Numbers: order, factors and remainders

GMAT Quantitative Reasoning

1 A street of numbers

- Every number has a spot on the line
- Zero is the middle; right is bigger
- Greater = further right, always
- Comparing = which house is further right?

IN THE LESSON

- Picture a very long straight street, with a house at every number.
- Zero is the town square in the middle.
- Walk to the right and the house numbers grow: one, two, three, and on and on.
- Walk to the left and you pass minus one, minus two, minus three.
- Every number you will ever meet, whole numbers, fractions and decimals, has its own spot on this street, which we call the number line.
- Here is the one rule that matters.
- Order means position.
- The number further to the right is the greater one, always.
- Every comparison question on the exam is secretly just asking which of two houses lies further right.

2 Five numbers on the line

- −5/6
- −2/3
- 0
- 1/2
- 3/4

IN THE LESSON

- Here is the street drawn as a line, with the smaller numbers on the left and the greater on the right.
- Look at the five marks.
- Minus five sixths sits far to the left.
- Minus two thirds is a little closer to the town square.
- Zero is in the middle.
- One half sits to the right of zero, and three quarters is further right still.
- Notice something about the two negatives.
- Five sixths is bigger than two thirds, yet minus five sixths is the smaller number, because it is further from zero in the negative direction.
- Keep this picture in your head.
- Whenever a comparison feels confusing, sketch the line and drop the numbers onto it.

3 Negatives flip the picture

- Negative: further from zero = smaller
- −500 is less than −200 (a bigger debt)
- Compare the sizes, then reverse the answer
- Every negative is below every positive

IN THE LESSON

- With positive numbers, the bigger digits mean the bigger number, and that habit is exactly what the exam tests.
- Once you go below zero, the picture flips.
- Think of debt.
- Owing five hundred dollars is worse than owing two hundred, so minus five hundred is less than minus two hundred, even though five hundred is the bigger digit.
- The further a negative number is from zero, the smaller it is.
- A neat way to handle this is to compare the sizes as if they were positive, then reverse the answer.
- Another useful fact: any negative number is smaller than any positive number, and zero is neither positive nor negative.

4 Example 1: put four negatives in order

Least to greatest: −5/6, −3/4, −0.7, −2/3

−5/6	size 0.833...	**1st (least)**
−3/4	size 0.75	**2nd**
−0.7	size 0.7	**3rd**
−2/3	size 0.667...	**4th (greatest)**

Biggest size = smallest negative number

IN THE LESSON

- Example one, a plain ordering question.
- Put minus five sixths, minus three quarters, minus zero point seven and minus two thirds in order, from least to greatest.
- Step one: all four are negative, so compare their sizes and reverse.
- Step two: turn each into a decimal.
- Five sixths is about zero point eight three three.
- Three quarters is exactly zero point seven five.
- Zero point seven is itself.
- Two thirds is about zero point six six seven.
- Step three: sort the sizes from biggest to smallest, because the biggest size is the smallest number.
- That gives five sixths, three quarters, zero point seven, two thirds.
- So the order, least to greatest, starts with minus five sixths and finishes with minus two thirds.

5 Four ways to compare fractions

- 1. Decimals 2. Common denominator
- 3. Benchmark: 1/2 or 1
- 4. Same top: smaller bottom is bigger
- Add the same to top and bottom: below 1 grows

IN THE LESSON

- Now fractions, where the exam hides most of its speed tests.
- You have four tools.
- Tool one: turn both into decimals.
- Tool two: use a common denominator, then compare the tops.
- Tool three: use a benchmark, such as one half or one, and ask which fraction is above it and by how much.
- Tool four, a rarer one: if the tops are equal, the smaller bottom makes the larger fraction.
- Three fifths beats three sevenths, because fifths are bigger pieces than sevenths.
- There is also a neat trick.
- Add the same number to the top and bottom of a fraction below one, and it grows, so three quarters is less than five sixths.
- Above one it shrinks, so five quarters beats seven sixths.

6 Example 2: find the greatest fraction by benchmark

Greatest of 5/9, 4/7, 6/11, 7/13?

5/9	1/2 + 1/18	**extra 1/18**
4/7	1/2 + 1/14	**extra 1/14**
6/11	1/2 + 1/22	**extra 1/22**
7/13	1/2 + 1/26	**extra 1/26**

Smallest bottom in the extra = biggest extra

IN THE LESSON

- Example two, a typical question.
- Which is the greatest of five ninths, four sevenths, six elevenths and seven thirteenths?
- Use the benchmark one half instead of long division.
- Each fraction is just above one half, so ask by how much.
- Five ninths is one half plus one eighteenth.
- Four sevenths is one half plus one fourteenth.
- Six elevenths is one half plus one twenty second.
- Seven thirteenths is one half plus one twenty sixth.
- Now compare the extras: one over a smaller number is the bigger extra.
- One fourteenth is the largest, so the greatest fraction is four sevenths.
- Two clean steps and no decimals.

7 Absolute value is distance

$|x - a| = d \rightarrow x = a + d$ or $x = a - d$

The number we want (x)	Unknown: where we land
The centre (a)	Centre: the fixed point
The distance (d)	How far away, never below 0

IN THE LESSON

- Now absolute value.
- The bars around a number mean one thing only: how far it is from zero, in either direction.
- So the absolute value of minus four is four, and the absolute value of four is four.
- The bars never make a number negative.
- The real power comes when we write the absolute value of x minus a.
- That is the distance between x and a on the line.
- If it equals d, then x sits d steps from a, so it could be on the right, at a plus d, or on the left, at a minus d.
- Two directions mean two answers.
- Let's take each term.
- The letter a is the centre.
- The letter d is the distance, and it can never be negative.
- And x is the number we want.

8 Seven steps from three

- $x = -4$
- centre 3
- $x = 10$

IN THE LESSON

- Let's draw the picture for the absolute value of x minus three equals seven.
- The middle mark is three, the centre.
- The bars say distance, so we ask for points that are seven away from three.
- Walk seven steps to the right, and you land on ten.
- Walk seven steps to the left, and you land on minus four.
- Both are correct, and the two answers sit the same distance either side of the centre.
- If the right side had been zero, there would be a single answer, the centre itself.
- If it had been a negative number, there would be no answer at all, because a distance cannot be negative.

9 Example 3: the trap, two solutions

Sum of all x with |2x − 5| = 9?

Case one	2x − 5 = 9	**x = 7**
Case two	2x − 5 = −9	**x = −2**
Check x = 7	|14 − 5|	**9**
Check x = −2	|−4 − 5|	**9**
Sum of solutions	7 + (−2)	**5**

IN THE LESSON

- Example three: what is the sum of all values of x for which the absolute value of two x minus five equals nine?
- Same idea, but now the inside is not a plain x.
- Step one: the inside can be nine or minus nine, so write two cases.
- Case one: two x minus five equals nine, so two x is fourteen and x is seven.
- Case two: two x minus five equals minus nine, so two x is minus four and x is minus two.
- Step two: check both.
- Two times seven minus five is nine, good.
- Two times minus two minus five is minus nine, and its absolute value is nine, good.
- Step three: add the solutions.
- Seven plus minus two is five.
- A student who stops after case one answers seven and walks into the trap.

10 Example 4: counting whole numbers within a distance

Integers x with |x − 2| < 3?

Read as distance	within 3 of the number 2	
Ends of the zone	2 − 3 = −1 and 2 + 3 = 5	
Strictly inside	0, 1, 2, 3, 4	**5 integers**
If it were ≤ 3	−1 to 5 included	**7 integers**

IN THE LESSON

- Example four, a counting question.
- How many integers x satisfy the absolute value of x minus two being less than three?
- Read it aloud as distance: x is fewer than three steps from two.
- Start at two, and go three steps left to minus one, and three steps right to five.
- Now the trap.
- The word is less than, not less than or equal to, so the end points minus one and five are not allowed.
- The integers strictly inside are zero, one, two, three and four.
- That is five of them.
- Had the sign been less than or equal to, we would include minus one and five too, and the answer would be seven.
- Always check whether the ends are in.

11 The fastest path on the number line

- Order: sketch the line, benchmark fractions
- Absolute value: say it as a distance
- Two answers, unless the distance is zero
- 10 − |x − 3| is greatest when |x − 3| = 0

IN THE LESSON

- Let's collect the fast methods.
- For ordering, sketch the line, or convert to decimals only when the numbers are friendly, such as halves, quarters and thirds.
- For fractions, reach for a benchmark first.
- For absolute value, always say it as distance, and give two answers unless the distance is zero.
- And for a question that asks for the greatest value of something like ten minus the absolute value of x minus three, remember that the absolute value part can never go below zero.
- So to make the whole thing as big as possible, make that part zero, which happens at x equal to three, and the greatest value is ten.
- Thinking in distances saves minutes.

12 Building blocks: factors, multiples and primes

- Factor: divides n exactly (12: 1, 2, 3, 4, 6, 12)
- Multiple: n times a whole number
- Prime: exactly two factors, 1 and itself
- 2 is the only even prime; 1 is not prime

IN THE LESSON

- Now a different idea: how numbers are built.
- Think of a tray of twelve chocolates.
- You can arrange them in one row of twelve, two rows of six, three rows of four, and so on.
- The numbers that fit exactly, one, two, three, four, six and twelve, are the factors of twelve.
- A factor divides a number with nothing left over.
- A multiple goes the other way: it is the number times a whole number, so the multiples of twelve are twelve, twenty four, thirty six and so on.
- Some numbers can only be arranged in a single row or a single column.
- Those are the primes, numbers with exactly two factors, one and themselves.
- Two is the only even prime, and one is not prime.

13 Quick divisibility tests

Look at the digits, not the whole number

The quick test	Tells you
Last digit is even	by 2
Digits add to a multiple of 3	by 3
Last two digits divide by 4	by 4
Last digit is 0 or 5	by 5

IN THE LESSON

- Before we dig deeper, here are the quick tests that save you from dividing by hand.
- A number is even if its last digit is even, and that means it divides by two.
- It divides by three if its digits add to a multiple of three.
- It divides by four if its last two digits divide by four.
- It divides by five if the last digit is zero or five.
- Nine works the same way: the digits must add to a multiple of nine.
- And to test for six, simply check both two and three.
- For example, take the number four hundred and twelve.
- Its digits add to seven, so three is out, but the last two digits, twelve, divide by four, so four is in.

14 Prime factorisation: the master tool

Break 360 into primes, smallest prime first

360 ÷ 2	= 180	
180 ÷ 2, then ÷ 2	= 90, then 45	
45 ÷ 3, then ÷ 3	= 15, then 5	
5 is prime	stop	
As powers of primes	$2^3 \times 3^2 \times 5$	**= 360**

IN THE LESSON

- Every whole number above one can be written in exactly one way as a product of primes.
- This is the master tool for the whole section.
- Break the number down with a factor tree.
- Take three hundred and sixty.
- Divide by two, and you get one hundred and eighty.
- Divide by two again, ninety.
- Again, forty five.
- Now two no longer fits, so try three: forty five over three is fifteen, and fifteen over three is five, and five is prime.
- So three hundred and sixty is two cubed, times three squared, times five.
- Count the twos, the threes and the fives, and write each as a power.
- From that single line you can read off its factors, common factors, multiples and divisors.

15 GCF and LCM from prime powers

GCF = lowest powers LCM = highest powers
GCF × LCM = a × b

Greatest common factor	Shared primes, lowest power
Least common multiple	All primes, highest power
First number	The first number (a)
Second number	The second number (b)

IN THE LESSON

- Here are the two big ideas, both read straight from the prime powers.
- The greatest common factor, the G C F, is the biggest number that divides both.
- Take each prime they share, and use its lowest power.
- The least common multiple, the L C M, is the smallest number that both divide into.
- Take every prime that appears in either number, and use its highest power.
- There is also a lovely check.
- For two positive whole numbers, the G C F times the L C M equals the first number times the second.
- And if the G C F is one, we call the numbers co prime, so the L C M is just their product, as with eight and fifteen.

16 Example 5: GCF and LCM of 18 and 24

GCF and LCM of 18 and 24

Primes	$18 = 2 \times 3^2$, $24 = 2^3 \times 3$	
GCF: lowest powers	$2^1 \times 3^1$	**6**
LCM: highest powers	$2^3 \times 3^2$	**72**
Check: GCF × LCM	6 × 72	**432**
Check: a × b	18 × 24	**432**

IN THE LESSON

- Example five.
- Find the G C F and L C M of eighteen and twenty four.
- Step one: write each as primes.
- Eighteen is two times three squared.
- Twenty four is two cubed times three.
- Step two, the G C F.
- Both share the primes two and three.
- The lowest power of two is one, the lowest power of three is one, so the G C F is two times three, which is six.
- Step three, the L C M.
- The highest power of two is two cubed, which is eight.
- The highest power of three is three squared, which is nine.
- Eight times nine is seventy two.
- Step four, the check.
- Six times seventy two is four hundred and thirty two, and eighteen times twenty four is also four hundred and thirty two.

17 Which tool? Cut it up, or wait for it

Splitting into equal pieces

Biggest piece that fits both → Use the GCF

Cycles that repeat

First time they meet again → Use the LCM

IN THE LESSON

- Word problems on this topic come in two flavours, and choosing the right tool is half the battle.
- In the first flavour, you are splitting something into equal pieces, such as cutting a sheet into square tiles, or sharing items into identical bags.
- You want the biggest piece that fits everything exactly, so the answer is the greatest common factor.
- In the second flavour, things repeat on a cycle, such as buses that leave every so many minutes, or lights that flash at different rates.
- You want the first moment they line up again, so the answer is the least common multiple.
- Ask yourself one question: am I cutting down, or waiting for a meeting?

18 Example 6: two buses, an LCM problem

Buses every 12 and 20 minutes, together at 8:00

12 as primes	$2^2 \times 3$	
20 as primes	$2^2 \times 5$	
LCM: highest powers	$2^2 \times 3 \times 5$	**60**
Next time together	8:00 plus 60 minutes	**9:00**
Trap: just multiply	12×20	**240, too big**

IN THE LESSON

- Example six.
- Two buses leave the depot together at eight o'clock.
- One leaves every twelve minutes, the other every twenty.
- When do they next leave together?
- This is a waiting problem, so it is the L C M.
- Twelve is two squared times three.
- Twenty is two squared times five.
- Take the highest powers: two squared, three and five.
- Four times three times five is sixty.
- So they meet again after sixty minutes, at nine o'clock.
- Now the trap.
- Many people multiply twelve by twenty to get two hundred and forty, which is a common multiple, but not the least one.
- A product is only the L C M when the numbers share no primes.
- Here they share two squared, so the product is too big.

19 Example 7: tiles, a GCF problem

84 cm by 126 cm, biggest square tiles, no waste

84 as primes	$2^2 \times 3 \times 7$	
126 as primes	$2 \times 3^2 \times 7$	
GCF = side of a square	$2 \times 3 \times 7$	**42 cm**
Squares along each side	84 ÷ 42 and 126 ÷ 42	**2 and 3**
Total squares	2×3	**6**

IN THE LESSON

- Example seven.
- A sheet measures eighty four centimetres by one hundred and twenty six.
- It must be cut, with no waste, into identical squares that are as large as possible.
- How many squares?
- This is a cutting problem, so it is the G C F.
- Eighty four is two squared times three times seven.
- One hundred and twenty six is two times three squared times seven.
- The shared primes are two, three and seven, each at its lowest power of one.
- Two times three times seven is forty two.
- So each square is forty two centimetres a side.
- Along the short side we fit two squares, along the long side three.
- Two times three is six squares in total.

20 Counting divisors

Divisors of p^a × q^b = (a + 1)(b + 1)

Each exponent	Exponent of each prime
Add one to each	Add one to each exponent
Number of divisors	Multiply to count divisors

IN THE LESSON

- How many positive divisors does a number have?
- Listing them is slow and easy to get wrong.
- Use the exponents instead.
- Write the number as prime powers.
- Add one to each exponent.
- Then multiply those results together.
- Why does it work?
- For a prime with exponent three, a divisor can use that prime zero, one, two or three times, which is four choices.
- The choices for different primes are independent, so we multiply.
- Take seventy two, which is two cubed times three squared.
- We get four times three, which is twelve divisors.
- For the odd divisors only, ignore the prime two entirely.
- That leaves three choices for the threes, so there are three odd divisors: one, three and nine.
- The other nine of the twelve are even.

21 Example 8: divisors of 360, odd and even

Divisors of 360 = $2^3 \times 3^2 \times 5$

Exponents plus one	(3+1)(2+1)(1+1)	**24 divisors**
Odd: drop the prime 2	(2+1)(1+1)	**6 odd**
Even = all minus odd	24 − 6	**18 even**

IN THE LESSON

- Example eight.
- How many positive divisors does three hundred and sixty have, and how many are even?
- We know three hundred and sixty is two cubed, times three squared, times five.
- Add one to each exponent: four, three and two.
- Multiply: four times three times two is twenty four divisors.
- Now for the odd ones.
- An odd divisor cannot use the prime two at all, so drop it.
- We have three squared times five left.
- Three times two is six odd divisors.
- The even divisors are everything else: twenty four minus six is eighteen.
- Count the odd ones first, then subtract.
- This is a favourite exam setting.

22 Example 9: the trap, n squared versus n

n^2 divisible by 12: what must divide n?

12 as primes	$2^2 \times 3$	
n^2 has even prime powers	so it holds at least $2^2 \times 3^2$	
So n holds at least	2×3	**6**
Test n = 6	$36 = 3 \times 12$, but $6 \div 12$ fails	**12 not forced**

IN THE LESSON

- Example nine, the trap one.
- The positive integer n is such that n squared is divisible by twelve.
- What is the largest number that must divide n?
- Many choose twelve.
- But twelve is two squared times three.
- Inside n squared, every prime appears with an even power.
- To contain two squared, n needs at least one two.
- To contain three, the power of three in n squared must be even, so at least three squared, which means n needs at least one three.
- So n must be divisible by two times three, which is six.
- Test it.
- Let n be six.
- Then n squared is thirty six, which is three twelves, yet six is not divisible by twelve.
- So n squared divisible by k does not mean n is divisible by k.

23 What is left over: remainders

- 17 sweets, 5 friends: 3 each, 2 left
- $17 = 3 \times 5 + 2$
- The remainder is at least 0 and below the divisor
- Cycles and days of the week are remainders

IN THE LESSON

- Seventeen sweets are shared among five friends.
- Each friend gets three, and two are left over.
- That leftover is the remainder.
- The number of sweets each friend got is the quotient.
- We can write the whole story as one line: seventeen equals three times five, plus two.
- The key rule is that the remainder is a whole number, at least zero, and always less than the divisor you divided by.
- You could never have five or more left over, because you would just hand out another round.
- Remainders appear whenever a question talks about leaving a remainder, repeating cycles, days of the week, or last digits.
- They are secretly questions about leftovers.

24 The remainder equation

$n = q \times d + r$ with $0 \le r < d$

The number (n)	The number being divided
The divisor (d)	What we divide by
The quotient (q)	How many whole times it fits
The remainder (r)	The leftover, below d

IN THE LESSON

- Here is the equation behind every remainder question.
- The number n equals the quotient q times the divisor d, plus the remainder r.
- Let's take the terms one at a time.
- The number n is the thing being divided.
- The divisor d is what we divide by.
- The quotient q is how many whole times d fits.
- And r is what is left over, which must be at least zero and less than d.
- This lets you build numbers.
- A number that leaves remainder four when divided by seven could be four, eleven, eighteen, twenty five and so on, always four more than a multiple of seven.
- And you can do arithmetic on the remainders directly.
- Add, subtract or multiply the remainders, then reduce again.

25 Example 10: remainders of expressions

n leaves remainder 4 when divided by 7

Plug in the smallest: n = 4		**4**
3n	12 = 1 × 7 + 5	**5**
n + 10	14 = 2 × 7 + 0	**0**
n²	16 = 2 × 7 + 2	**2**
Second test: n = 11, 3n	33 = 4 × 7 + 5	**5**

IN THE LESSON

- Example ten.
- When n is divided by seven, the remainder is four.
- Find the remainder when three n is divided by seven.
- The fastest sound path is to plug in the smallest number that fits.
- The smallest is four itself.
- Three times four is twelve.
- Twelve divided by seven is one with five left over.
- So the remainder is five.
- We can test with another number to be sure.
- Eleven also leaves four.
- Three times eleven is thirty three, which is four sevens, twenty eight, plus five.
- Same answer.
- Now two more in the same family.
- For n plus ten, four plus ten is fourteen, which is exactly two sevens, so the remainder is zero.
- The trap is saying fourteen, a remainder can never reach seven.
- For n squared, sixteen leaves two.

26 Example 11: two remainder conditions at once

Remainder 3 on ÷5 and remainder 2 on ÷4

Remainder 3 on ÷ 5	3, 8, 13, 18, 23	
Remainder 2 on ÷ 4	2, 6, 10, 14, 18	
First number in both lists		**18**
Pattern repeats every	LCM of 5 and 4	**20**
Shortcut: n + 2 is a multiple	of 20, so n = 20k − 2	**18, 38, 58**

IN THE LESSON

- Example eleven.
- A positive integer leaves a remainder of three when divided by five, and a remainder of two when divided by four.
- What is the smallest such integer?
- List the first condition: three, eight, thirteen, eighteen, twenty three.
- List the second: two, six, ten, fourteen, eighteen.
- The first number on both lists is eighteen.
- Here is why that works as a pattern.
- The two conditions repeat together every twenty, the least common multiple of five and four.
- So the answers are eighteen, thirty eight, fifty eight and so on.
- There is an even faster way.
- Notice that both remainders are short of the divisor by two.
- So n plus two is a multiple of both five and four, hence a multiple of twenty, and n is eighteen.

27 Odd and even rules

Parity: even or odd, in one line each

Combination	Result
even + even	even
odd + odd	even
even + odd	odd
even × anything	even

IN THE LESSON

- Odd and even numbers follow rules that you can see with small examples.
- Even plus even is even, as two plus four is six.
- Odd plus odd is also even, as three plus five is eight, because the two odd leftovers pair up.
- Even plus odd is odd.
- For products, one even factor is enough to make the whole product even.
- Four times seven is twenty eight.
- Only if every factor is odd is the product odd.
- These are remainder facts in disguise: even means remainder zero when dividing by two, and odd means remainder one.
- When unsure, test one even and one odd example.

28 Example 12: which statement must be odd?

a + b is odd. Which must be odd?

Try a = 2, b = 3	one even, one odd	
a × b	6, even (even × odd)	**never odd**
a − b, and a² + b²	−1 and 13	**must be odd**
2a + b	7 if a=2, 8 if a=3	**can be either**

IN THE LESSON

- Example twelve.
- The integers a and b are such that a plus b is odd.
- Which of the following must be odd?
- Start with what we know.
- A sum is odd only if one of them is even and the other is odd.
- So try a equals two and b equals three.
- Then the product, a times b, is six, which is even, so a times b is never odd here.
- The difference, a minus b, is minus one, which is odd, and it always will be.
- The sum of squares, four plus nine, is thirteen, odd again, since an even square is even and an odd square is odd.
- Now two a plus b.
- With a equal two and b equal three, it is seven, odd.
- But swap them, a three and b two, and it is eight, even.
- So that one does not have to be odd.

29 Consecutive integers: count times middle

Sum = count × middle value

Count of numbers	How many numbers in the run
Middle value	Average of first and last
Their sum	The total of the run

IN THE LESSON

- Consecutive integers are whole numbers that follow each other, such as seventeen, eighteen, nineteen.
- They are evenly spaced, and that gives a great shortcut.
- The sum of a run of consecutive integers equals the count of numbers times the middle value.
- The middle value is also the average of the first and the last.
- Each term: the count is how many numbers there are, the middle value is the one in the centre or the halfway point if the count is even, and the sum is the total.
- A second useful fact: among any k consecutive integers, exactly one is a multiple of k.
- Among any three in a row, one is a multiple of three.
- Among any two, one is even.
- That is why a product of three consecutive integers is always divisible by six.

30 Example 13: five consecutive integers

Five consecutive integers add to 85. Largest?

Middle = sum ÷ count	85 ÷ 5	**17**
Spread out from the middle	15, 16, 17, 18, 19	
Largest of the five		**19**
Check the sum	15 + 16 + 17 + 18 + 19	**85**

IN THE LESSON

- Example thirteen.
- The sum of five consecutive integers is eighty five.
- What is the largest?
- Use the formula backwards.
- The sum is the count times the middle, so the middle is eighty five divided by five, which is seventeen.
- Five numbers centred on seventeen are fifteen, sixteen, seventeen, eighteen and nineteen.
- The largest is nineteen.
- Check by adding them up: fifteen plus sixteen is thirty one, plus seventeen is forty eight, plus eighteen is sixty six, plus nineteen is eighty five.
- This is much faster than setting up x, x plus one, x plus two and so on.

31 Example 14: counting multiples in a range

Multiples of 7 from 100 to 200

First multiple at or above 100	7 × 15	**105**
Last multiple at or below 200	7 × 28	**196**
Gaps between them	(196 − 105) ÷ 7	**13**
Add one for the first number	13 + 1	**14 multiples**

IN THE LESSON

- Example fourteen, a counting problem.
- How many multiples of seven lie between one hundred and two hundred, including both ends if they qualify?
- Find the first multiple at or above one hundred.
- Seven times fourteen is ninety eight, too small, so seven times fifteen, which is one hundred and five.
- Find the last multiple at or below two hundred.
- Seven times twenty eight is one hundred and ninety six.
- Now count the gaps between them: one hundred and ninety six minus one hundred and five is ninety one, divided by seven, is thirteen steps.
- Thirteen steps connect fourteen numbers, so add one.
- The answer is fourteen.
- The fence post trap is forgetting that final plus one.

32 Mistakes to avoid

- Negatives: bigger digits, smaller number
- |x − a| = d has two answers
- Product of two numbers is not always the LCM
- Remainder below the divisor; 1 is not prime
- n^2 by k does not mean n; ranges: add one

IN THE LESSON

- Mistakes that cost marks on this whole topic.
- One.
- With negatives, the bigger digit is the smaller number.
- Minus five sixths is less than minus two thirds.
- Two.
- Absolute value equations usually have two answers, so find both.
- Three.
- A product is not the least common multiple unless the numbers share no primes.
- Four.
- One is not prime, and two is the only even prime.
- A remainder is always less than the divisor, so reduce it.
- Five.
- If n squared is divisible by k, that does not make n divisible by k.
- And when you count a range, add one for the first number.

33 Recap: five toolkits for numbers

Numbers: five ideas to remember

Order	Right is greater; negatives flip
Distance	|x − a| = d: two answers
Factors	Prime powers: GCF, LCM, divisors
Remainders	n = q × d + r, r below d
Consecutive	Sum = count × middle

IN THE LESSON

- Let's pull it together.
- Order: the greater number is further right on the line, negatives flip, and fractions go through decimals or a benchmark.
- Distance: the absolute value of x minus a is a distance, so expect two answers.
- Factors: write numbers as prime powers, take lowest powers for the G C F, highest for the L C M, and add one to each exponent to count divisors.
- Remainders: n equals q times d plus r, with r below d, and you can work with remainders directly.
- Consecutive integers: the sum is the count times the middle, and among k in a row exactly one is a multiple of k.
- Plug in a small number to check any answer.

34 Try three quick questions

1. Which is greater: −7/8 or −5/6?
   Answer: −5/6, it is closer to zero (0.833 against 0.875).
2. How many positive divisors does 100 have?
   Answer: 100 = $2^2 \times 5^2$, so (2+1)(2+1) = 9.
3. n leaves remainder 5 on ÷9. Remainder of 2n?
   Answer: Try n = 5: 2n = 10, which leaves 1.

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- Which is greater, minus seven eighths, or minus five sixths?
- Seven eighths is zero point eight seven five, five sixths is about zero point eight three three.
- Minus five sixths has the smaller size, so it is the greater number.
- Question two.
- How many positive divisors does one hundred have?
- One hundred is two squared times five squared, so add one to each exponent: three times three is nine.
- Question three.
- A number leaves a remainder of five when divided by nine.
- What is the remainder when two n is divided by nine?
- Plug in five, double it to ten, and ten leaves one.
- Well done.