CHARTA LEARNING · LESSON NOTES

Powers, decimals and fractions by hand

GMAT Quantitative Reasoning

1 A story: the chessboard

- An exponent counts copies: $2^5 = 2\times2\times2\times2\times2 = 32$
- Doubling grows enormous: 2^{63} grains by square 64
- Exam numbers are too big to compute
- So rewrite with rules, never multiply out

IN THE LESSON

- Here is an old story.
- A clever courtier asks a king for one grain of rice on the first square of a chessboard, two on the second, four on the third, and so on, doubling each time.
- It sounds like nothing.
- But by the last square the pile is sixty three twos multiplied together, more rice than a kingdom could grow.
- That repeated multiplication is what an exponent records.
- The small raised number tells you how many copies of the base to multiply.
- On the exam, the numbers get huge fast, so you never work them out.
- You use rules to rewrite them, and the answer falls out of the pattern.

2 The exponent rules

$a^m \times a^n = a^{m+n}$
$a^m \div a^n = a^{m-n}$
$(a^m)^n = a^{mn}$

Multiply, same base	Multiply: add the exponents
Divide, same base	Divide: subtract the exponents
Power of a power	Power of a power: multiply
Zero and negative	$a^0 = 1$ and $a^{-n} = 1 \div a^n$

IN THE LESSON

- Here are the rules.
- Each one comes from the idea of counting copies.
- When two powers share a base and you multiply them, you are just stacking copies together, so you add the exponents.
- Two cubed times two to the fourth is two to the seventh.
- When you divide, copies cancel, so you subtract the exponents.
- A power raised to a power means copies of copies, so you multiply the exponents.
- And there are two special cases.
- Anything to the power zero is one.
- A negative exponent means take the reciprocal, so two to the minus three is one over two cubed, which is one eighth.
- These rules only work when the bases match, and that is the whole game.

3 Example 1: find a common base

What is $2^5 \times 4^3$?

Spot a common base	$4 = 2^2$	**base 2**
Rewrite	$4^3 = (2^2)^3$	**2^6**
Same base: add	$2^5 \times 2^6$	**2^{11}**
Compute last	2^{11}	**2,048**
Check	32×64	**2,048**

IN THE LESSON

- Example one, a plain one.
- What is two to the fifth, times four cubed?
- The choices are numbers like two thousand and forty eight, so the answer must be a single number.
- The rules need the same base, so the first move is always to look for one.
- Four is two squared.
- So four cubed is two squared, cubed, and a power of a power multiplies the exponents: two to the sixth.
- Now both factors have base two, so add the exponents.
- Five plus six is eleven.
- Two to the eleventh is two thousand and forty eight.
- A quick check: thirty two times sixty four is also two thousand and forty eight.

4 Example 2: equal powers added

$9^n + 9^n + 9^n = 3^{25}$. Find n.

Equal terms add up	3×9^n	**3 copies**
Base 3	$9^n = 3^{2n}$	**3^{2n}**
Multiply: add exponents	$3^1 \times 3^{2n}$	**3^{2n+1}**
Match exponents	$2n + 1 = 25$	**n = 12**

Equal powers added are a multiple, not an exponent change

IN THE LESSON

- Example two, the typical one, with a trap.
- If nine to the n, plus nine to the n, plus nine to the n equals three to the twenty fifth, what is n?
- The trap is to treat the left side as an exponent problem.
- It is an addition problem.
- Three equal terms add up to three times one of them, so the left side is three times nine to the n.
- Now get a common base of three.
- Nine to the n is three to the two n.
- And three times three to the two n is three to the two n plus one, because three is three to the first.
- So two n plus one equals twenty five, and n is twelve.
- Never add exponents when the problem adds the numbers.

5 Roots are powers too

- √9 = 3, because 3 × 3 = 9
- A square root is the power one half: √a = a^(1/2)
- Roots of a product split: √50 = √25 × √2 = 5√2
- Look for the biggest perfect square inside

IN THE LESSON

- Now roots.
- A square root asks the question: what number, multiplied by itself, gives this?
- The square root of nine is three.
- Here is the idea that makes roots easy on the exam.
- A square root is the power one half.
- Why?
- Because half plus half is one, and adding exponents on the same base fits the multiply rule.
- So every power rule you know also works for roots.
- The most useful consequence is that a root of a product splits into a product of roots.
- That lets you pull out perfect squares.
- The square root of fifty is the square root of twenty five times two, which is five times the square root of two.
- Always look for the biggest perfect square inside.

6 The root rules

√(a×b) = √a × √b
√a = a^(1/2)

Product splits	Root of a product splits
Root as a power	Square root = power one half
Like roots only	Add only like roots
No split over a sum	√(a+b) is not √a + √b

IN THE LESSON

- Here are the root rules.
- For positive numbers, the root of a times b equals the root of a, times the root of b.
- This is the splitting rule that pulls out squares.
- A square root is also the exponent one half, so you can combine roots with the power rules.
- Now the part where marks are lost.
- You can add roots only when they are like roots, meaning the same number under the root sign, just as you can add three apples and five apples but not three apples and five pears.
- So three root two plus five root two is eight root two.
- Finally, the root of a sum is not the sum of the roots.
- Root of nine plus sixteen is five, not three plus four.

7 Example 3: a sum of roots

What is √50 + √18 ?

Split out squares	√50 = √25 × √2	**5√2**
Same for the second	√18 = √9 × √2	**3√2**
Now like roots: add	5√2 + 3√2	**8√2**
Check, √2 ≈ 1.41	8 × 1.41	**≈ 11.3**
Trap answer	√(50 + 18) = √68	**≈ 8.2**

IN THE LESSON

- Example three.
- What is the square root of fifty, plus the square root of eighteen?
- The two roots look different, so a careless solver adds the numbers under the root and gets the square root of sixty eight.
- That is wrong.
- Instead, simplify each root first.
- Fifty is twenty five times two, so its root is five root two.
- Eighteen is nine times two, so its root is three root two.
- Now they are like roots, and five root two plus three root two is eight root two.
- As a check, root two is about one point four one, and eight times that is about eleven point three, while root fifty is about seven point one and root eighteen is about four point two.
- Together that is eleven point three.
- It matches.

8 Compare big powers: same exponent

Base, once every exponent is 10

2^{60}	64
3^{40}	81
5^{30}	125
7^{20}	49

Equal exponents: the bigger base wins

IN THE LESSON

- Comparing powers is the last idea in this first skill, and here is a picture of it.
- Which is largest of two to the sixty, three to the fortieth, five to the thirtieth and seven to the twentieth?
- The exponents are different, so you cannot compare the bases directly.
- The trick is to find a common exponent.
- Every exponent here is a multiple of ten, so write each as something to the tenth.
- Two to the sixty is two to the sixth, all to the tenth, which is sixty four to the tenth.
- Three to the fortieth is eighty one to the tenth.
- Five to the thirtieth is one hundred and twenty five to the tenth, and seven to the twentieth is forty nine to the tenth.
- With equal exponents, the bigger base wins, so five to the thirtieth is the largest.
- Each bar shows the base left inside.

9 Example 4: x plus one over x

$x + 1/x = 3$. Find $x^2 + 1/x^2$.

Square both sides	$(x + 1/x)^2 = 3^2$	9
Expand the left	$x^2 + 2 + 1/x^2$	= 9
Middle term is x × 1/x	2×1	2
Subtract the 2	$9 - 2$	7

IN THE LESSON

- Example four, a trap one.
- If x plus one over x equals three, what is x squared plus one over x squared?
- You do not need to find x, which would involve an ugly root.
- Instead square both sides.
- Squaring x plus one over x gives x squared, plus two times x times one over x, plus one over x squared.
- The middle term is x times one over x, which is one, so it is two.
- So the square is x squared, plus two, plus one over x squared.
- The left side squared is nine.
- So x squared plus one over x squared is nine minus two, which is seven.
- The trap is to answer nine, forgetting the middle term of two, or to guess that squaring each part gives nine.

10 The fastest sound path for powers

Equation or simplify

Write all in one base →
Add, subtract or multiply exponents →
Match exponents, solve

Compare

Common factor of exponents →
Rewrite with that exponent →
Bigger base wins

IN THE LESSON

- Here is the method as a picture.
- For an equation or a simplification, read along the top row.
- First, look at the bases and ask whether they are all powers of one small number: two, three, five.
- Rewrite every term in that base.
- Then use the rules: add exponents on multiplication, subtract on division, multiply for a power of a power.
- Last, match exponents on both sides and solve.
- For a comparison, read the bottom row.
- Find the common factor of the exponents, rewrite each number with that common exponent, and let the bases decide.
- At the end, a quick estimate guards against a slip: the sign and rough size of your answer should make sense.

11 Mistakes to avoid: powers and roots

- Different bases: do not add exponents
- $2^{2^o} + 2^{2^o} = 2^{2^1}$, not 2^{4^o}
- $\sqrt{a} + \sqrt{b}$ is not $\sqrt{(a + b)}$
- Bigger exponent does not mean bigger number

IN THE LESSON

- Let us pause and gather the traps from the first skill.
- The first is adding exponents across different bases, as if two squared times three cubed were six to the fifth.
- It is not, because the bases differ.
- Rewrite first, or just compute.
- The second is turning an addition into an exponent change.
- Two to the twentieth plus two to the twentieth is two times two to the twentieth, which is two to the twenty first, not two to the fortieth.
- The third is adding the numbers under two root signs.
- The fourth is believing that a bigger exponent always means a bigger number.
- Two to the sixty is smaller than five to the thirtieth.
- When in doubt, plug in small numbers to test a rule.

12 A story: money and place value

- Each place is ten times the one on its right
- In 4,725.183: the 2 is in the tens place
- The 1 is in the tenths place
- Know the names: tenths, hundredths, thousandths

IN THE LESSON

- Now decimals.
- Think of money.
- In a price of four dollars and seventy five cents, the seven is worth seventy cents and the five is worth five cents.
- The same digit is worth ten times less each step to the right of the point.
- That is all place value is.
- Every digit is named by its power of ten.
- In four thousand, seven hundred and twenty five, point one eight three, the digit two sits in the tens place, worth twenty.
- The digit one is the first place after the point, the tenths place, worth one tenth.
- Exam questions ask which digit is in which place, and they ask for a number to be rounded to a given place, so the names matter.

13 Place value of 4,725.183

Each digit is named by its power of ten

Digit	Place and worth
4	thousands: 4,000
7	hundreds: 700
2	tens: 20
1	tenths: 0.1

IN THE LESSON

- Here is the number laid out, place by place.
- The four is in the thousands place, so it is worth four thousand.
- The seven is in the hundreds, the two in the tens, the five in the units.
- After the point, the one is in the tenths, the eight is in the hundredths, and the three is in the thousandths.
- Notice the symmetry: tens on the left matches tenths on the right, and hundreds matches hundredths.
- A useful habit is to write a decimal as a power of ten.
- A thousandth is ten to the minus three.
- So zero point zero zero three is three times ten to the minus three.
- Moving the point one place left divides by ten, and one place right multiplies by ten.

14 Multiplying and dividing decimals

Multiply: count places
Divide: shift both points

Count the places	Places in answer = places added
Shift both points	Divide: shift both points alike
Powers of ten	$0.003 = 3 \times 10^{-3}$

IN THE LESSON

- Here are the two methods.
- To multiply decimals, ignore the points and multiply as whole numbers.
- Then count the decimal places in the two numbers you started with, add them up, and put that many places in the answer.
- Zero point three times zero point zero two: three times two is six, and the places are one plus two, which is three, so the answer is zero point zero zero six.
- The same thing in powers of ten is three times two times ten to the minus three.
- To divide by a decimal, shift the point in both numbers the same number of places to the right until the divisor is whole.
- This does not change the value, because you multiplied top and bottom by the same power of ten.

15 Example 5: a chain of decimals

0.06 × 0.004 ÷ 0.012 = ?

Whole numbers	6 × 4	**24**
Count places: 2 + 3	0.06 × 0.004	**0.00024**
Shift both by 3	0.00024 ÷ 0.012	**0.24 ÷ 12**
Divide	0.24 ÷ 12	**0.02**

IN THE LESSON

- Example five, a typical one.
- What is zero point zero six, times zero point zero zero four, divided by zero point zero one two?
- Take the multiplication first.
- Six times four is twenty four.
- The places are two plus three, which is five, so the product is zero point zero zero zero two four.
- Now divide by zero point zero one two.
- The divisor has three decimal places, so shift both points three places to the right.
- The divisor becomes twelve, and the top becomes zero point two four.
- Zero point two four divided by twelve is zero point zero two.
- Check by estimating: the top is about two ten thousandths, the bottom is about one hundredth, and two over a hundred is a few hundredths.
- The size is right.

16 When does a fraction end?

a/b ends if b = $2^p \times 5^q$
places = larger of p, q

Lowest terms	Reduce to lowest terms first
Only twos and fives	Denominator: only 2s and 5s
Places	Places = larger exponent
Otherwise a cycle	Any other prime: it repeats

IN THE LESSON

- Why does one half give zero point five and then stop, while one third goes on forever?
- There is a clean rule.
- First put the fraction in lowest terms, by cancelling common factors.
- Then look at the denominator.
- If its only prime factors are two and five, the decimal terminates.
- That is because ten, the base of our number system, is two times five, so only those primes divide into a power of ten.
- The number of decimal places is the larger of the two exponents.
- For example, forty is two cubed times five, so seven over forty ends after three places.
- If any other prime appears, such as three or seven, the decimal repeats in a cycle instead.

17 Example 6: ends or repeats?

Which fractions terminate, and after how many places?

7/40	$40 = 2^3 \times 5$	**3 places: 0.175**
9/16	$16 = 2^4$	**4 places: 0.5625**
6/35	$35 = 5 \times 7$	**repeats**
12/30	reduces to 2/5	**1 place: 0.4**

IN THE LESSON

- Example six.
- Decide which of these fractions end, and how many places.
- Seven over forty: the denominator is two cubed times five, so it ends, with three places.
- Check: seven divided by forty is zero point one seven five.
- Nine over sixteen: sixteen is two to the fourth, so it ends with four places, and it is zero point five six two five.
- Now the trap one.
- Six over thirty five.
- The denominator is five times seven, so it should repeat.
- But what about twelve over thirty?
- Twelve over thirty reduces to two over five, which ends, with one place.
- So always reduce first, and only then look at the denominator.

18 Repeating decimals and the hundredth digit

- 1/7 = 0.142857 142857 … (cycle of 6 digits)
- Divide the position by the cycle length
- The remainder is the place inside the cycle
- Remainder 0 means the last digit of the cycle

IN THE LESSON

- When a decimal repeats, it repeats in a cycle, and the exam loves to ask about a digit far along, such as the hundredth.
- You do not write a hundred digits.
- You find the cycle length and use a remainder.
- Take one seventh.
- It is zero point one four two eight five seven, and then those six digits repeat forever.
- The cycle length is six.
- To find the hundredth digit, divide one hundred by six.
- Six goes into one hundred sixteen times, with remainder four.
- The remainder tells you the position inside the cycle.
- Remainder four means the fourth digit of the cycle.
- If the remainder is zero, it is the last digit of the cycle.

19 Example 7: the 100th digit

100th digit of 1/7 = 0.142857 142857 …

Cycle	1 4 2 8 5 7	**length 6**
Divide	100 ÷ 6	**16 rem 4**
Fourth digit of cycle	1, 4, 2, 8	**8**
Test: 12th digit	12 ÷ 6 = 2 rem 0	**7**

IN THE LESSON

- Example seven.
- What is the hundredth digit after the decimal point of one seventh?
- The cycle is one, four, two, eight, five, seven, so the length is six.
- A hundred divided by six is sixteen, with remainder four, because six times sixteen is ninety six.
- The remainder is four, so the hundredth digit is the fourth digit of the cycle, which is eight.
- Let us test the method on a smaller case.
- The twelfth digit: twelve divided by six has remainder zero, so it is the last digit of the cycle, seven.
- Count it directly: the digits two cycles in end with a seven.
- Good.
- A second quick one, three elevenths is zero point two seven two seven, a cycle of two, and a hundred is even, so the hundredth digit is seven.

20 Rounding once, from the exact value

Round once

7.245 exact → Next digit is 4 → 7.2 correct

Round twice

7.245 exact → Round early: 7.25 → 7.3 wrong

IN THE LESSON

- Rounding is where careless marks go.
- To round to a given place, look at the single digit just after it.
- If that digit is five or more, round up.
- If it is four or less, leave it.
- The trap is rounding in stages.
- Take seven point two four five to the nearest tenth.
- Read along the top row.
- The digit after the tenths place is four, which is less than five, so the answer is seven point two.
- Now the bottom row.
- If you first round to seven point two five, you create a five that was never there, and rounding again gives seven point three.
- That is wrong.
- The honest check is distance: seven point two four five is nearer seven point two than seven point three.
- Round once, and only from the exact value.

21 Example 8: estimate, then round

0.0745 × 40 ÷ 0.4, to the nearest whole number

Estimate first	0.07 × 40 ÷ 0.4	**about 7**
Multiply exactly	0.0745 × 40	**2.98**
Divide: shift one place	29.8 ÷ 4	**7.45**
Round once	tenths digit is 4	**7**
Trap: round twice	7.45 → 7.5 → 8	**wrong**

IN THE LESSON

- Example eight, a trap one.
- What is zero point zero seven four five, times forty, divided by zero point four, to the nearest whole number?
- Start with an estimate, because the choices will include answers that are ten times too big or too small.
- Zero point zero seven is about seven hundredths, times forty is about two point eight, and dividing by zero point four is about seven.
- Now exactly.
- Seven hundred and forty five times four is two thousand nine hundred and eighty, and with four places, then one more for the forty, the product is two point nine eight.
- Divided by zero point four, shift one place: twenty nine point eight over four is seven point four five.
- Round to the nearest whole number by looking at the tenths digit, which is four.
- The answer is seven, not eight.

22 Mistakes to avoid: decimals

- Places in a product: add them, then sanity check
- Division: shift both points the same amount
- Round once, from the exact value
- Reduce a fraction before testing for 2s and 5s
- Cycle remainder 0 = last digit

IN THE LESSON

- Here are the traps in this section.
- The first is miscounting decimal places when multiplying.
- Add the places of both numbers, and check the size with an estimate.
- A product of two numbers less than one must be smaller than each of them.
- The second is shifting only one point when dividing.
- Shift both by the same amount.
- The third is double rounding.
- The fourth is calling a fraction non terminating without reducing it first, or the opposite, forgetting that a three or a seven in the reduced denominator makes it repeat.
- The fifth is the cycle remainder.
- A remainder of zero means the last digit of the cycle, not the first.
- Slow down on these, they are free marks.

23 A story: sharing a pizza

- A fraction is always a fraction of something
- Ate 1/3, then 1/4 of what was left
- 1/4 of 2/3 is 1/6, not 1/4 of the whole
- Name the whole before you calculate

IN THE LESSON

- Now fractions.
- Suppose you eat a third of a pizza, and your friend eats a quarter of what is left.
- Many people answer that together you ate a third plus a quarter, which is seven twelfths.
- But the second fraction applies to the leftover, not to the whole pizza.
- A quarter of two thirds is a sixth.
- So together you ate a third plus a sixth, which is a half.
- This is the single most important idea in fraction word problems.
- A fraction always means a fraction of something, and you must ask what.
- Before any arithmetic, name the whole for every fraction.
- The arithmetic itself is three short rules, and we will see them first.

24 Adding and dividing by hand

Add

`3/4 + 5/6` → `9/12 + 10/12` → `19/12`

Divide

`3/5 ÷ 9/10` → `3/5 × 10/9` → `2/3`

IN THE LESSON

- Here are the operations as a picture.
- In the top row, adding: three quarters plus five sixths.
- The bottoms differ, so find a common bottom, twelve.
- Three quarters is nine twelfths and five sixths is ten twelfths.
- Add only the tops: nineteen twelfths.
- Never add the bottoms.
- In the middle row, dividing: three fifths divided by nine tenths.
- Dividing by a fraction means multiplying by its flip, the reciprocal.
- So three fifths times ten ninths.
- Cancel where you can, and you get two thirds.
- Multiplying is the easiest of all: straight across, tops times tops and bottoms times bottoms.
- Nested fractions we will meet in a moment.

25 Comparing fractions: cross multiply

a/b > c/d when a × d > b × c

Top left times bottom right	a × d, from the first top
Bottom left times top right	b × c, from the second top
The verdict	Bigger product: bigger fraction

IN THE LESSON

- To compare two positive fractions without a calculator, cross multiply.
- Take a over b and c over d.
- Multiply the top of the first by the bottom of the second, a times d.
- Then multiply the bottom of the first by the top of the second, b times c.
- Whichever product is bigger sits with the bigger fraction.
- If a times d is bigger than b times c, then a over b is bigger than c over d.
- Why does it work?
- You are rewriting both fractions over the common bottom b times d, and then comparing the tops.
- The method is safe because the denominators are positive.
- It saves you from dividing out long decimals, and the numbers stay small.

26 Example 9: which is larger?

Which is larger: 11/18 or 16/27 ?

First product	11 × 27	**297**
Second product	16 × 18	**288**
Compare	297 > 288	**11/18 wins**
Check as decimals	0.611 vs 0.593	**agrees**

IN THE LESSON

- Example nine.
- Which is larger: eleven eighteenths, or sixteen twenty sevenths?
- Decimals would be long.
- Cross multiply instead.
- Eleven times twenty seven is two hundred and ninety seven.
- Sixteen times eighteen is two hundred and eighty eight.
- The first product is bigger, and it came from the top of the first fraction, so eleven eighteenths is the larger.
- As a check, eleven eighteenths is about zero point six one, and sixteen twenty sevenths is about zero point five nine.
- The trap is to compare the tops and bottoms separately, and say sixteen is more than eleven so the second is bigger.
- The difference between the two fractions is tiny, about two hundredths, which is why a quick eyeball is risky.

27 Example 10: a fraction of what is left

Mia has $240. Rent 1/4, then 2/5 of the remainder on food.

After rent	3/4 × 240	**$180**
Food	2/5 × 180	**$72**
Left	180 − 72	**$108**
Shortcut: fraction left	3/4 × 3/5	**9/20**
Shortcut: amount	9/20 × 240	**$108**

IN THE LESSON

- Example ten, the word problem.
- Mia has two hundred and forty dollars.
- Mia spends a quarter of it on rent.
- Then Mia spends two fifths of what remains on food.
- How much is left?
- The second fraction applies to the remainder, not to the original.
- After rent, Mia has three quarters of two hundred and forty, which is one hundred and eighty.
- Two fifths of one hundred and eighty is seventy two.
- So Mia has one hundred and eighty minus seventy two, which is one hundred and eight dollars.
- A quicker way is to multiply the fractions that remain: three quarters times three fifths is nine twentieths, and nine twentieths of two hundred and forty is also one hundred and eight.

28 The whole shrinks at every step

Money left ($)

Start	**240**
After rent	**180**
After food	**108**

Each fraction applies to the bar before it

IN THE LESSON

- Here is the same problem as bars, because a picture shows why the fraction of what question matters.
- The first bar is what Mia started with, two hundred and forty dollars.
- After rent, a quarter is gone and the bar drops to one hundred and eighty.
- The second fraction, two fifths, is a fraction of that shorter bar, not of the tall one.
- Two fifths of one hundred and eighty is seventy two, so the last bar is one hundred and eight.
- Had you taken two fifths of the original, you would have removed ninety six dollars, and your answer would be too small by twenty four.
- The wrong choice is usually on the exam, because that is the mistake people make.

29 Example 11: a nested fraction

1 ÷ (1 + 1 ÷ (1 + 1/2)) = ?

Innermost	1 + 1/2	**3/2**
Next layer	1 ÷ 3/2 = 2/3	**2/3**
Add one	1 + 2/3	**5/3**
Outside	1 ÷ 5/3 = 3/5	**3/5**

IN THE LESSON

- Example eleven, a nested expression.
- What is one divided by one plus one divided by one plus one half?
- Work from the inside out, one layer at a time.
- The innermost piece is one plus one half, which is three halves.
- Next, one divided by three halves is two thirds, by multiplying by the flip.
- Then one plus two thirds is five thirds.
- Finally, one divided by five thirds is three fifths.
- The trap is to rush from the outside, or to add the tops and bottoms across.
- Write each layer on its own line, and replace the inner piece with its value before moving out.
- If you do this, a scary looking expression takes four short steps.

30 Mistakes to avoid: fractions

- Do not add tops and bottoms: 1/2 + 1/3 ≠ 2/5
- Later fractions apply to what remains
- A fraction of a group is not a fraction of the whole
- Flip only the fraction you divide by

IN THE LESSON

- Here are the fraction traps.
- The first is adding tops and bottoms separately: a half plus a third is not two fifths.
- Find the common bottom.
- The second is applying a later fraction to the original whole, instead of to what remains.
- The third is confusing a fraction of a group with a fraction of the whole.
- If two thirds of the members are adults, and one quarter of the adults are teachers, the teachers are one sixth of all members, not one quarter of everyone.
- The fourth is flipping the wrong fraction when dividing.
- Flip only the one you divide by.
- Before you answer, ask whether the size makes sense.
- Adding two positive fractions always makes something bigger than each.

31 Recap: three skills

Three skills, one habit: rewrite before you compute

Powers	Common base, then the rules
Roots	Pull squares out, like roots only
Decimals	Count places, round once
Fractions	Common bottom, cross multiply
Of what	Ask: a fraction of what?

IN THE LESSON

- Let us pull it together.
- For powers, rewrite everything in one base, then add exponents for a product, subtract for a quotient, and multiply for a power of a power.
- Pull squares out of roots, and add only like roots.
- For comparing big powers, give them a common exponent and let the bases decide.
- For decimals, name the place of each digit, count places when multiplying, and shift both points when dividing.
- A reduced fraction ends only if its denominator has just twos and fives.
- Repeating decimals use a remainder, and you round once.
- For fractions, use a common bottom to add, flip to divide, cross multiply to compare, and always ask: a fraction of what?

32 Try three quick questions

1. $3^5 \times 9^2$ as a power of 3?
   Answer: $9^2 = 3^4$, so $3^5 \times 3^4 = 3^9$.
2. Tenths digit of 7/40?
   Answer: 7/40 = 0.175, so the tenths digit is 1.
3. Larger: 5/7 or 7/10?
   Answer: 5×10 = 50 > 7×7 = 49, so 5/7.

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- What is three to the fifth, times nine squared, written as a power of three?
- Nine squared is three to the fourth, and five plus four is nine, so the answer is three to the ninth.
- Question two.
- What is the digit in the tenths place of seven over forty?
- Seven over forty is zero point one seven five, and the tenths digit is one.
- Question three.
- Which is larger, five sevenths or seven tenths?
- Five times ten is fifty, seven times seven is forty nine, so five sevenths is larger.
- Well done.