Charta | LESSON NOTES

Percent and ratio shortcuts: multiply, do not subtract

Quantitative Reasoning

1 Four shortcuts

Percent changes are multiplications

1. Percent change	up 20%: x 1.2; down 25%: x 0.75	**multiplier**
2. Unknown start	pick 100, or the ratio's numbers	**pick**
3. Changing ratio	change each part, then compare	**parts**
4. Mixtures	balance distances to the target	**balance**

IN THE LESSON

- Here are the four shortcuts we will use.
- First, turn every percent change into a multiplier.
- A rise of twenty percent multiplies by one point two, and a fall of twenty five percent multiplies by zero point seven five.
- Second, when a quantity is not given, pick one hundred for it, or the numbers of the ratio, and let the percentages act on that.
- The answer will hold for any starting value.
- Third, for a ratio that changes, apply the change to each part separately, then compare the new parts.
- Fourth, for a mixture or a weighted average, balance the distances: the amount of each ingredient is in inverse proportion to how far its strength is from the target.
- All four replace long algebra with a few short lines.

2 Multipliers you should know

Percent change to multiplier, and back

Change	Multiplier
Rise of 20%	x 1.2
Fall of 20%	x 0.8
Fall of 25%	x 0.75
Rise 20%, then fall 20%	1.2 x 0.8 = 0.96

IN THE LESSON

- Learn to switch between percent changes and multipliers instantly.
- A rise of ten percent is a multiplier of one point one.
- A rise of twenty percent is one point two.
- A rise of fifty percent is one point five.
- A fall of ten percent is zero point nine, a fall of twenty percent is zero point eight, a fall of twenty five percent is zero point seven five.
- Successive changes multiply.
- A rise of twenty percent followed by a fall of twenty percent is one point two times zero point eight, which is zero point nine six, a net fall of four percent, not zero.
- And to undo a change, divide by its multiplier, or multiply by its reciprocal.
- To undo a rise of twenty percent, divide by one point two.

Charta Learning · charta-learning.com

© Charta Learning · charta-learning.com · For your own study. Please do not share or redistribute.

3 Percent points and percent change

- Percentage points: simply subtract
- Percent change: divide by the starting value
- The base is the thing the question compares with
- Always ask: percent of what?

IN THE LESSON

- The most common trap in these questions is confusing a percentage point with a percent.
- If a pass rate rises from forty percent to fifty five percent, it has risen by fifteen percentage points.
- But as a percent change, it has risen by fifteen over forty, which is thirty seven and a half percent.
- Both are correct statements about the same facts, and a question will ask for one of them.
- The base of a percent change is always the starting value, the one the question compares against.
- If the question asks about a fall in the fail rate, the starting value is the old fail rate, not the pass rate and not the new rate.
- Read the words after the word of carefully: percent of what?

4 Ratios that change

- A ratio 3 : 4 means 3k and 4k
- Apply each change to its own part
- The k cancels in the new ratio
- Simplify; keep the order the question uses

IN THE LESSON

- A ratio does not tell you how many there are, only how they compare.
- Three to four means three parts and four parts, so you may take three k and four k, where k is any positive number.
- When a question changes one or both quantities, apply the change to each part.
- If the first part rises by twenty percent, multiply its three k by one point two.
- If the second part falls by twenty five percent, multiply its four k by zero point seven five.
- Then the new ratio is the new parts compared, and the k cancels.
- You never need to know how many there are.
- Finish by simplifying the ratio, and make sure you write it in the order the question asks for, boys to girls and not girls to boys.

5 Mixtures: balance the distances

- Target strength lies between the two strengths
- Volume x distance balances on both sides
- Far from the target: use a little
- Check by counting the pure ingredient

IN THE LESSON

- For mixtures, think of a seesaw.
- Two solutions of different strengths are mixed to make a target strength between them.
- The target sits some distance from each strength.
- The mixture balances when the volume of each solution times its distance from the target is equal on both sides.
- So the amounts of the two solutions are in inverse proportion to the distances.
- A solution that is far from the target is needed in a small amount, and one that is close to the target is needed in a large amount.
- This is called alligation, and it saves setting up an equation.
- You can always check by counting the amount of the pure ingredient in each solution and in the mix.

Charta Learning · charta-learning.com

© Charta Learning · charta-learning.com · For your own study. Please do not share or redistribute.

6 Question 1: a fall in the fail rate

- In a school exam the pass rate rose from 40% to 55%. By what percent did the fail rate fall?
- A. 75%
- B. 37.5%
- C. 33 1/3%
- D. 25%
- E. 15%

IN THE LESSON

- Question one is an easy one.
- Read it with me, and try it yourself first.
- Notice that the numbers given are pass rates, but the question asks about something else, and you will need to find it.
- In a school exam the pass rate rose from forty percent to fifty five percent.
- By what percent did the fail rate fall?
- Choice A.
- seventy five percent.
- Choice B.
- thirty seven and a half percent.
- Choice C.
- thirty three and a third percent.
- Choice D.
- twenty five percent.
- Choice E.
- fifteen percent.
- Careful: the question asks about the fail rate, and not the pass rate.

7 Question 1: the fail rate, then its change

Pass rate 40% to 55%: the fail rate is what falls

Old fail rate	100% - 40%	**60%**
New fail rate	100% - 55%	**45%**
Fall	60 - 45	**15 points**
Percent fall, base 60	15 / 60	**25%**

IN THE LESSON

- First, change the point of view.
- The question is about the fail rate, so work out the fail rates.
- Forty percent passed, so sixty percent failed.
- Then fifty five percent passed, so forty five percent failed.
- The fail rate fell from sixty percent to forty five percent.
- That is a fall of fifteen percentage points.
- Now the percent change.
- The base is the old fail rate, sixty.
- So divide the fall, fifteen, by sixty.
- Fifteen over sixty is one quarter, which is twenty five percent.
- The fail rate fell by twenty five percent.
- A good habit is to check with a different number: imagine one hundred students.
- Sixty failed before, and forty five now.
- Fifteen fewer students failed, out of sixty who had failed.
- A quarter.

Charta Learning · charta-learning.com

© Charta Learning · charta-learning.com · For your own study. Please do not share or redistribute.

8 Question 1: the traps in the choices

Which rate? Which base?

A: 75%	45 as a percent of 60	**new over old**
B: 37.5%	the rise in the pass rate	**wrong rate**
C: 33 1/3%	15 over the new rate 45	**wrong base**
D: 25%	15 over the old rate 60	**the answer**
E: 15%	the 15 percentage points	**points not percent**

IN THE LESSON

- Look at the traps, because each is a different misreading.
- Choice A gives the new fail rate as a percent of the old one: forty five over sixty, which is seventy five percent.
- That is what is left, not what fell.
- Choice B is the percent rise in the pass rate, fifteen over forty.
- It answers a question about the wrong rate.
- Choice C divides the fall by the new fail rate, forty five, instead of the old one.
- Choice E reports the fifteen percentage points as a percent.
- The answer is D, twenty five percent.
- Every wrong choice can be reached from the same four numbers, forty, fifty five, sixty and forty five, which is why you must say, aloud or on paper, which rate and which base.

9 Question 2: a mixture

- How many litres of a 30% alcohol solution must be mixed with 12 litres of a 10% alcohol solution to produce a 15% alcohol solution?
- A. 2
- B. 4
- C. 12
- D. 16
- E. 36

IN THE LESSON

- Question two is an easy one, and a nice use of balancing.
- Read it with me.
- Notice that the two solutions are on either side of the strength we want, and that we know the volume of only one of them.
- How many litres of a thirty percent alcohol solution must be mixed with twelve litres of a ten percent alcohol solution to produce a fifteen percent alcohol solution?
- Choice A.
- two.
- Choice B.
- four.
- Choice C.
- twelve.
- Choice D.
- sixteen.
- Choice E.
- thirty six.
- The target is fifteen percent: between ten and thirty.

Charta Learning · charta-learning.com

© Charta Learning · charta-learning.com · For your own study. Please do not share or redistribute.

10 Question 2: balance the distances

Weak 10% (12 litres), strong 30% (x litres), target 15%

Weak is below the target by	15 - 10	**5**
Strong is above the target by	30 - 15	**15**
Balance	12 x 5 = x x 15	**60 = 15x**
Solve	x = 60 / 15	**4 litres**

IN THE LESSON

- Draw a seesaw with the strengths on a line.
- The weak solution is at ten percent, the strong one at thirty percent, and the target is at fifteen percent.
- The target is five points above the weak solution and fifteen points below the strong one.
- Balance the volumes with the distances: the weak solution, twelve litres, times its distance, five, must equal the strong solution, x litres, times its distance, fifteen.
- So twelve times five, which is sixty, equals fifteen times x.
- X is sixty over fifteen, which is four.
- Notice it makes sense: the target is close to the weak strength, so most of the mix is weak, and we need only a little strong solution.
- Four litres of strong, against twelve of weak, is one to three.

11 Question 2: check by counting alcohol

Alcohol in: 1.2 + 1.2 litres, in 16 litres

Alcohol in 12 litres of 10%	0.10 x 12	**1.2 litres**
Alcohol in 4 litres of 30%	0.30 x 4	**1.2 litres**
Total alcohol, total volume	2.4 litres in 16 litres	**15%**
Trap: equal volumes	12 and 12 litres give 20%	**choice C**

IN THE LESSON

- Always check a mixture by counting the pure alcohol.
- In the twelve litres of ten percent solution there are one point two litres of alcohol.
- In the four litres of thirty percent solution there are one point two litres as well: thirty percent of four.
- The total alcohol is two point four litres.
- The total volume is twelve plus four, which is sixteen litres.
- Two point four out of sixteen is fifteen hundredths, which is fifteen percent.
- It works.
- Now see where the traps come from.
- If you take twelve litres of each, the mix is twenty percent, which is choice C, not fifteen.
- If you forget that the added litres increase the volume, you get two litres, which is choice A.
- And swapping the distances gives thirty six, choice E.

Charta Learning · charta-learning.com

© Charta Learning · charta-learning.com · For your own study. Please do not share or redistribute.

12 Question 2: the traps in the choices

Balance, then check by counting alcohol

A: 2	leaves out the added litres	**wrong total**
B: 4	12 x 5 = 15 x 4	**the answer**
C: 12	equal volumes: gives 20%	**equal volumes**
D: 16	the total volume	**wrong quantity**
E: 36	swaps the two distances	**swapped weights**

IN THE LESSON

- Now the traps.
- Choice A leaves the added litres out of the final volume, which would make one point two plus point three x equal to point one five times twelve, and gives two.
- The added litres are part of the mixture, so they must be counted in the volume.
- Choice C assumes equal volumes.
- Twelve litres of each would give twenty percent, which is too strong.
- Choice D is the total volume, sixteen litres, not the amount of strong solution.
- Choice E swaps the weights, matching twelve litres to the wrong distance.
- The answer is B, four litres.
- The seesaw gave the answer in two lines, and the alcohol count confirmed it.

13 Question 3: a changing ratio

- At a school, the ratio of boys to girls is 3:4. The number of boys then rises by 20% and the number of girls falls by 25%. What is the new ratio of boys to girls?
- A. 3:4
- B. 5:6
- C. 6:5
- D. 75:64
- E. 18:5

IN THE LESSON

- Question three is a medium one.
- Read it with me.
- We do not know how many boys and girls there are, and we do not need to.
- At a school, the ratio of boys to girls is three to four.
- The number of boys then rises by twenty percent and the number of girls falls by twenty five percent.
- What is the new ratio of boys to girls?
- Choice A.
- three to four.
- Choice B.
- five to six.
- Choice C.
- six to five.
- Choice D.
- seventy five to sixty four.
- Choice E.
- eighteen to five.
- Take three k boys and four k girls.

Charta Learning · charta-learning.com

© Charta Learning · charta-learning.com · For your own study. Please do not share or redistribute.

14 Question 3: change each part

3k boys and 4k girls, then x 1.2 and x 0.75

Boys after +20%	3k x 1.2	**3.6k**
Girls after -25%	4k x 0.75	**3k**
New ratio	3.6k : 3k	**3.6 : 3**
Simplify	36 : 30, then divide by 6	**6 : 5**

IN THE LESSON

- The ratio is three to four, so take three k boys and four k girls, where k stands for the size of one part.
- The boys rise by twenty percent, so multiply by one point two: three k times one point two is three point six k.
- The girls fall by twenty five percent, so multiply by zero point seven five.
- Four k times three quarters is three k.
- The new ratio of boys to girls is three point six k to three k.
- The k cancels, leaving three point six to three.
- Multiply both sides by ten, to clear the decimal: thirty six to thirty.
- Divide by six: six to five.
- The new ratio is six to five.
- Notice that the answer is the other way round from the start.
- Boys now outnumber girls, which fits the changes.

15 Question 3: the traps in the choices

Keep the order: boys to girls

A: 3:4	assumes the ratio is unchanged	**no change**
B: 5:6	girls to boys, inverted	**wrong order**
C: 6:5	3.6k : 3k	**the answer**
D: 75:64	swaps the two percentages	**swapped changes**
E: 18:5	cuts girls to 25%, not by 25%	**fall misread**

IN THE LESSON

- Look at the traps.
- Choice A assumes that a rise of twenty percent and a fall of twenty five percent leave the ratio unchanged.
- They do not, because they act on different quantities.
- Choice B inverts the ratio, writing girls to boys.
- Check the order the question uses, boys to girls.
- Choice D swaps the percentages, raising the boys by twenty five and cutting the girls by twenty.
- Choice E cuts the girls to twenty five percent of their number, instead of reducing them by twenty five percent.
- That makes girls one k, and boys three point six k, which is eighteen to five.
- The answer is C, six to five.
- When ratio and percent meet, work on the parts.

Charta Learning · charta-learning.com

© Charta Learning · charta-learning.com · For your own study. Please do not share or redistribute.

16 Question 4: price and volume

- A shop raises the price of an item by 20% and its sales volume then falls by 20%. By what percent must the volume rise from its new level for the shop's revenue to return to its original level?
- A. 4%
- B. 4 1/6%
- C. 16 2/3%
- D. 20%
- E. 25%

IN THE LESSON

- Question four is the hard one.
- Read it with me.
- Revenue is price times volume.
- A shop raises the price of an item by twenty percent, and its sales volume then falls by twenty percent.
- By what percent must the volume rise from its new level for the shop's revenue to return to its original level?
- Choice A.
- four percent.
- Choice B.
- four and a sixth percent.
- Choice C.
- sixteen and two thirds percent.
- Choice D.
- twenty percent.
- Choice E.
- twenty five percent.
- The question asks for the rise needed from the new level.

17 Question 4: revenue as a product

Revenue = price x volume; multipliers multiply

Price: up 20%	multiplier	**1.2**
Volume: down 20%	multiplier	**0.8**
Revenue now	1.2 x 0.8	**0.96**
Needed on the volume	1 / 0.96 = 25/24	**1 + 1/24**

IN THE LESSON

- Revenue is price times volume, so the changes multiply.
- The price rises by twenty percent: multiplier one point two.
- The volume falls by twenty percent: multiplier zero point eight.
- Revenue is now one point two times zero point eight of what it was, which is zero point nine six.
- Revenue has fallen by four percent.
- Now we want to bring revenue back to one, and only the volume can move.
- The price multiplier stays at one point two.
- So the volume must be multiplied by whatever turns zero point nine six back into one.
- That is one divided by zero point nine six, which is twenty five twenty fourths.
- Twenty five twenty fourths is one plus one twenty fourth.
- So the volume must rise by one twenty fourth of its new level.

Charta Learning · charta-learning.com

© Charta Learning · charta-learning.com · For your own study. Please do not share or redistribute.

18 Question 4: turn it into a percent

1/24 is 4 and 1/6 percent

Rise needed	1/24 x 100	**4 1/6 %**
Start: price 100, volume 100	revenue	**10,000**
After: price 120, volume 80	revenue	**9,600**
Volume 80 x 25/24 = 83.3	120 x 83.3	**10,000**

IN THE LESSON

- One twenty fourth, as a percent, is one hundred over twenty four, which is four and a sixth.
- So the volume must rise by four and one sixth percent.
- Check with a hundred.
- Start with price one hundred, and volume one hundred, so revenue is ten thousand.
- After the changes, price is one hundred twenty and volume eighty, so revenue is nine thousand six hundred.
- Multiply the volume of eighty by twenty five twenty fourths: eighty times twenty five is two thousand, over twenty four, is about eighty three point three.
- Revenue is one hundred twenty times eighty three point three, which is ten thousand.
- It works.
- The traps: four percent is the fall in revenue, but a rise applies to a smaller base.
- Twenty five percent restores the volume, but the price is still high.

19 Question 4: the traps in the choices

To undo a multiplier, divide

A: 4%	the fall in revenue, wrong base	**wrong base**
B: 4 1/6%	1/0.96 - 1 = 1/24	**the answer**
C: 16 2/3%	20 as a part of 120	**wrong base**
D: 20%	reuses the stem figure	**echo**
E: 25%	restores volume only	**overshoots**

IN THE LESSON

- Now the traps.
- Choice A notices that revenue fell by four percent and uses four percent as the rise needed.
- But a percent rise applies to the new, smaller revenue, so it needs a little more than four.
- Choice C takes twenty out of the new price of one hundred twenty, which is the wrong base for a change in volume.
- Choice D reuses the twenty percent from the stem.
- Choice E restores the volume to its original level, which overshoots, because the price is still twenty percent higher.
- The answer is B, four and a sixth percent.
- The key insight is that percent changes are multipliers, and to undo a multiplier you divide, not add or subtract percentages.
- This is the single most useful idea in this lesson.

Charta Learning · charta-learning.com

© Charta Learning · charta-learning.com · For your own study. Please do not share or redistribute.

20 Accurate and quick

- Percent becomes a multiplier
- No start given? Pick 100
- Say which value is the starting value
- Mixtures: seesaw, then count the pure part
- Does your answer match a wrong-base trap?

IN THE LESSON

- Now habits that make percent and ratio questions fast.
- First, translate every percent into a multiplier, and write it beside the quantity.
- Second, if there is no starting number, pick one hundred, or the numbers of the ratio.
- Third, before you divide for a percent change, say which value is the starting value.
- Fourth, write rates separately, as we did with pass and fail.
- Fifth, for mixtures, draw the seesaw, and check by counting the pure ingredient.
- Sixth, compare your answer with the traps in the choices.
- If your answer matches a number you got by a wrong base, stop and recheck.
- Seventh, remember that the first sensible number you get can be a trap.
- And spend about two minutes a question.
- Percent questions look wordy, but the calculations are short.

21 Quick check

1. Up 10%, then up 10% again: total rise?
   Answer: 21 percent: 1.1 x 1.1 = 1.21.
2. A rate from 20% to 30%: percent change?
   Answer: 50 percent: 10 points over the start, 20.
3. Equal volumes of 10% and 30%?
   Answer: 20 percent: the midpoint.

IN THE LESSON

- Now a quick check.
- Pause after each question and answer it yourself.
- Question one.
- A price rises by ten percent, and then by ten percent again.
- What is the total rise?
- The answer: twenty one percent, because one point one times one point one is one point two one.
- Question two.
- A rate goes from twenty percent to thirty percent.
- What is the percent change?
- The answer: fifty percent, because the rise of ten points is divided by the starting value, twenty.
- Question three.
- You mix equal volumes of a ten percent solution and a thirty percent solution.
- What is the strength?
- The answer: twenty percent, the middle, because the volumes are equal.

Charta Learning · charta-learning.com

© Charta Learning · charta-learning.com · For your own study. Please do not share or redistribute.

22 Recap: percent and ratio shortcuts

- Percent change is a multiplier
- Successive changes multiply; undo by dividing
- Percent change: divide by the starting value
- Ratios: change each part, k cancels
- Mixtures: balance the distances, then check

IN THE LESSON

- Let us recap.
- Turn every percent change into a multiplier: a rise of twenty percent is one point two, and a fall of twenty five percent is zero point seven five.
- Successive changes multiply, and to undo a change you divide by its multiplier.
- A percent change is always the change divided by the starting value, so say which value that is.
- Do not confuse percentage points with percent.
- When there is no starting value, pick one hundred, or use the parts of the ratio.
- For a changing ratio, apply each change to its own part, and the common factor cancels.
- For a mixture, balance the volumes and the distances from the target strength, and check by counting the pure ingredient.
- These shortcuts turn long questions into short ones.

Summary notes for the Charta whiteboard lesson. Figures are from the lesson's made-up examples.

Charta Learning · charta-learning.com

© Charta Learning · charta-learning.com · For your own study. Please do not share or redistribute.