Charta | LESSON NOTES

Data sufficiency: one method for the five answers

Data Insights

1 What sufficient means

- Sufficient: one certain answer, however the facts fall
- Value question: exactly one possible value
- Yes or no question: always yes, or always no
- A definite no is sufficient; sometimes yes is not

IN THE LESSON

- Start with the one idea that everything else rests on.
- A statement is sufficient when it lets you answer the question with certainty, and it must be the same answer every way the facts could turn out.
- There are two kinds of question.
- A value question asks for a number, such as how many child tickets were sold.
- Sufficient means exactly one possible value.
- A yes or no question asks something like, are at least a third of the plants tomatoes.
- Here sufficient means always yes, or always no.
- A definite no is just as good as a definite yes.
- What is not sufficient is sometimes yes and sometimes no.
- So you never need the actual number in a yes or no question.
- You need the answer to be certain.

2 The five answers, always in this order

The answer choices are the same in every question

Answer	What it says
A	Statement one alone works, two does not
B	Statement two alone works, one does not
C	Only both together work
D	Each statement alone works
E	Even both together do not work

IN THE LESSON

- The five answer choices never change, and they come in a fixed order, so learn it once.
- Answer A: statement one alone is sufficient, but statement two alone is not.
- Answer B is the reverse: statement two alone is sufficient, but statement one alone is not.
- Answer C: neither statement is enough alone, but both together are.
- Answer D: each statement alone is sufficient, so you could use either one.
- Answer E: even both together are not sufficient.
- Notice what the answers describe.
- They never ask for the value.
- They only say which statements do the job.
- That is why you can stop calculating the moment you know the answer is certain, or that it is not.

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3 The order of checking

Never let one statement borrow from the other

1. The question	value or yes or no?	**decide**
2. Statement one	test it alone	**enough?**
3. Statement two	test it alone	**enough?**
4. Together	only if neither works	**C or E**

IN THE LESSON

- Here is the order that keeps you from being fooled.
- First, read the question, and decide whether it asks for a value or for yes or no.
- Say to yourself what a certain answer would look like.
- Second, take statement one and forget that statement two exists.
- Test it by itself.
- Third, do the same for statement two, forgetting statement one.
- Only if neither works alone do you combine them.
- This protects you from a common mistake: a statement that looks useless, borrowing help from the other one.
- The words together belong to answers C and E only.
- If one statement works alone, you have already ruled out C and E, and the rest is simple.

4 Two statements, one answer

Fill this in as you test

Statement one alone	Statement two alone	Answer
Enough	Not enough	A
Not enough	Enough	B
Enough	Enough	D
Not enough	Not enough	Try both: C or E

IN THE LESSON

- Now turn the checks into an answer, with no guessing.
- Test statement one alone and write yes or no.
- Then test statement two alone and write yes or no.
- If one alone works and two does not, the answer is A.
- If one does not work and two does, the answer is B.
- If both work alone, the answer is D.
- If neither works alone, you combine them, and the answer is C if together they work, and E if they do not.
- So after the two solo tests you are down to a single answer, or to a choice between C and E.
- This small table is worth drawing on your scratch board at the start of every question, and filling in as you go.

5 Test with real numbers

- Build two situations that fit the statement
- Different answers: not sufficient
- Same answer every time: sufficient
- Real things are whole numbers; assume nothing else

IN THE LESSON

- How do you tell whether a statement is sufficient?
- In a word problem, test it with real numbers.
- Try to build two different situations that both obey the statement, and see whether the answer changes.
- If you can build two situations with different answers, the statement is not sufficient, and you have the proof.
- If every situation you can build gives the same answer, it is sufficient.
- Remember that these are real things.
- People, tickets and marbles come in whole numbers, so you cannot sell half a ticket.
- And use only what the question tells you.
- If it does not say a number is whole, or that it is more than one, do not quietly assume it.
- Facts that are written down are all you get.

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6 Question 1: Dana's wage

- Dana is paid the same amount for every hour she works. What is Dana's hourly wage?
- (1) Dana earns more per hour than Eli does.
- (2) Dana earned $136 for one 8-hour shift.

IN THE LESSON

- Question one is an easy one.
- Read it with me, and try it yourself first.
- Cover the answer choices, and cover your idea of what the wage might be.
- Look only at the two statements, and ask what each one really tells you.
- Dana is paid the same amount for every hour she works.
- What is Dana's hourly wage?
- , one, Dana earns more per hour than Eli does.
- , two, Dana earned one hundred thirty six dollars for one eight-hour shift.
- This is a value question: we need one hourly wage, and not a guess.

7 Question 1: test each statement

Dana earns the same every hour: what is her hourly wage?

Question	one hourly wage	**value**
(1) more than Eli	no amount, any wage fits	**not enough**
(2) 136 for 8 hours	136 divided by 8	**17 an hour**
Result	(1) no, (2) yes	**B**

IN THE LESSON

- Step one, the question.
- It asks for Dana's hourly wage, so a sufficient statement must fix one number.
- Step two, statement one alone.
- Dana earns more per hour than Eli does.
- That compares her with someone else, and gives no amount at all.
- Dana could earn ten, or twenty, and still earn more than Eli.
- Not sufficient.
- So the answer is not A and not D.
- It is B, C or E.
- Step three, statement two alone.
- She earned one hundred thirty six dollars for one eight hour shift, at the same pay every hour.
- Divide: one hundred thirty six divided by eight is seventeen.
- One value, seventeen dollars an hour.
- Sufficient, so we need not combine.

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8 Question 1: check the five answers

Statement two alone fixes the wage at seventeen dollars

A: (1) alone works	a comparison gives no amount	**fails**
B: (2) alone works, (1) not	17 an hour is certain	**the answer**
C: only both together	(2) already does it alone	**not needed**
D: each alone	(1) alone does not work	**fails**
E: not even together	(2) alone is enough	**too gloomy**

IN THE LESSON

- Now check the five answers against what we found.
- Answer A says statement one alone is sufficient.
- It is not: a comparison with Eli gives no amount.
- Answer B says statement two alone is sufficient, and statement one is not.
- That is exactly what we found.
- The wage is seventeen dollars an hour.
- Answer C says only both together work.
- But statement two works alone, so combining is not needed.
- Answer D says each alone works.
- Statement one does not.
- Answer E says even together the data are not sufficient, but statement two settles it.
- The answer is B.
- Notice we never needed to see the other statement help.
- That is the order of checking doing its job.

9 Question 2: the allotment plants

- An allotment plot holds 30 plants, each of them a tomato plant or a bean plant. Are at least one third of the plants tomato plants?
- (1) There are 9 tomato plants on the plot.
- (2) There are more bean plants than tomato plants on the plot.

IN THE LESSON

- Question two is a yes or no question, and it is easy once you know what a certain answer looks like.
- An allotment plot holds thirty plants, each of them a tomato plant or a bean plant.
- Are at least one third of the plants tomato plants?
- , one, There are nine tomato plants on the plot.
- , two, There are more bean plants than tomato plants on the plot.
- One third of thirty plants is how many?
- Hold that number.

10 Question 2: a certain no counts

Are at least one third of the 30 plants tomato plants?

Question	ten or more tomato plants?	**yes or no**
(1) nine tomato plants	nine is fewer than ten	**always no**
(2) more beans than tomato	12 gives yes, 5 gives no	**sometimes yes**
Result	(1) yes, (2) no	**A**

IN THE LESSON

- Step one, the question: are at least one third of the thirty plants tomato plants?
- One third of thirty is ten.
- So the question is really: are there ten or more tomato plants?
- We need a certain yes or a certain no.
- Statement one alone: there are nine tomato plants.
- Nine is fewer than ten, so the answer is a definite no.
- It is a no every time, so statement one is sufficient.
- Statement two alone: more bean plants than tomato plants.
- That allows up to fourteen tomato plants.
- With twelve tomato plants the answer is yes.
- With five it is no.
- Sometimes yes, sometimes no: not sufficient.
- Statement one works alone, and two does not, so the answer is A.

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11 Question 2: check the five answers

A certain no is sufficient

A: (1) alone works, (2) not	nine is a certain no	**the answer**
B: (2) alone works	twelve and five disagree	**fails**
C: only both together	(1) works alone already	**not needed**
D: each alone works	(2) alone does not work	**fails**
E: not even together	(1) alone settles it	**too gloomy**

IN THE LESSON

- Check the five answers.
- Answer A says statement one alone is sufficient and statement two is not.
- Statement one gives a certain no, and statement two gives yes for twelve tomato plants but no for five.
- So A is exactly right.
- Answer B has it backwards.
- Answer C says only both together work, but one works alone.
- Answer D says each works alone, and two does not.
- Answer E says nothing works, which ignores the certain no.
- Here is the lesson inside this question.
- Many learners feel that a no answer means the statement did not help, and so they call it not sufficient.
- That is a trap.
- A certain no answers the question just as well as a certain yes.
- The answer is A.

12 Question 3: marbles in a bag

- A bag contains only red and blue marbles. How many marbles are in the bag?
- (1) Exactly 3/7 of the marbles are red.
- (2) There are more than 15 and fewer than 25 marbles in the bag.

IN THE LESSON

- Question three is a medium one.
- Read it with me, and look closely at what each statement leaves open.
- One of them talks about a fraction, and the other talks about a range, and neither one mentions the exact number.
- A bag contains only red and blue marbles.
- How many marbles are in the bag?
- , one, Exactly three sevenths of the marbles are red.
- , two, There are more than fifteen and fewer than twenty five marbles in the bag.
- Another value question: we need the exact number of marbles.

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13 Question 3: neither alone, both together

How many marbles are in the bag?

(1) three sevenths red	totals: 7, 14, 21, 28 ...	**not enough**
(2) between 15 and 25	totals: 16 to 24	**not enough**
Together	multiple of 7 in 16 to 24	**21 only**
Result	(1) no, (2) no, both yes	**C**

IN THE LESSON

- Step one, the question: how many marbles are in the bag?
- We need exactly one total.
- Statement one alone: exactly three sevenths of the marbles are red.
- For that to be a whole number of marbles, the total must be a multiple of seven: seven, fourteen, twenty one, and so on.
- Many totals fit.
- Not sufficient.
- Statement two alone: more than fifteen and fewer than twenty five.
- That allows every total from sixteen to twenty four.
- Not sufficient.
- Neither alone, so we combine them.
- The totals must be multiples of seven, and between fifteen and twenty five.
- Look through the multiples of seven.
- Fourteen is too small.
- Twenty one fits.
- Twenty eight is too big.
- So the total is twenty one, exactly one value.
- Together they are sufficient: answer C.

14 Question 3: check the five answers

Together the total is exactly twenty one

A: (1) alone works	7, 14, 21 ... all fit	**fails**
B: (2) alone works	any total from 16 to 24	**fails**
C: only both together	together the total is 21	**the answer**
D: each alone works	neither does	**fails**
E: not even together	together they give 21	**too gloomy**

IN THE LESSON

- Check the five answers.
- Answer A says statement one alone is enough.
- It is not: seven, fourteen, twenty one and more all fit three sevenths.
- Answer B says statement two alone is enough.
- It is not: sixteen to twenty four all fit.
- Answer C says only both together work.
- We found that together the only total is twenty one.
- That is the answer.
- Answer D says each alone works, which is false twice over.
- Answer E says even together they fail, but twenty one is a single value.
- Notice how the two statements cover each other's weakness.
- One gives the shape, the multiples of seven, and the other gives the range.
- Neither could finish the job alone.
- The answer is C.

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15 Question 4: football and tennis

- A class has 30 students. Of these, 18 play football and 15 play tennis, and each student plays football, tennis, both or neither. How many students play neither?
- (1) Exactly 3 students play both.
- (2) Every student plays at least one of the two sports.

IN THE LESSON

- Question four is a medium one, with a surprise.
- Read it carefully, because the numbers are chosen to work together.
- Notice that the class has thirty students, and that eighteen and fifteen are both quite large numbers.
- A class has thirty students.
- Of these, eighteen play football and fifteen play tennis, and each student plays football, tennis, both or neither.
- How many students play neither?
- , one, Exactly three students play both.
- , two, Every student plays at least one of the two sports.
- We need the number who play neither sport.

16 Question 4: each statement alone

30 students: 18 football, 15 tennis, how many play neither?

Question	how many play neither?	value
(1) three play both	18 + 15 - 3 = 30 play a sport	**neither: 0**
(2) all play at least one	nobody plays neither	**neither: 0**
Result	(1) yes, (2) yes	**D**

IN THE LESSON

- Step one, the question: how many students play neither sport?
- Statement one alone: exactly three students play both.
- Count the students who play at least one sport.
- Eighteen play football and fifteen play tennis, which adds to thirty three.
- But three students were counted twice, because they play both.
- So thirty three take away three leaves thirty students who play at least one sport.
- There are only thirty students in the class.
- So nobody plays neither.
- The answer is zero, one value.
- Sufficient.
- Statement two alone: every student plays at least one of the two sports.
- Then nobody plays neither, so again zero.
- Sufficient.
- Each statement works on its own, so the answer is D.
- Remember, sufficient never means that the answer is large.
- Here the answer is zero.

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17 Question 4: check the five answers

Each statement alone gives zero: sufficient

A: (1) alone, (2) does not	(2) also gives zero	**(2) works**
B: (2) alone, (1) does not	(1) also gives zero	**(1) works**
C: only both together	each works without help	**not needed**
D: each alone works	both give zero	**the answer**
E: not even together	neither each alone is enough	**too gloomy**

IN THE LESSON

- Check the five answers.
- Answer A says statement one works and statement two does not.
- But statement two works too, so A is wrong.
- Answer B fails for the opposite reason, because statement one works as well.
- Answer C says only both together work, but each is sufficient without help.
- Answer D says each statement by itself is sufficient.
- That is exactly right.
- Answer E says the data are not sufficient even together, which is false, since the number is zero.
- Two cautions come out of this question.
- First, zero is a perfectly good value.
- Second, when both statements are sufficient, they must never disagree.
- If you ever find two statements giving different answers, check your arithmetic.
- The answer is D.

18 Question 5: cinema tickets

- A family spent exactly $60 on cinema tickets, at $8 for each adult ticket and $6 for each child ticket. How many child tickets did the family buy?
- (1) The family bought at least one adult ticket and at least one child ticket.
- (2) The family bought fewer than 10 tickets.

IN THE LESSON

- Question five is a hard one.
- It tests whether you can be honest about a statement that adds nothing.
- A family spent exactly sixty dollars on cinema tickets, at eight dollars for each adult ticket and six dollars for each child ticket.
- How many child tickets did the family buy?
- , one, The family bought at least one adult ticket and at least one child ticket.
- , two, The family bought fewer than ten tickets.
- A value question again: we need the number of child tickets.

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19 Question 5: list every possibility

$8 adult, $6 child, exactly $60: how many child tickets?

All ways to make 60	(0,10) (3,6) (6,2)	**adults, child**
(1) at least one of each	removes (0,10)	**6 or 2**
(2) fewer than 10 tickets	removes (0,10) only	**6 or 2**
Together	the same case removed twice	**still 6 or 2**

IN THE LESSON

- Step one, the question: how many child tickets?
- When the facts are a total and two prices, list the possibilities.
- Eight dollars for an adult ticket and six for a child ticket must total sixty.
- Try zero adult tickets: ten child tickets.
- Try three adult tickets, that is twenty four dollars, leaving thirty six, which is six child tickets.
- Try six adult tickets, forty eight dollars, leaving twelve, which is two child tickets.
- No other count of adult tickets works.
- So the families are: zero and ten, three and six, six and two.
- Statement one removes the case with no adult tickets, leaving six or two child tickets.
- Statement two, fewer than ten tickets, removes only the ten child ticket case.
- Both leave six or two.

20 Question 5: check the five answers

Together they still allow six or two child tickets

A: (1) alone works	6 or 2 child tickets	**fails**
B: (2) alone works	6 or 2 child tickets	**fails**
C: only both together	together still 6 or 2	**fails**
D: each alone works	neither does	**fails**
E: not even together	6 and 2 both survive	**the answer**

IN THE LESSON

- Check the five answers.
- Answers A and B each claim one statement alone is sufficient.
- Both leave six or two child tickets.
- Answer C says both together are sufficient.
- Here is the trap.
- When two statements are both about the same thing, they can remove the same case and nothing else, and then together they add nothing.
- Answer D says each alone is sufficient, which is false.
- So the answer is E: even together, the statements leave both six and two child tickets possible.
- In the exam, this is the hardest decision: accepting that the data are not enough.
- Do not force an answer out of them.
- Two statements are not always better than one.
- The answer is E.

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21 Traps to watch for

- Each statement alone first
- A certain no is sufficient
- Use the facts in the question itself
- Two cases with different answers: not sufficient
- You never need the final value

IN THE LESSON

- Let us collect the traps.
- One, letting a statement borrow from the other.
- Test each alone first.
- Two, calling a certain no not sufficient.
- A certain no is sufficient.
- Three, forgetting the facts in the question itself.
- The prices, the totals and the whole numbers are part of the data.
- Four, stopping at the first case that works.
- To prove a statement is sufficient, you need to be sure that no other case could break it.
- Five, forcing an answer.
- If you can build two different answers from the data, the answer is E, or the statement is not enough, and you should say so honestly.
- Six, spending ages on the arithmetic.
- You only need enough to decide, and you never need the final value.

22 Accurate and quick

- Value, or yes or no?
- Write: one, two, together
- Break each statement with a second case
- Decide; do not finish the sum
- About two minutes, then move on

IN THE LESSON

- Now a few habits that save time.
- First, read the question carefully, and decide in a few words whether it is a value or a yes or no question.
- Second, write the little table on your scratch board: one, two, together.
- Third, for each statement, look for the quickest way to break it, by building a second case.
- If you find one, stop: not sufficient.
- Fourth, do not solve more than you need.
- You only need to know that one value exists, not what it is.
- Fifth, spend about two minutes a question.
- If the decision is hard, use the answer order to choose: eliminate the answers you can rule out, then pick the best of the rest, and move on.

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23 Quick check

1. One statement gives a certain no; the other fails. Answer?
   Answer: A or B: a certain no is sufficient.
2. Neither works alone; together one value. Answer?
   Answer: C.
3. Neither works alone; together two answers still fit. Answer?
   Answer: E.

IN THE LESSON

- Now a quick check.
- Pause after each question and answer it yourself.
- Question one.
- Statement one alone gives a certain no to a yes or no question, and statement two does not work alone.
- Which answer?
- The answer is A, because a certain no is sufficient.
- Question two.
- Neither statement works alone, but together they give one value.
- Which answer?
- The answer is C.
- Question three.
- Both statements are about the same fact, and neither works alone.
- Together they still allow two different answers.
- Which answer?
- The answer is E.
- When the statements repeat each other, combining them adds nothing.

24 Recap: the data sufficiency method

- Value, or yes or no?
- Statement one alone, then statement two alone
- Both alone: D. One alone: A or B
- Neither: together, C or E
- Try to build two different answers

IN THE LESSON

- Let us recap the technique.
- Decide whether the question asks for a value or for yes or no.
- Test statement one alone, then statement two alone, and fill in the little table.
- If one alone works and the other does not, the answer is A or B.
- If both work alone, the answer is D.
- Only if neither works do you combine, and then the answer is C if together they are enough, and E if they are not.
- Test every statement by trying to build two different answers.
- A certain no counts as sufficient, and zero is a perfectly good value.
- Never let a statement borrow from the other, and never assume a fact the question does not give.

Summary notes for the Charta whiteboard lesson. Figures are from the lesson's made-up examples.

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