CHARTA LEARNING · LESSON NOTES

Pricing forwards and futures: arbitrage and the cost of carry

Derivatives

1 A story: the promise to sell a bike

- A friend promises to sell you a bike in a year
- He does not guess next year's bike price
- He asks: what does waiting cost me?
- Price = what it costs to carry the bike

IN THE LESSON

- Imagine your neighbour has a bike worth one hundred dollars, and he agrees to sell it to you in exactly one year, at a price you both fix today.
- You pay nothing now.
- Why would he pick one price rather than another?
- Not because he guesses what bikes will cost next year.
- He asks a simpler question.
- If I simply kept the bike and waited, what would waiting cost me?
- Money tied up in a bike could instead earn interest in a bank.
- So the price he asks next year has to make up for that lost interest.
- Pricing by the cost of waiting, and not by prediction, is the heart of this whole lesson.

2 What a forward contract is

- Forward: buy or sell later at a price fixed today
- Underlying: the thing traded. Spot: price today
- Long agrees to buy. Short agrees to sell.
- Nothing is paid at the start. Desk: Kestrel Capital

IN THE LESSON

- Let us name the pieces.
- A forward contract is an agreement made today to buy or sell something at a fixed price on a fixed date in the future.
- The thing being traded is the underlying.
- The price agreed today is the forward price.
- The price for immediate delivery is the spot price.
- The buyer, who agrees to buy later, is the long.
- The seller is the short.
- Nothing is paid at the start.
- We will follow one trading desk, Kestrel Capital, and every price and rate is a made-up round number for teaching.
- The first question is always the same: what price is fair?

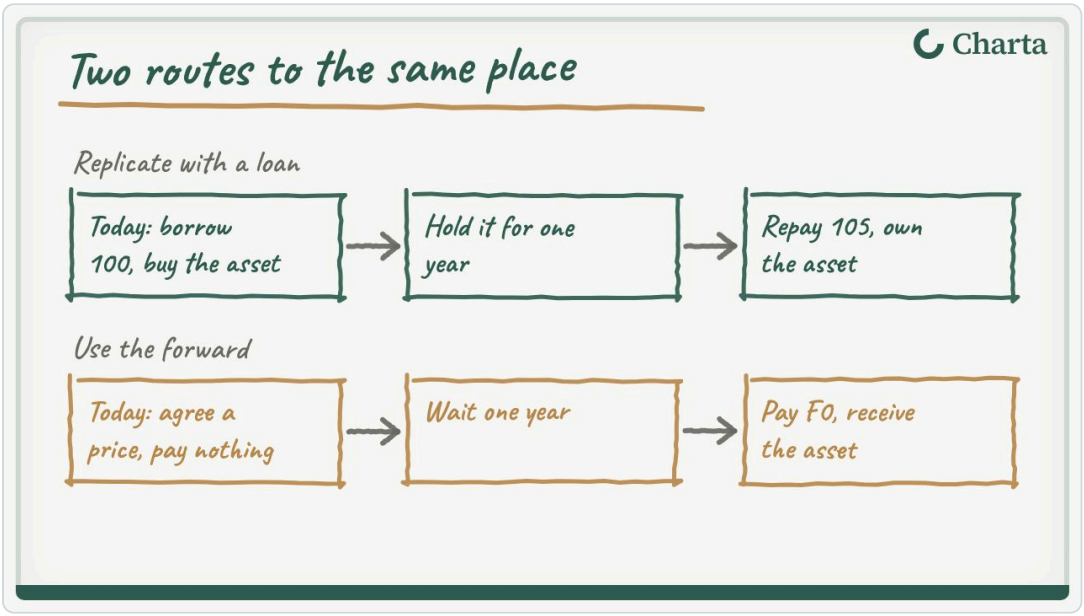
3 No arbitrage: the rule behind the price

- Arbitrage: a sure profit with no risk, no own money
- Real markets close such gaps fast, so assume none
- Replicate: borrow, buy the asset, hold to the date
- Forward price must equal that cost to replicate

IN THE LESSON

- Here is the rule.
- An arbitrage is a way of making a certain profit, with no risk and no money of your own put in.
- Traders spot such chances and close them quickly.
- We therefore assume there is no arbitrage.
- That one assumption fixes the forward price.
- Suppose you want the same result as a forward, which is to own the asset at a future date.
- You could instead borrow money today, buy the asset now, and hold it.
- That is replicating the forward.
- If the forward cost less than the replication, everyone would buy it, and if it cost more, everyone would sell it.
- So the forward price must equal the cost of replicating.

4 Two routes to the same place



IN THE LESSON

- Here are the two routes drawn out.
- The top row is the replication.
- Today, Kestrel borrows one hundred dollars at five per cent and uses it to buy the asset.
- It holds the asset for a year.
- At the end, it repays the loan with interest, which is one hundred and five dollars, and it owns the asset.
- The bottom row is the forward.
- Today, nothing happens, and the price is agreed.
- At the end, Kestrel pays that price and receives the asset.
- Both routes finish with Kestrel owning the asset in a year.
- The first route cost one hundred and five dollars in all, so the second must cost the same.

5 The simplest forward price

Forward price F0 = S0 × (1 + r)^T

Spot price, S0	Price for delivery today
Risk-free rate, r	Safe rate for one year
Time, T years	Years until delivery
Forward price, F0	Fair price agreed today

IN THE LESSON

- Now the formula.
- With no income and no storage cost, the forward price is the spot price compounded at the risk-free rate for the life of the contract.
- The risk-free rate is what a completely safe borrower pays or earns, and it is the rate Kestrel uses for the loan.
- In symbols, F zero equals S zero times one plus r, all raised to the power T.
- S zero is today's spot price.
- The letter r is the rate for one year, and T is the time to delivery in years.
- Notice that the formula compounds upward, from today to the future.
- It uses no forecast of the future price, only the cost of carrying the asset.
- At Level One we use discrete compounding, not continuous.

6 Example 1: the fair forward price

Spot 100, risk-free rate 5% a year, no income, no storage

Spot price	today	**100**
One-year forward	100 × 1.05	**105**
Two-year forward	100 × 1.05²	**110.25**

IN THE LESSON

- Example one.
- Kestrel quotes a one-year forward on an asset that pays nothing and costs nothing to hold.
- The spot price is one hundred dollars and the risk-free rate is five per cent a year.
- The forward price is one hundred times one point zero five, which is one hundred and five dollars.
- For a two-year forward, we compound twice.
- One hundred times one point zero five squared is one hundred and ten point two five dollars.
- A longer contract has a higher forward price, because there is more waiting to pay for.
- Notice that nobody said anything about whether the asset will go up or down.
- That does not matter, and it is the most common thing that trips people up.

7 Example 2: the forward is quoted too high

Quoted forward 108, but fair is 105

Today: sell forward at 108	borrow 100, buy asset	**0 cash**
Year end: deliver the asset	receive forward price	**+108**
Year end: repay the loan	100 × 1.05	**− 105**
Riskless profit	108 − 105	**$3**

IN THE LESSON

- Example two, an arbitrage.
- Suppose another dealer quotes the one-year forward at one hundred and eight dollars, above the fair one hundred and five.
- Kestrel takes this free money.
- Today, it sells the forward at one hundred and eight, borrows one hundred dollars and buys the asset, with none of its own money.
- In a year, it delivers the asset and receives one hundred and eight.
- It repays the loan of one hundred and five.
- What is left is three dollars, a certain profit, whatever the asset did in between.
- This is called a cash and carry trade, and the rush of such sellers pushes the forward price back down.

8 Example 3: the forward is quoted too low

Quoted forward 102, but fair is 105

Today: buy forward at 102	short asset, lend 100	**0 cash**
Year end: loan repaid	100 × 1.05	**+105**
Year end: pay under the forward	buy asset to return	**− 102**
Riskless profit	105 − 102	**$3**

IN THE LESSON

- Example three, the other direction.
- Now another dealer quotes the one-year forward at one hundred and two, below the fair one hundred and five.
- Kestrel does the reverse.
- It buys the forward at one hundred and two.
- It sells the asset short, which means borrowing one and selling it, and lends the one hundred dollars it receives at five per cent.
- In a year, the lending returns one hundred and five.
- Kestrel pays one hundred and two under the forward and hands the asset back to its lender.
- It keeps three dollars, again with no risk.
- Buyers of the cheap forward push its price back up.
- This is called a reverse cash and carry.

9 A story: owning costs money, and pays you back

- Owning a cottage costs upkeep and insurance
- It also gives rent or enjoyment
- Friend buys later: you charge the costs you paid
- And you deduct the benefits he did not get

IN THE LESSON

- Think about owning a holiday cottage for a year.
- You pay for upkeep and insurance.
- Those are costs of holding it.
- But you might also rent it out for some weeks, or enjoy living in it yourself.
- Those are benefits of holding it.
- Now suppose a friend wants the cottage in a year, and wants a price fixed today.
- You would add to the price the costs you will have paid, and you would deduct the benefits you will have enjoyed, because your friend will not have had them.
- Assets work just the same way.
- Together, the costs and benefits of holding are called the cost of carry, and they shift the forward price.

10 Costs raise the price, benefits lower it

- Raise the price: interest, storage, insurance
- Lower the price: dividends and coupons
- Lower the price: convenience yield (physical goods)
- Net carry decides if forward is above or below spot

IN THE LESSON

- Let us sort the carrying items into two groups.
- The first group are costs, which raise the forward price.
- Interest is the first, since money tied up in the asset could have been earning.
- Storage and insurance for physical goods like gold or oil also raise it.
- The second group are benefits, which lower the forward price.
- Dividends on a share, and coupons on a bond, are cash paid to whoever holds the asset today.
- The forward buyer does not receive them, so the price comes down.
- The last benefit is convenience yield, the advantage of having the physical thing on hand, for example a factory that needs oil next week.
- Convenience yield applies to physical commodities.

11 The forward price with costs and benefits

F0 = (S0 − PV benefits + PV costs) × (1 + r)^T

Spot price, S0	Price today
Benefits, PV	Dividends, coupons, convenience
Costs, PV	Storage, insurance
Risk-free rate, r	Safe rate for one year

IN THE LESSON

- Here is the full formula.
- Take today's spot price.
- Subtract the present value of the benefits, which are the dividends or coupons, expressed in today's money.
- Add the present value of the costs, such as storage.
- Then compound the result at the risk-free rate for the life of the contract.
- Written out, F zero equals S zero minus gamma plus theta, all multiplied by one plus r to the power T.
- Gamma is the present value of the benefits, and theta is the present value of the costs.
- The present value of a payment is that payment divided by one plus r, raised to the power of the years until it is made.
- This still contains no forecast of the future spot price.

12 Example 4: a share that pays a dividend

Spot 100, dividend 2 at delivery, rate 5%, one year

PV of the dividend	2 ÷ 1.05	**1.90**
Spot minus PV of dividend	100 − 1.90	**98.10**
Compound one year	98.10 × 1.05	**103.00**
Check	105 − 2	**103**

IN THE LESSON

- Example four.
- Kestrel prices a one-year forward on the shares of Alder Corporation.
- The spot price is one hundred dollars.
- Alder will pay a dividend of two dollars on the delivery date, in one year, and the risk-free rate is five per cent.
- The present value of the dividend is two divided by one point zero five, which is one point nine zero dollars.
- Take that from the spot price and you get ninety eight point one zero.
- Compound for a year, multiplying by one point zero five, and the forward price is one hundred and three point zero zero dollars.
- A quick check: the no-dividend price of one hundred and five, minus the two dollar dividend, is also one hundred and three.
- The dividend lowered the price.

13 Example 5: gold with storage costs

Gold spot 2,000, storage 20 at year end, rate 5%

PV of storage	20 ÷ 1.05	**19.05**
Fair forward	(2,000 + 19.05) × 1.05	**2,120**
Quoted forward	market quote	**2,090**
Gap, buy the forward	2,120 − 2,090	**$30**

IN THE LESSON

- Example five, gold.
- The spot price is two thousand dollars an ounce.
- Storing and insuring it for a year costs twenty dollars, paid at the end, and the risk-free rate is five per cent.
- There are no benefits.
- The present value of the storage is twenty over one point zero five, about nineteen point zero five.
- Add it to the spot price, and compound by one point zero five.
- The fair forward is two thousand one hundred and twenty dollars.
- That is two thousand one hundred, plus the storage of twenty.
- Now suppose the market quotes the forward at only two thousand and ninety.
- That is thirty dollars below fair value.
- Kestrel buys the cheap forward, sells gold it holds, lends the proceeds, and collects thirty dollars with no risk.

14 Example 6: oil and convenience yield

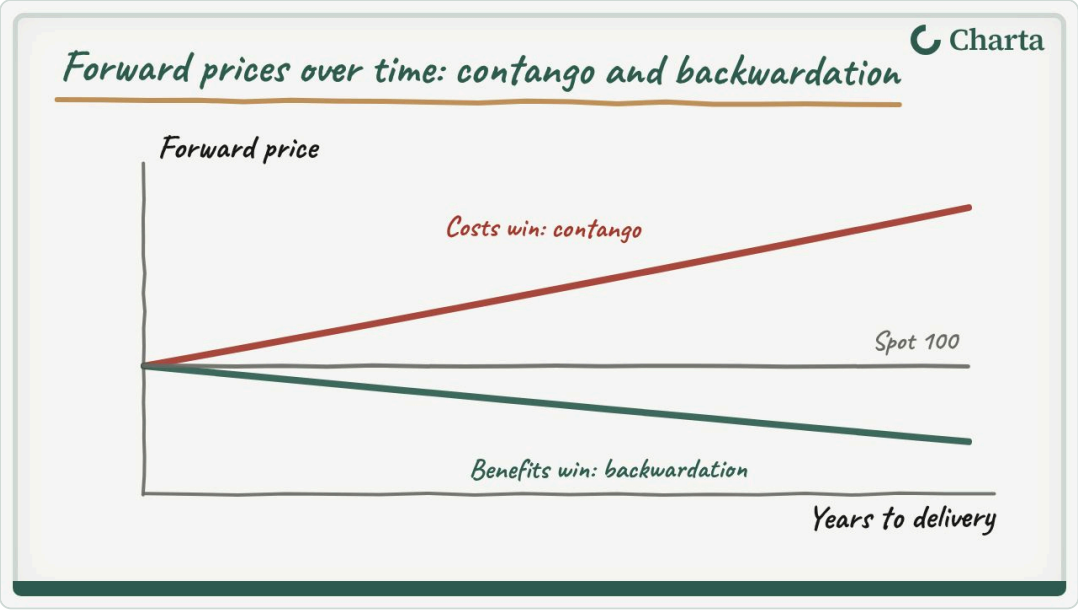
Oil spot 80, PV storage 1, PV convenience yield 6, rate 5%

Spot, less benefit, plus cost	80 − 6 + 1	**75**
Compound one year	75 × 1.05	**78.75**
Compared with spot	78.75 vs 80	**below**
Name for this	forward below spot	**backwardation**

IN THE LESSON

- Example six, oil, where something unusual can happen.
- The spot price is eighty dollars a barrel.
- Storage costs have a present value of one dollar.
- Refiners value having oil on hand, and the present value of that convenience yield is six dollars.
- The risk-free rate is five per cent.
- Start with eighty, subtract the benefit of six, add the cost of one, and you have seventy-five.
- Compound for a year, and the forward price is seventy eight point seven five dollars.
- That is below the spot price of eighty.
- When a forward sits below spot, because the benefits of holding exceed the financing and storage costs, the market is said to be in backwardation.
- The opposite, with the forward above spot, is called contango.

15 Forward prices over time: contango and backwardation



IN THE LESSON

- Here is the same idea as a picture.
- Time to delivery runs along the bottom, from today out to two years.
- The forward price is on the side.
- The flat line is the spot price, one hundred dollars.
- Now the two curves.
- The upper curve starts at spot and climbs, reaching one hundred and twelve at two years.
- Here the costs of carry, mainly interest and storage, are bigger than the benefits, so a longer wait means a higher price.
- That shape is contango.
- The lower curve falls away, to about ninety-four at two years.
- There, benefits such as dividends or convenience yield are larger than the costs, so a longer wait means a lower price.
- That shape is backwardation.
- The spot price itself has not moved.

16 A story: the price is fixed, the worth is not

- Price: the number in the contract. It never changes
- Value: what the contract is worth today. It moves
- At the start the value is zero for both sides
- Later, one side gains and the other loses

IN THE LESSON

- Go back to the bike.
- You and your neighbour signed a deal for a hundred and four dollars in a year.
- Six months later, bikes have become scarce and the same bike is now worth a lot more.
- You still pay only the agreed price.
- Your deal has become valuable, and his has become a burden, even though neither of you has paid anything.
- This is the key distinction in forward contracts.
- The price is the number written in the contract, and it never changes.
- The value is what the contract is worth today to each side, and it changes all the time.
- At the start, the two are set so that the value is exactly zero for both sides.
- After that, the value drifts.

17 The value of a forward before it ends

Vt (long) = [Ft − F0] ÷ (1 + r)^(T − t)

Value of long, Vt	What the long holds today
Forward price now, Ft	Forward price for time left
Price contracted, F0	Price signed at the start
Discount to today	Years left, at the safe rate

IN THE LESSON

- Here is how to value a forward after it has started.
- Step one, find what the forward price would be today for the remaining time.
- Call it F t.
- For an asset with no income or costs, that is today's spot price compounded for the time left.
- Step two, compare it with the old price you signed, F zero.
- If F t is higher, the long gains by the difference.
- That difference is paid at delivery, which is in the future, so step three is to bring it back to today by dividing by one plus r, to the power of the time left.
- The value to the long is that present value.
- The value to the short is exactly the same number with the opposite sign.

18 Example 7: valuing a forward at six months

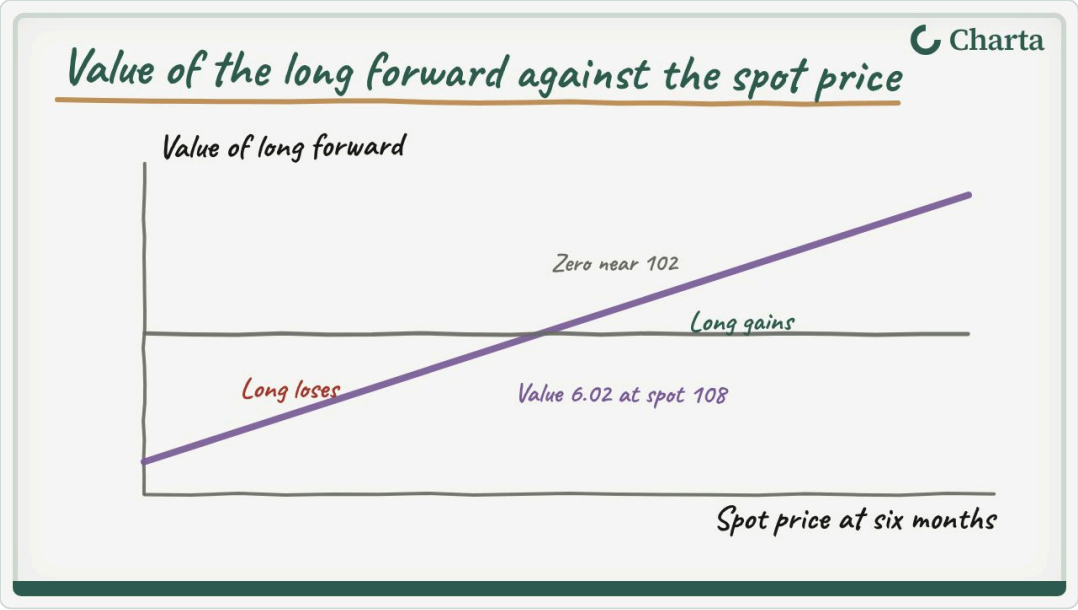
Signed at 104. Now spot 108, rate 4%, half a year left.

Forward for time left	108 × 1.04^0.5	**110.14**
Gap to signed price	110.14 − 104	**6.14**
Discount half a year	6.14 ÷ 1.04^0.5	**6.02**
Value to the short	−6.02	**−6.02**

IN THE LESSON

- Example seven.
- Kestrel went long a one-year forward on an asset with no income.
- Spot was one hundred dollars and the risk-free rate is four per cent, so the contracted price was one hundred and four.
- Six months later, the spot price is one hundred and eight, and the rate is still four per cent.
- Step one, the forward price for the remaining half year is one hundred and eight times one point zero four to the power of a half, which is one hundred and ten point one four.
- Step two, the gap against the signed price of one hundred and four is six point one four.
- Step three, discount the gap for half a year, and the long's position is worth six point zero two dollars.
- The short's is minus six point zero two.
- At expiry no time is left, so the value is simply the spot price then, minus the signed price.

19 Value of the long forward against the spot price



IN THE LESSON

- Here is the value drawn out.
- Along the bottom is the spot price six months in, from ninety to one hundred and fifteen.
- Up the side is the value of the long forward.
- The horizontal line is zero.
- The sloping line is the value.
- It is a straight line with a slope of one, since every extra dollar on the spot price adds a dollar to the long.
- It crosses zero at about one hundred and two, which is the signed price of one hundred and four, discounted for half a year.
- Left of that crossing, the long is losing.
- Right of it, the long is winning.
- At one hundred and eight, the value is the six dollars and two cents we just found.
- The short's picture is the mirror image.

20 Mistakes to avoid so far

- 1. The forward price is not a forecast
- 2. Benefits lower the price. Costs raise it.
- 3. Price is fixed. Value starts at zero and moves.
- 4. Compare with Ft, not spot. Then discount.

IN THE LESSON

- Let us stop and collect the mistakes that cost marks.
- One.
- Treating the forward price as a prediction of the future spot price.
- It is a carrying cost, not a forecast.
- Two.
- Subtracting benefits and adding costs the wrong way round.
- Dividends and convenience yield lower the price, storage and insurance raise it.
- Three.
- Mixing up price and value.
- The price is fixed at the start, and the value is zero at the start and moves afterwards.
- Four.
- Comparing the signed price with the spot price, instead of with the current forward price for the same delivery date.
- Five.
- Forgetting the last step, which is to discount the gain back to today.

21 A story: locking in next season's loan rate

- Redwood Foods will borrow 10m in 3 months
- It fears rates will rise before then
- A forward rate agreement fixes a rate today
- The question: what rate goes in the contract?

IN THE LESSON

- Picture a bakery called Redwood Foods that knows it will need to borrow ten million dollars in three months, for three months, to buy flour in bulk.
- Today, the going rate for that loan is unknown.
- If rates jump, the bakery's costs jump with them.
- So the owner wants to fix the rate today, for borrowing that starts in the future.
- The contract that does this is called a forward rate agreement, or F R A.
- It settles in cash, and no loan is made under it.
- It is a bet on a future interest rate, used as insurance.
- The question for us is, what rate should be written into the contract?

22 Where the forward rate comes from

- Lend 180 days, or lend 90 days then 90 more
- Both routes must end with the same money
- The second 90-day rate is the implied forward rate
- FRA rate = implied forward rate. Value zero at start.

IN THE LESSON

- The answer is the same no-arbitrage idea again.
- Suppose you can borrow or lend for ninety days, or for one hundred and eighty days, at known rates today.
- You could lend for one hundred and eighty days directly.
- Or you could lend for ninety days, and then lend again for the next ninety, at a rate you agree today.
- If the second route gave more, everyone would use it, so the two must match.
- The rate for that second ninety days, fixed today, is called the implied forward rate.
- The F R A rate is set equal to it.
- That makes the contract worth zero when signed, because no rate other than the implied one can be free of arbitrage.

23 The forward rate formula

Forward rate = [(1 + Rb × b/360) ÷ (1 + Ra × a/360) − 1] × 360 ÷ (b − a)

Short rate, Ra	Rate for the shorter time
Long rate, Rb	Rate for the longer time
Short days, a	Days to the start
Long days, b	Days to the end

IN THE LESSON

- The formula for money market rates.
- A money market rate is quoted on a three hundred and sixty day year, and I will say so each time.
- Take the short rate R a for a days, and the long rate R b for b days, with b larger than a.
- The growth of one dollar over the long period is one plus R b times b over three hundred and sixty.
- The growth over the short period is one plus R a times a over three hundred and sixty.
- Divide the first by the second, subtract one, and you have the growth over the gap.
- Finally multiply by three hundred and sixty over b minus a, to annualise.

24 Example 9: the rate for the second ninety days

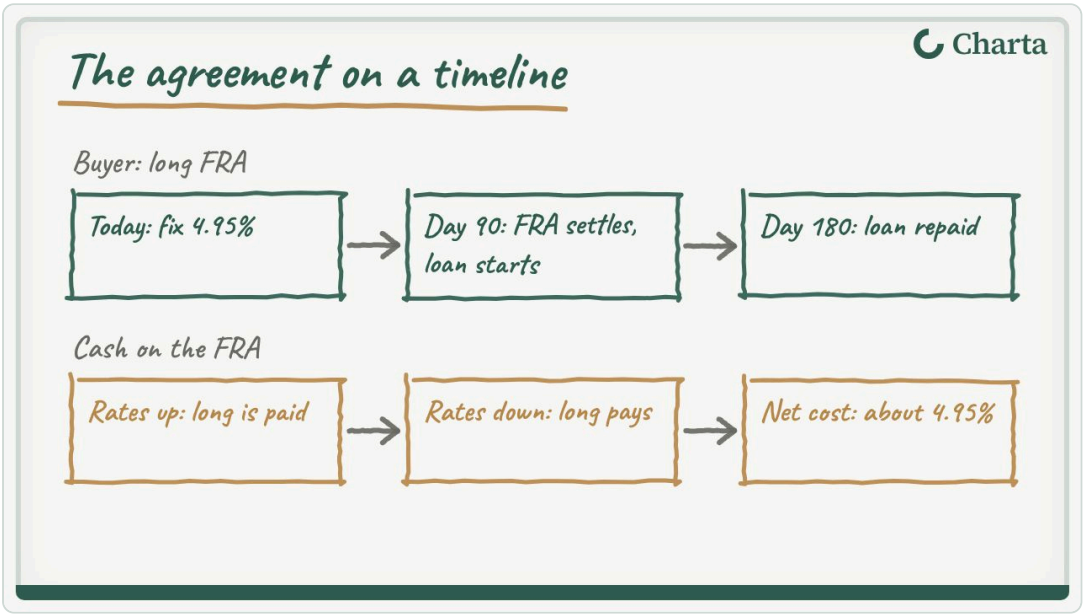
90 days 4.0%, 180 days 4.5%. Quotes on a 360 day year.

Growth over 180 days	1 + 0.045 × 180/360	**1.0225**
Growth over 90 days	1 + 0.04 × 90/360	**1.0100**
Rate for the gap	1.0225 ÷ 1.01 − 1	**1.2376%**
Annualised, 360 days	1.2376% × 360/90	**4.95%**

IN THE LESSON

- Example nine.
- The ninety day rate is four per cent, and the one hundred and eighty day rate is four point five per cent, both quoted on a three hundred and sixty day year.
- One dollar lent for the full one hundred and eighty days grows to one plus point zero four five times a half, which is one point zero two two five.
- One dollar lent for ninety days grows to one plus point zero four times a quarter, which is one point zero one.
- Divide one point zero two two five by one point zero one, subtract one, and you get one point two three eight per cent for the ninety days.
- Annualise on three hundred and sixty days, by multiplying by four, and the rate is four point nine five per cent.

25 The agreement on a timeline



IN THE LESSON

- This is the timeline for Redwood Foods.
- Today, the bakery buys a three by six forward rate agreement, meaning it starts in three months and covers the following three months.
- The rate is fixed at four point nine five per cent.
- Ninety days from now, the contract settles in cash, against the rate that is then really in the market.
- At that same moment, Redwood borrows its ten million dollars in the real market at the going rate.
- One hundred and eighty days from today, the loan is repaid with interest.
- The agreement does not make the loan.
- It pays the bakery cash if rates went up, and takes cash from it if they went down, so the cost lands on the locked rate.

26 Example 10: settling the agreement

Notional 10m, FRA rate 4.95%, rate at day 90 is 5.5%

Rate difference	5.50% − 4.9505%	**0.5495%**
Owed at day 180	0.5495% × 90/360 × 10m	**$13,738**
Discount to day 90	÷ (1 + 0.055 × 90/360)	**÷ 1.01375**
Paid to the long	13,738 ÷ 1.01375	**$13,551**

IN THE LESSON

- Example ten, the settlement.
- Redwood holds the three by six agreement on ten million dollars at four point nine five per cent.
- At day ninety, the ninety day rate in the market is five point five per cent, so rates have risen.
- The long gains the difference.
- Five point five minus four point nine five is zero point five five per cent.
- For ninety days, on ten million, that gap comes to thirteen thousand seven hundred and thirty eight dollars, at the end of the loan period.
- But the agreement settles at the start of the period, so we discount it, using the market rate of five point five per cent over ninety days.
- That gives thirteen thousand five hundred and fifty one dollars paid to the long today.
- Had rates fallen, the long would pay it.

27 Example 11: the same idea with years

One-year rate 3%, two-year rate 4%, annual compounding

Two-year growth	1.04^2	**1.0816**
One-year growth	1 + 0.03	**1.03**
Forward rate, year two	1.0816 ÷ 1.03 − 1	**5.01%**

IN THE LESSON

- Example eleven, the same idea with whole years.
- The rate for one year is three per cent, and for two years is four per cent, compounded annually.
- What is the rate for the second year, agreed today?
- Lending for two years grows one dollar to one point zero four squared, which is one point zero eight one six.
- Lending for one year grows it to one point zero three.
- Divide one by the other and subtract one, and you get five point zero one per cent.
- This is the one year rate, one year forward.
- Check whether a question gives money market days or annual rates, and use the matching formula.

28 Who uses an FRA, and what to remember

- Long: pays fixed, gets floating. Gains if rates rise.
- Short: gets fixed, pays floating. Gains if rates fall.
- Long suits a future borrower. Short, a future lender.
- Zero at signing. Cash at start of period, discounted.

IN THE LESSON

- A quick summary on who does what.
- The long, who pays the fixed rate and receives the floating rate, gains when rates rise.
- That suits a future borrower, like Redwood, who wants to lock in the cost.
- The short, who receives the fixed rate and pays the floating rate, gains when rates fall.
- That suits a future lender who wants to lock in a return.
- The agreement is worth zero when signed, because the fixed rate is the implied forward rate.
- As time passes, its value moves as the market's view of that forward rate changes.
- And remember it settles in cash, at the start of the period, and so the payment is discounted.

29 A story: pay as you go, or settle at the end

- Futures: standard contract traded on an exchange
- Gains and losses are settled in cash every day
- That daily settling is marking to market
- Forward: all settled once, at the end

IN THE LESSON

- Back to the neighbour and his bike.
- Imagine a different version of the deal.
- Instead of waiting a year, the two of you settle up every evening.
- If the bike price rose today, he hands you the day's gain in cash.
- If it fell, you hand him the loss.
- At the end of each day the deal is reset to zero.
- That is how a futures contract works, and the daily settling is called marking to market.
- A futures contract is a standard forward traded on an exchange, with a clearing house in the middle, and margin posted as a deposit.
- A forward settles once, at the very end.
- So what difference could the timing of cash make to the price?

30 Forwards and futures side by side

Two ways to lock in a price

Forward versus futures	Forward / Futures
Terms	Tailored / Standard
Where traded	Private / Exchange
Cash settled	End / Daily
Credit risk	Other side / Clearing house

IN THE LESSON

- Here are the differences, and in a moment the one that matters for price.
- A forward is private, tailored between two parties, with terms to suit, and carries the credit risk that the other side may not pay.
- A futures contract is standardised, traded on an exchange, and guaranteed by the clearing house.
- It is marked to market each day, and margin is posted to cover losses.
- That daily process removes most of the credit risk, but it creates a cash flow every day.
- Both are priced from the same cost of carry, so we expect the same price, yet there are reasons it may differ.

31 Why the two prices may differ

- Gains are invested daily; losses are funded daily
- Constant rates: the effects cancel, prices equal
- Rates move with the price: the effects do not cancel
- Taxes, liquidity and credit risk also differ

IN THE LESSON

- So why might the prices differ?
- Reason one is interest rates.
- A futures gain arrives in cash straight away, and can be invested.
- A futures loss must be funded straight away.
- A forward gain or loss arrives only at the end.
- If interest rates were constant, those little investing and funding effects would cancel exactly, and the two prices would be equal.
- Reason two is the extra features: taxes, liquidity and credit risk differ between the two contracts.
- At Level One, the main point is the first.
- The gap exists only when interest rates change in step with the underlying asset's price.
- In practice the gap is small, and you normally use the forward formula for both.

32 Example 12: why the long futures is worth more

±$10,000 move. Rates 6% if price up, 2% if down. 6 months.

Price up, rates 6%	10,000 × 6% × ½	**+$300**
Price down, rates 2%	10,000 × 2% × ½	**−$100**
Average edge for futures	(300 − 100) ÷ 2	**+$100**

IN THE LESSON

- Example twelve, an illustration.
- Suppose the price of an asset moves up or down by ten thousand dollars on a long position, six months before expiry.
- It tends to rise when interest rates rise to six per cent, and to fall when they fall to two per cent.
- If the price rises, the futures long gets ten thousand in cash now, and invests it for six months at six per cent, earning three hundred dollars.
- If it falls, the long pays ten thousand now, financed at two per cent, costing one hundred dollars.
- Taking each path as equally likely, the long futures position earns on average one hundred dollars more than the forward.
- The forward gets none of this.
- So buyers will pay more for futures, and the futures price is above the forward price.
- If the correlation is negative, as with bonds, the edge reverses and the futures price is lower.
- With no correlation, the prices are equal.

33 Recap: the formulas to remember

F0 = (S0 − PV benefits + PV costs) × (1 + r)^T
Vt = [Ft − F0] ÷ (1 + r)^(T − t)

Spot price, S0	Price today
Benefits, PV	Lower the forward price
Costs, PV	Raise the forward price
Value of long, Vt	Zero at start, then moves
Forward price now, Ft	Forward price for the time left

IN THE LESSON

- Let us pull it together.
- The forward price is the spot price carried forward: spot, less the present value of benefits, plus the present value of costs, all compounded at the risk-free rate.
- No forecast is needed, because arbitrage enforces it.
- Benefits like dividends and convenience yield lower the price, and costs like storage raise it.
- The value of an existing forward is the present value of the difference between the current forward price for the remaining time, and the price signed.
- It starts at zero.
- A forward rate agreement carries the implied forward rate from today's spot rates, and settles in cash, discounted.
- And futures and forwards differ because of daily settlement, only when rates and prices move together.

34 Try three quick questions

1. Spot 50, rate 4%, dividend 1 at delivery in one year. Forward?
   Answer: 50 × 1.04 − 1 = $51.
2. Signed at 100. Forward now 106, discount factor 0.99. Value to long?
   Answer: (106 − 100) × 0.99 = 5.94.
3. Rates and price negatively correlated. Futures vs forward?
   Answer: Futures price is lower.

IN THE LESSON

- Now try three quick questions yourself.
- Question one.
- A share trades at fifty dollars.
- The risk-free rate is four per cent, and it pays one dollar on the delivery date, one year away.
- What is the fair forward price?
- Fifty times one point zero four is fifty-two, minus the one dollar dividend, gives fifty-one dollars.
- Question two.
- A forward was signed at one hundred.
- The forward price today for the same date is one hundred and six, and the discount factor is zero point nine nine.
- What is the long's value?
- Six times zero point nine nine is five dollars and ninety-four cents.
- Question three.
- Rates and the price are negatively correlated.
- Is the futures price above or below the forward price?
- Below.