest above 25?

(1) smallest is 5: try	5, 5, 5, 5, 80	**80: yes**
(1) and again	5, 23, 24, 24, 24	**24: no**
(2) median 18: try	18, 18, 18, 23, 23	**23: no**
(2) and again	1, 1, 18, 40, 40	**40: yes**
Verdict so far	neither works alone	**test both**

IN THE LESSON

- Example eleven, a hard one.
- Five positive integers have a mean of twenty.
- Is the largest greater than twenty five?
- Five numbers with a mean of twenty must sum to one hundred.
- Do not try every list.
- Ask for the extremes instead.
- Statement one: the smallest is five.
- Try five, five, five, five and eighty.
- The sum is one hundred, and the largest is eighty, so the answer is yes.
- Try five, twenty three, twenty four, twenty four, twenty four.
- That sums to one hundred as well, and the largest is twenty four, so the answer is no.
- Not sufficient.
- Statement two: the median is eighteen.
- Try eighteen, eighteen, eighteen, twenty three, twenty three, which sums to one hundred, largest twenty three, so no.
- But one, one, eighteen, forty, forty also works, largest forty, so yes.
- Not sufficient.

26 Example 11: both together

Order: 5, x, 18, y, z with x ≤ 18 and y ≤ z

Known sum of the other three	100 − 5 − 18	**77**
x is at most	x ≤ 18	**18**
So y + z is at least	77 − 18	**59**
z at least	59 ÷ 2 = 29.5, whole	**30**
Answer	30 > 25 always	**C**

IN THE LESSON

- Now put both statements together.
- The numbers in order are five, then x, then eighteen, then y and z, with y at most z.
- They add to one hundred.
- So x plus y plus z equals one hundred minus five minus eighteen, which is seventy seven.
- How small can the largest, z, be?
- Make the other two as large as possible.
- The number x is at most eighteen, since it comes before the median.
- That leaves y plus z at least fifty nine.
- If y and z are as close as possible, z is at least twenty nine and a half, so as a whole number z is at least thirty.
- Thirty is above twenty five.
- So the largest is always above twenty five, every time.
- Sufficient together, choice C.
- We never tested every list.
- We only pushed the largest down as far as it could go.

27 Mistakes to avoid in logic questions

- 1. Do not compute what was not asked
- 2. Test odd cases: ties, zero, extremes
- 3. Every item: true at both extremes too
- 4. Order tells you nothing about distance
- 5. Cannot build the no case? Explain why

IN THE LESSON

- Some last mistakes before we finish.
- One.
- Doing arithmetic the question did not ask for.
- If you only need to know whether the answer is always yes, find the extremes and stop.
- Two.
- Testing one friendly example.
- The exam hides its no case in an unfriendly arrangement, such as ties, equal values, zero, or a very large number.
- Three.
- Forgetting that a statement about every item in a group also holds for the biggest and smallest.
- Four.
- Assuming order means distance.
- Knowing Ana is older than Cara does not say by how much.
- Five.
- Giving up on a counterexample too soon.
- If you cannot build the second case, explain why it is impossible.
- If you can explain it, the statement is sufficient.
- If you cannot, keep looking.

28 Recap: one method, three families

Sufficient = exactly one answer

The method	Alone, alone, then together
Number properties	Test cases, remainder patterns
Algebra and words	Translate, count, check signs
Logic	Chains, extremes, counterexamples

IN THE LESSON

- Let us pull it together.
- First, the method.
- Decide whether the question is value or yes or no.
- Test statement one alone, then statement two alone, then both together only if you must.
- Sufficient means one value, or always yes, or always no.
- Second, number properties.
- Test cases, aiming for one yes and one no, and write remainders as patterns.
- Third, algebra and word problems.
- Translate, count, watch for squares with two signs, watch for a combination that one equation can fix, and use whole numbers from the story.
- Fourth, logic.
- Chain the order, add overlapping groups against the whole, push averages to their extremes, and try to build a counterexample.
- The habit behind all four is the same: look for a second possible answer.

29 Try three quick questions

1. Is integer n even, given n + 3 is odd? Sufficient?
   Answer: Yes: n must be even, always yes.
2. What is x, given $x^2 = 49$ and $x > 0$? Sufficient?
   Answer: Yes: only x = 7 is left.
3. Is $m > 5$, given $m^2 = 16$? Sufficient?
   Answer: Yes: m is 4 or −4, always no.

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- Is the integer n even, given that n plus three is odd?
- The answer is yes, sufficient.
- If n plus three is odd, n must be even every time, so the answer is always yes.
- Question two.
- What is x, given that x squared equals forty nine and x is greater than zero?
- The answer is sufficient.
- The square allows seven and minus seven, but the positive condition removes minus seven, leaving exactly seven.
- Question three.
- Is m greater than five, given that m squared equals sixteen?
- The answer is sufficient.
- M is four or minus four, and neither is greater than five, so the answer is always no.
- A definite no counts.