CHARTA LEARNING · LESSON NOTES

Discounted cash flow and growth models

Equity Investments

1 A tree and its fruit

- A share is a tree; dividends are the fruit
- Intrinsic value = what the fruit is worth to you
- Market price = what the screen says
- r = the return you require for the risk

IN THE LESSON

- Picture an apple tree.
- What would you pay for it?
- Not for the wood, but for the fruit it will give you, year after year.
- You would add up the fruit you expect, and you would pay less for fruit that comes late, because waiting has a cost.
- A share works the same way.
- Its fruit is the dividend, the cash a company pays out to its owners.
- The value you work out this way is called intrinsic value.
- The price on the screen is the market price.
- If your value is above the price, the share may be cheap.
- The rate you use to shrink late fruit is the required return, written r.
- It is what you demand for taking the risk of owning this share.

2 The dividend discount model

$V0 = D1/(1+r) + D2/(1+r)^2 + D3/(1+r)^3 + \ldots$

Value today, V0	What the share is worth now
Dividend in year t	Cash paid to owners in year t
Required return, r	Return you demand each year
Year number, t	1, 2, 3 … years from now

IN THE LESSON

- The dividend discount model, which we will call the D D M, says a share is worth the present value of every dividend it will ever pay.
- Present value means what a future payment is worth today.
- Value today is written V zero.
- You take each dividend and divide it by one plus r, raised to the power of its year.
- The first dividend is pulled back one year, the second two years, and so on without end.
- Dt is the dividend in year t.
- The letter r is your required return, and t simply counts the years.
- Why dividends and not profits?
- Because a dividend is the only cash that actually reaches you.
- One snag: the list never ends, and nobody can forecast for ever.
- So we need a shortcut.

3 The Gordon growth shortcut

V0 = D1 ÷ (r − g)

Value today, V0	What the share is worth now
Next year's dividend, D1	D0 × (1 + g), received next year
Required return, r	Return you demand each year
Growth rate, g	Steady dividend growth, for ever

IN THE LESSON

- Here is the shortcut.
- If dividends grow at the same rate, g, every year for ever, the endless sum collapses into one neat fraction.
- This is the Gordon growth model.
- Value today equals next year's dividend, divided by r minus g.
- Take care with the first term.
- It is D one, the dividend you will receive next year, not the one just paid.
- If you know the dividend just paid, called D zero, multiply it by one plus g to get D one.
- Think back to a perpetuity, which is a payment divided by a rate.
- Growth works like a discount on the rate.
- The faster dividends grow, the smaller the gap, and the bigger the value.
- And r must be larger than g, or the formula breaks.

4 Example 1: what is Larkspur worth?

Larkspur: dividend just paid $1.80, growth 5%, required return 9%

Dividend just paid, D0	this year's, already gone	**$1.80**
Next year's dividend, D1	1.80 × 1.05	**$1.89**
The gap, r − g	0.09 − 0.05	**0.04**
Value today, V0	1.89 ÷ 0.04	**$47.25**
Market price	on the screen	**$42.00**

IN THE LESSON

- Meet Larkspur Foods, a steady maker of packaged goods.
- It has just paid a dividend of one dollar eighty a share.
- Dividends have grown five per cent a year, and we expect that to continue.
- You require nine per cent.
- Step one, next year's dividend.
- One dollar eighty times one point zero five is one dollar eighty-nine.
- Step two, the gap between r and g.
- Nine per cent minus five per cent is four per cent, or zero point zero four.
- Step three, divide.
- One dollar eighty-nine over zero point zero four is forty-seven dollars twenty-five.
- The screen price is forty-two dollars.
- So the share trades below our value, and looks cheap, if our inputs are right.

5 Who the model suits

- Best: stable firms with steady dividends
- Poor: firms that pay little or nothing
- Poor: dividends that jump around
- V0 sits one year before D1 arrives

IN THE LESSON

- When is this model a good tool?
- When dividends are the thing you can forecast.
- Think of a mature company with steady profits that pays out a regular share each year.
- Its dividends are predictable, so growing them at one rate is a fair story.
- The model is a poor tool for the opposite cases.
- A young company that pays nothing has no dividends to discount.
- A company whose dividends jump around, or are cut and restored, will not fit a single growth rate.
- Also remember the timing.
- The formula gives the value one year before the first dividend it uses.
- That is why D one sits on top, and why V zero is today's value.

6 What the Gordon model takes for granted

- Dividends grow at ONE rate, for ever
- g must sit strictly below r
- g cannot beat the economy for ever
- r is assumed stable for ever

IN THE LESSON

- Every shortcut has a price.
- The Gordon model assumes dividends grow at one rate, for ever.
- That is a strong claim.
- Think of a snowball rolling down a hill.
- It cannot keep growing faster than the hill allows.
- In the same way, no company can grow faster than the whole economy for ever, because it would eventually be bigger than the economy.
- So in practice g is capped near the long run growth of the economy, in nominal terms.
- Second, g must be strictly below r.
- If g creeps toward r, the gap shrinks toward zero and the value races toward infinity.
- If g passes r, the answer turns negative, which is nonsense.
- Third, r itself is assumed steady for ever.

7 Where does g come from?

g = b × ROE where b = 1 − payout ratio

Retention ratio, b	Share of earnings kept, not paid
Return on equity, ROE	Profit earned on owners' money
Sustainable growth, g	Pace funded from within

IN THE LESSON

- So where does a growth rate come from?
- A company can only grow if it keeps some of its profit and reinvests it.
- The share of earnings it keeps is the retention ratio, b.
- The share it pays out is the payout ratio, and the two add up to one.
- What the reinvested money earns is the return on equity, written R O E, which is profit divided by shareholders' money.
- Multiply the two and you get the sustainable growth rate, the pace the firm can fund from its own profit.
- For instance, a firm earning twelve per cent on equity that keeps half its profit can grow six per cent.
- This is a starting point, not a promise.
- Competition can pull R O E down, and then growth fades.

8 Example 2: growth from the firm's own profit

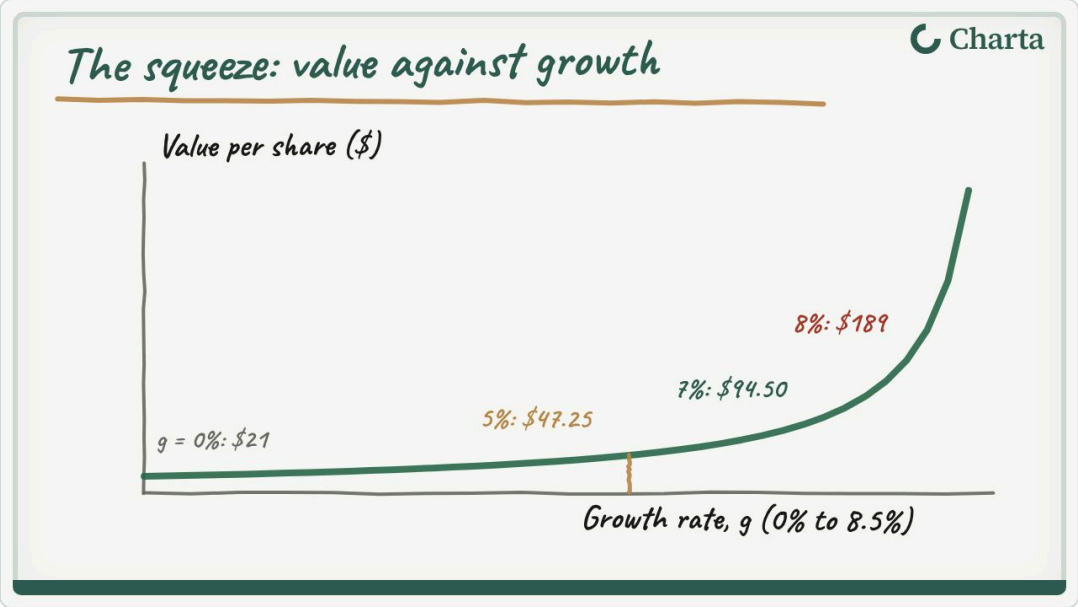
Larkspur: pays out 60% of earnings, ROE 12.5%, earnings now $3.00

Payout ratio	dividends ÷ earnings	**60%**
Retention ratio, b	1 − 0.60	**40%**
Sustainable growth, g	0.40 × 0.125	**5%**
Earnings per share next year	3.00 × 1.05	**$3.15**
D1 = payout × earnings	0.60 × 3.15	**$1.89**

IN THE LESSON

- Back to Larkspur.
- Its earnings per share this year are three dollars, and it paid one dollar eighty, so the payout ratio is sixty per cent.
- That means it keeps forty per cent.
- Its return on equity is twelve and a half per cent.
- Multiply forty per cent by twelve and a half per cent and you get five per cent.
- So the five per cent we used in Example one was not plucked from the air.
- The firm can fund it from its own profit.
- Now check the dividend.
- Earnings next year are three dollars times one point zero five, which is three dollars fifteen.
- Sixty per cent of that is one dollar eighty-nine.
- It matches D one exactly.

9 The squeeze: value against growth



IN THE LESSON

- Now the picture.
- Growth runs along the bottom, from zero up to eight and a half per cent.
- Value per share runs up the side.
- We keep next year's dividend fixed at one dollar eighty-nine and r at nine per cent, and change only g.
- The curve starts low on the left.
- With no growth the share is worth twenty-one dollars.
- It climbs gently at first.
- At five per cent, our Larkspur case, it is forty-seven dollars twenty-five.
- Then the curve turns almost straight up.
- At seven per cent the value is ninety-four dollars fifty, and at eight per cent it is one hundred and eighty-nine dollars.
- The gap between r and g is almost gone, and the value runs away.

10 Example 3: one point of growth

D1 = $1.89 fixed, r = 9%: value at each growth rate

Growth rate, g	Value per share
4%	$37.80
5%	$47.25
6%	$63.00
7%	$94.50

IN THE LESSON

- Here are the same numbers as a table.
- Next year's dividend stays at one dollar eighty-nine and r stays at nine per cent.
- At four per cent growth the value is thirty-seven dollars eighty.
- At five per cent, forty-seven dollars twenty-five.
- At six per cent, sixty-three dollars.
- At seven per cent, ninety-four dollars fifty.
- Look at the step from six to seven.
- One point of growth adds half as much value again.
- And at nine per cent the formula has nothing to say.
- This is why analysts show a table like this, not a single number.
- You can also run it backwards.
- If the price is forty-two dollars, the growth it implies is four and a half per cent.

11 When growth changes gear

- Stage 1: forecast each dividend one by one
- Stage 2: terminal value once growth settles
- Value = PV of dividends + PV of terminal value
- Two-stage drops growth; H model fades it

IN THE LESSON

- Few companies grow at one speed for ever.
- Think of a child.
- Growth is fast for years, then it levels off.
- A young company is similar.
- It may grow dividends quickly while it expands, then settle to a steady pace as it matures.
- For that we use a multi-stage dividend discount model.
- Step one, forecast the dividends one by one through the fast stage.
- Step two, when growth has settled, use the Gordon formula to value everything after that point.
- That value is the terminal value.
- One trap: the Gordon formula gives a value one year before the dividend it uses.
- So a terminal value at the end of year three uses the dividend of year four.
- A simple two-stage model drops growth abruptly.
- An H model lets it fade in a straight line, which is often more realistic.

12 The H model: growth that fades

V0 = D0 × (1 + gL) ÷ (r − gL) + D0 × H × (gS − gL) ÷ (r − gL)

Value today, V0	Value today by the H model
Dividend just paid, D0	Last dividend paid
Long-run growth, gL	Long-run growth, after the fade
Starting growth, gS	Starting growth, before the fade
Half-life, H	Half the fade period, in years

IN THE LESSON

- Here is the H model.
- Instead of dropping growth in one step, it lets growth fall in a straight line from a high starting rate to the long run rate.
- H is half the length of that fade in years, so a fade over four years has an H of two.
- The value has two parts.
- The first is the Gordon value, as if growth were the long run rate from today.
- The second is the extra value from the fast growth that fades.
- The dividend just paid, times H, times the gap between starting and long run growth, over the same denominator.
- It is an approximation.

13 Example 4: dividends and terminal value

Growth 10% for 3 years, then 5% for ever; r = 9%

Dividend, year 1	1.80 × 1.10	**$1.98**
Dividend, year 2	1.98 × 1.10	**$2.18**
Dividend, year 3	2.178 × 1.10	**$2.40**
Dividend, year 4	2.3958 × 1.05	**$2.52**
Terminal value, end of year 3	2.5156 ÷ 0.04	**$62.89**

IN THE LESSON

- Suppose Larkspur is launching a new range.
- Dividends grow ten per cent a year for three years, then five per cent for ever.
- We still require nine per cent.
- Start from the dividend just paid, one dollar eighty.
- Year one is one dollar ninety-eight.
- Year two is two dollars eighteen.
- Year three is two dollars forty, more precisely two dollars thirty-nine point six.
- Now growth settles at five per cent, so year four is that figure times one point zero five, which is about two dollars fifty-two.
- Now the terminal value at the end of year three.
- Divide the year four dividend by nine per cent minus five per cent.
- That gives sixty-two dollars eighty-nine.

14 Example 4: bring it all back to today

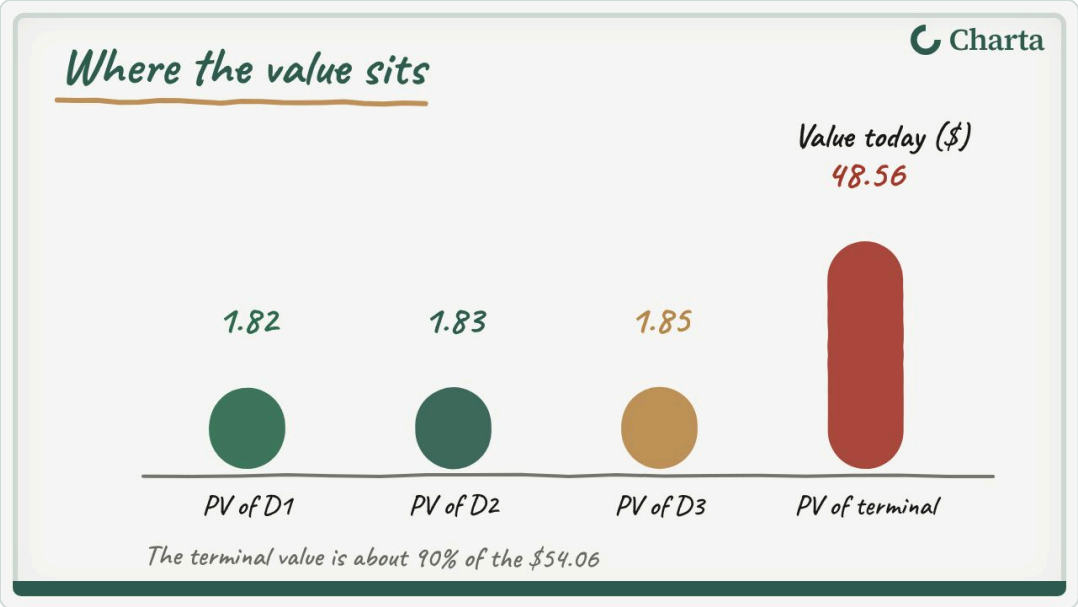
Discount each piece back to today at 9%

PV of dividend, year 1	1.98 ÷ 1.09	**$1.82**
PV of dividend, year 2	2.178 ÷ 1.09^2	**$1.83**
PV of dividend, year 3	2.3958 ÷ 1.09^3	**$1.85**
PV of terminal value	62.89 ÷ 1.09^3	**$48.56**
Value today, V0	sum of the four	**$54.06**

IN THE LESSON

- Now bring every piece back to today at nine per cent.
- Year one dividend, one dollar ninety-eight over one point zero nine, is one dollar eighty-two.
- Year two, divided by one point zero nine squared, is one dollar eighty-three.
- Year three, divided by one point zero nine cubed, is one dollar eighty-five.
- The terminal value of sixty-two dollars eighty-nine also sits at the end of year three, so it is divided by one point zero nine cubed too.
- Three years, not four.
- That gives forty-eight dollars fifty-six.
- Add the four pieces and the share is worth fifty-four dollars six cents today.
- Compare that with forty-seven twenty-five for steady five per cent growth.
- Three years of faster growth added nearly seven dollars.

15 Where the value sits



IN THE LESSON

- Here are the four pieces as bars.
- The left three are the present values of the dividends you worked so hard to forecast, each close to one dollar eighty-five.
- They are tiny.
- The tall bar on the right is the present value of the terminal value, forty-eight dollars fifty-six.
- Of the fifty-four dollars six cents total, that is about ninety per cent.
- In other cases it is three quarters or more, depending on the horizon, the growth and the discount rate.
- The lesson is where to spend your effort.
- Analysts often agonise over the first few dividends, yet one number sets most of the answer, the long run growth rate.
- That deserves the most scrutiny.

16 Example 5: one point on long-run growth

Same three dividends; long-run growth 6% instead of 5%

Terminal value at 5% growth	2.5156 ÷ 0.04	**$62.89**
Terminal value at 6% growth	2.3958 × 1.06 ÷ 0.03	**$84.65**
PV of the new terminal value	84.65 ÷ 1.09^3	**$65.37**
New value today	5.50 + 65.37	**$70.87**
Rise in value	70.87 − 54.06	**+$16.81**

IN THE LESSON

- Let us test that claim.
- Keep the same three dividends, but let long run growth be six per cent, not five.
- The terminal value becomes the year three dividend, times one point zero six, divided by nine per cent minus six per cent.
- That is eighty-four dollars sixty-five, up from sixty-two eighty-nine.
- Discount it three years and it is worth sixty-five dollars thirty-seven today.
- Add the present value of the three dividends, about five dollars fifty, and the share is worth seventy dollars eighty-seven.
- One point of long run growth has lifted the value by sixteen dollars eighty-one, about thirty-one per cent.
- We changed one cell, and the answer moved more than the first three years of dividends combined.

17 Four mistakes so far

- 1. Using D0 where the formula needs D1
- 2. Letting g reach or pass r
- 3. Terminal value: wrong dividend or wrong year
- 4. Quoting one value as if it were exact

IN THE LESSON

- Here are the traps that cost marks so far.
- One, using the dividend just paid where the formula wants next year's.
- Always grow D zero by one plus g first.
- Two, letting g reach or pass r.
- A growth rate equal to r divides by zero, and above r gives a negative, meaningless value.
- In the long run g should also stay below the growth of the economy.
- Three, mixing up the terminal value timing.
- A terminal value at the end of year three uses the year four dividend, and is discounted three years.
- Four, quoting one value as if it were exact.
- The inputs are guesses, so show a range.
- Check each of these before you trust any answer from these models.

18 Cash in the till

- Dividends = what the board CHOSE to pay
- FCFE = what the firm COULD pay its owners
- Use it when dividends are zero or unrelated
- Discount at the cost of equity, r

IN THE LESSON

- Imagine you and a friend own a shop.
- It earns a good profit, but your friend, who runs it, keeps most of the cash in the till and hands you very little.
- If you valued the shop by what you were handed, you would underrate it.
- What matters is what could be taken out without harming the shop.
- That is free cash flow to equity, which we will call F C F E.
- It is the cash left for owners after the business has paid its costs, interest and tax, reinvested what it needs, and after borrowing or repaying debt.
- Use it when a company pays no dividend, or pays one unrelated to what it can afford, or when you take a control view, where a buyer would choose the payout.
- It is discounted at the same required return, r.

19 Free cash flow to equity

FCFE = CFO − FCInv + net borrowing

Cash flow from operations	Cash earned after interest, tax
Fixed capital investment	Spending on plant and equipment
Net borrowing	New debt raised minus debt repaid
Free cash flow to equity	What could be paid to owners

IN THE LESSON

- Here is the formula.
- F C F E equals cash flow from operations, minus fixed capital investment, plus net borrowing.
- Cash flow from operations is the cash the business earns, and it is already after interest, tax and changes in working capital.
- Fixed capital investment is what the firm spends on plant and equipment.
- Net borrowing is new debt raised minus debt repaid.
- Borrowed money can reach owners, so it is added.
- If the firm repays debt, net borrowing is negative and it is subtracted.
- Notice what you do not do at the end.
- The cash flow is already the owners' share, so you do not subtract debt from the value.
- You discount it at the cost of equity and the answer is the value of the equity.

20 Example 6: Larkspur's free cash flow

Net income 60, depreciation 45, working capital up 7 ($ millions)

Cash flow from operations	60 + 45 − 7	**98**
Fixed capital investment	spent on equipment	**70**
Net borrowing	30 raised − 16 repaid	**14**
FCFE	98 − 70 + 14	**42**
FCFE per share	42 ÷ 20 million shares	**$2.10**

IN THE LESSON

- Let us work out Larkspur's F C F E, in millions of dollars.
- Net income was sixty.
- Depreciation of forty-five is a cost that did not use cash, so we add it back.
- Working capital rose by seven, which tied up cash, so we take it off.
- Cash flow from operations is sixty plus forty-five minus seven, which is ninety-eight.
- The firm spent seventy on equipment.
- It raised thirty of new debt and repaid sixteen, so net borrowing is fourteen.
- F C F E is ninety-eight minus seventy plus fourteen, which is forty-two.
- With twenty million shares, that is two dollars ten a share.
- The firm paid dividends of thirty-six million, which is the one dollar eighty a share.

21 Example 7: two ways to value the same share

Same share, r = 9%, g = 5%: capacity against payout

FCFE next year, per share	2.10 × 1.05	**$2.205**
Value from FCFE	2.205 ÷ 0.04	**$55.13**
Value from dividends	from Example 1	**$47.25**
Dividends as share of FCFE	36 ÷ 42	**86%**
Gap between the two	55.13 − 47.25	**$7.88**

IN THE LESSON

- Now value the same share two ways, with r at nine per cent and growth at five per cent.
- First the F C F E way.
- Next year's F C F E per share is two dollars ten times one point zero five, which is two dollars twenty and a half cents.
- Divide by nine per cent minus five per cent and you get fifty-five dollars thirteen.
- The dividend way gave forty-seven dollars twenty-five in Example one.
- The gap is about eight dollars.
- Why?
- Larkspur paid out thirty-six of a possible forty-two, which is eighty-six per cent.
- The rest stayed in the business.
- If you can only collect the dividend, the dividend model fits.
- If a buyer could pay out the full capacity, F C F E is the better guide.

22 Four models: what each one discounts

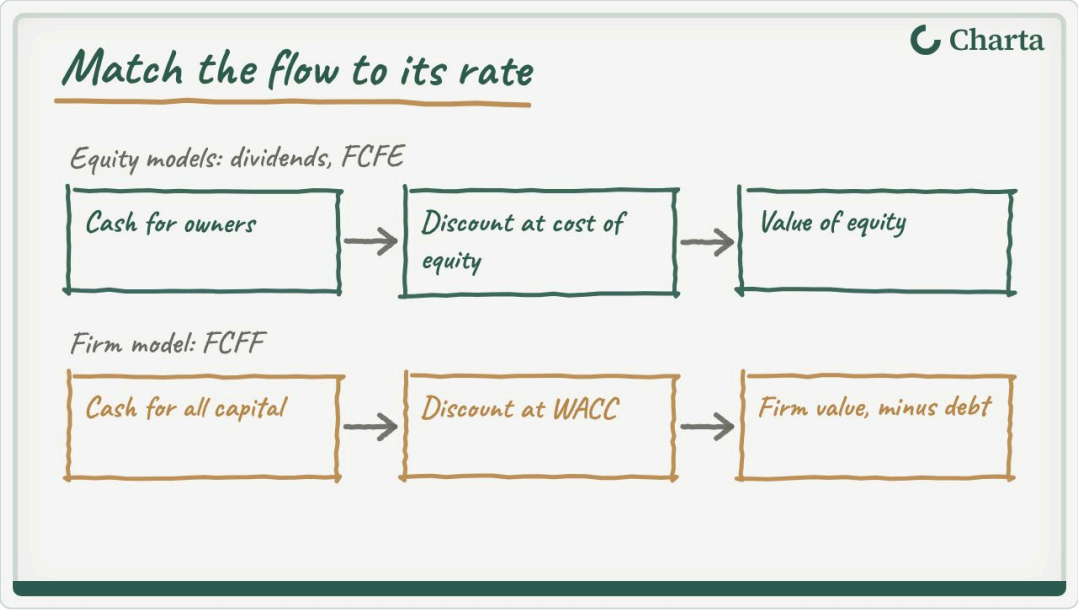
Which cash flow, at which rate

Model and what it discounts	Rate
DDM: dividends	Cost of equity
FCFE: cash for owners	Cost of equity
FCFF: cash for all capital	WACC
RI: earnings above the charge	Cost of equity

IN THE LESSON

- Now we line up four models side by side.
- Each differs in two things, which cash flow it discounts and at what rate.
- The dividend discount model uses dividends, at the cost of equity.
- The F C F E model uses free cash flow to equity, also at the cost of equity.
- The F C F F model uses free cash flow to the firm, which is cash available to all the capital providers, lenders and owners together.
- Because it belongs to both groups, it is discounted at the weighted average cost of capital, the W A C C.
- Residual income starts from book value and adds the present value of earnings above what owners require, again at the cost of equity.
- Which fits?
- Dividends for steady payers, F C F E for cash generators with low payouts, F C F F when debt is changing, and residual income when cash flows are negative or erratic.

23 Match the flow to its rate



IN THE LESSON

- The rule is simple.
- The rate must match the people who own the cash flow.
- In the top row, the flow belongs to owners alone.
- Dividends and F C F E are owners' cash, so you discount at the cost of equity, which is what owners require, and the answer is the value of the equity.
- In the bottom row, F C F F belongs to lenders and owners together.
- So you discount at the W A C C, a blend of what each group requires.
- The answer is the value of the whole firm, and only then do you subtract debt to reach equity.
- Mismatch them and you still get a number, just not a value.
- F C F E at the W A C C uses too low a rate and overstates the value.
- F C F F at the cost of equity understates it.

24 Free cash flow to the firm

FCFF = CFO + Int × (1 − t) − FCInv

Cash flow from operations	Cash earned after interest, tax
Interest after tax	Interest paid, less its tax saving
Fixed capital investment	Spending on plant and equipment
Free cash flow to the firm	Cash for lenders and owners

IN THE LESSON

- Here is free cash flow to the firm.
- F C F F equals cash flow from operations, plus interest after tax, minus fixed capital investment.
- Why add interest back?
- Cash flow from operations has had interest taken out, but interest is paid to lenders, and F C F F is the cash for lenders as well as owners.
- So we put it back.
- We add it after tax, because interest saves tax: the firm pays the interest, but its tax bill falls.
- The after tax interest is interest times one minus the tax rate.
- There is a handy bridge between the two flows.
- Take F C F F, subtract the after tax interest, and add net borrowing.
- That gives F C F E.

25 Example 8: from FCFF to the equity

Interest 12, tax 25%, WACC 8%, g = 5%, debt 200 ($ millions)

FCFF this year	98 + 12 × 0.75 − 70	**37**
Firm value	37 × 1.05 ÷ 0.03	**1,295**
Less debt		**200**
Equity value	1,295 − 200	**1,095**
Equity value per share	1,095 ÷ 20 million	**$54.75**

IN THE LESSON

- Larkspur again, in millions.
- Interest expense is twelve, and the tax rate is twenty-five per cent.
- So the after tax interest is nine.
- F C F F is ninety-eight plus nine minus seventy, which is thirty-seven.
- Check the bridge: thirty-seven, minus nine, plus fourteen of net borrowing, is forty-two, our F C F E.
- Now suppose the W A C C is eight per cent and growth is five per cent.
- Next year's F C F F is thirty-seven times one point zero five.
- Divide by eight per cent minus five per cent and the firm is worth one thousand two hundred and ninety-five.
- Debt is two hundred, so the equity is one thousand and ninety-five, or fifty-four dollars seventy-five a share.
- Close to the F C F E answer, though not identical.

26 Residual income

RI = (ROE − r) × B0
V0 = B0 + present value of future RI

Return on equity, ROE	Profit earned on owners' money
Required return, r	Return you demand each year
Opening book value, B0	Book value at start of year
Residual income, RI	Profit above the equity charge

IN THE LESSON

- The last model works differently.
- It does not discount cash.
- It starts from the book value of equity, which is what owners have put in plus the profit kept, and then adds extra value for earning more than owners require.
- Owners require r on their opening book value, B zero.
- That is the equity charge.
- If the firm earns more than the charge, the excess is residual income.
- A neat way to write it is R O E minus r, times opening book value.
- Note that opening figure; the usual convention is the book value at the start of the year.
- Value today is book value plus the present value of all future residual income.
- If the firm earns exactly what is required, residual income is zero and the share is worth its book value.

27 Example 9: residual income for Larkspur

Per share: ROE 12.5%, r = 9%, g = 5%, earnings next year $3.15

Opening book value, B0	3.15 ÷ 0.125	**$25.20**
Equity charge	0.09 × 25.20	**$2.27**
Residual income, RI	3.15 − 2.268	**$0.88**
PV of future RI	0.882 ÷ 0.04	**$22.05**
Value today, V0	25.20 + 22.05	**$47.25**

IN THE LESSON

- Now Larkspur per share.
- Earnings next year are three dollars fifteen, and R O E is twelve and a half per cent.
- So opening book value is three fifteen over zero point one two five, which is twenty-five dollars twenty.
- The equity charge is nine per cent of twenty-five twenty, about two dollars twenty-seven.
- Residual income is three fifteen minus two twenty-seven, which is eighty-eight cents.
- Residual income grows at five per cent, because book value grows five per cent.
- Its present value is eighty-eight cents over four per cent, twenty-two dollars five cents.
- Add the book value of twenty-five twenty and the value is forty-seven dollars twenty-five.
- Exactly the Gordon answer.
- Different route, same story, when the inputs agree.

28 Preferred shares: a fixed promise

V0 = D ÷ r

Fixed dividend, D	Level dividend, paid every year
Required return, r	Return investors require on this preferred

IN THE LESSON

- Last, preferred shares.
- Picture a bond with no end date.
- A preferred share pays a fixed dividend, and it is paid before any dividend on common shares.
- It usually has no growth and no vote.
- Take a preferred that is neither callable nor convertible.
- It is a perpetuity, a level payment for ever, and its value is the dividend divided by the required return.
- The return to use is the one investors require on this preferred, not the common share's return.
- It is usually lower, because preferred holders are paid first.
- A preferred with a fixed dividend of six dollars, where investors require eight per cent, is worth seventy-five.
- Value moves opposite to the required return, much like a bond price.

29 Example 10: Larkspur's preferred share

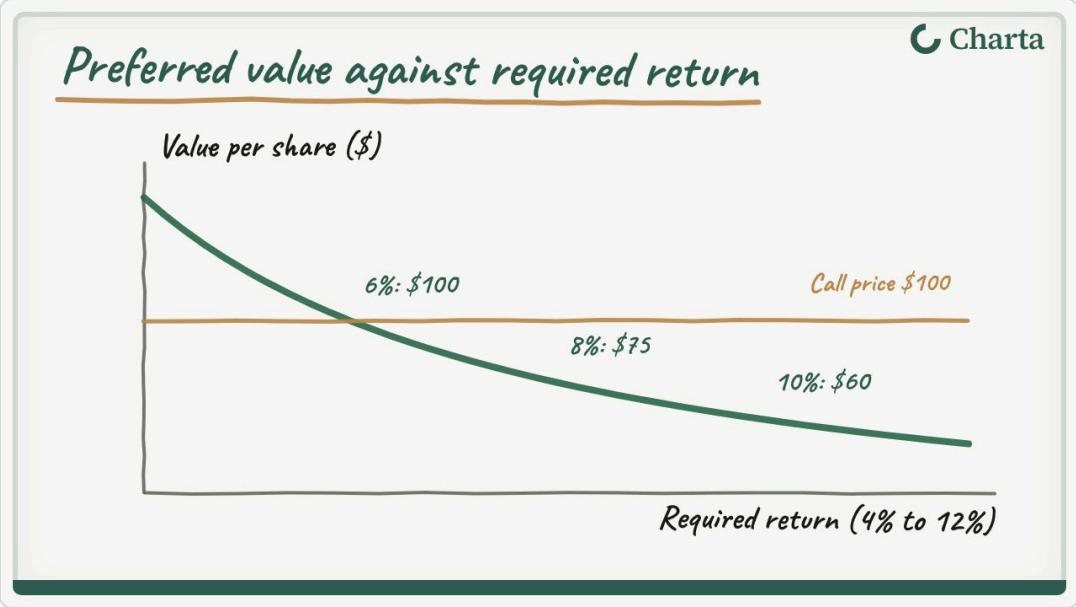
Preferred: $100 par, 6% dividend = $6 a year

Value at 8% required return	6 ÷ 0.08	**$75.00**
If the return rises to 10%	6 ÷ 0.10	**$60.00**
If it falls to 6%	6 ÷ 0.06	**$100.00**
Redeemable at $100 in 5 years	6 × 3.9927 + 100 ÷ 1.08^5	**$92.01**
PV of the $100 alone	100 ÷ 1.08^5	**$68.06**

IN THE LESSON

- Larkspur has a preferred share with a par value of one hundred dollars and a six per cent dividend, so it pays six dollars a year.
- At an eight per cent required return, it is worth six over zero point zero eight, which is seventy-five dollars.
- If the required return rises to ten per cent, it is worth sixty, a fall of twenty per cent.
- If it falls to six per cent, it is worth one hundred, a rise of thirty-three per cent.
- Now change the share: it will be redeemed at one hundred dollars in five years.
- That is valued like a bond.
- Six times the five year annuity factor of three point nine nine two seven, plus one hundred over one point zero eight to the fifth, gives ninety-two dollars one cent.

30 Preferred value against required return



IN THE LESSON

- Here is the picture.
- The required return runs along the bottom, from four to twelve per cent.
- Value per share runs up the side.
- The dividend never changes, six dollars, so the whole curve comes from the rate.
- It slopes down.
- At six per cent the value is one hundred dollars, at eight per cent seventy-five, at ten per cent sixty.
- See how the curve bends.
- It falls steeply at low rates and flattens at high ones.
- The flat gold line is a call price of one hundred dollars.
- If the issuer can call the share, then as rates fall and the value nears that line, the issuer can redeem it and refinance more cheaply.
- So a callable preferred is capped near its call price, and the holder loses the upside.

31 Features that move the value

Who holds the option decides the effect

Feature	Effect on value
Issuer call option	Lowers value
Holder put (retraction)	Raises value
Conversion into common	Raises value
Cumulative dividends	Raises value

IN THE LESSON

- Real preferred shares often come with extra features, and each shifts value toward whoever holds the option.
- A call option belongs to the issuer, who may redeem the share at a set price.
- That lowers the value, and caps the price when rates fall, as the picture showed.
- A put, sometimes called a retraction right, belongs to the holder, who can sell it back at a set price.
- That raises the value.
- A conversion right lets the holder swap into common shares if the common does well.
- That raises the value too.
- A cumulative feature means missed dividends must be paid before any common dividend.
- That also adds value.
- And a fixed redemption date makes the preferred value like a bond, as we saw.
- The simple D over r formula is for the plain case only.

32 Recap: the formulas

V0 = D1 ÷ (r − g) g = b × ROE
FCFE = CFO − FCInv + net borrowing RI = (ROE − r) × B0
FCFF = CFO + Int × (1 − t) − FCInv Preferred: V0 = D ÷ r

Gordon growth	Next dividend over r minus g
Growth from within	Keep a share, earn ROE on it
Terminal value	Terminal value: next D ÷ (r − g)
Matching	Owners: cost of equity; firm: WACC
Preferred share	Level dividend ÷ required return

IN THE LESSON

- Let us recap.
- The Gordon model values a share as next year's dividend over r minus g.
- Growth comes from within as retention times return on equity, and it must stay below r.
- A multi-stage model adds the present value of the forecast dividends to the present value of a terminal value, which is usually most of the answer.
- F C F E is cash flow from operations, minus fixed capital investment, plus net borrowing, discounted at the cost of equity.
- F C F F adds back interest after tax, is discounted at the W A C C, and debt is subtracted at the end.
- Residual income is R O E minus r, times opening book value.
- And a plain preferred is its dividend over the required return.
- Match each cash flow to its rate.

33 Try three quick questions

1. D1 = $2.10, r = 10%, g = 4%. What is the value of the share?
   Answer: 2.10 ÷ 0.06 = $35.00
2. CFO $80m, FCInv $50m, net borrowing −$10m. What is FCFE?
   Answer: 80 − 50 − 10 = $20m
3. A plain preferred pays $4 a year; investors require 8%. Its value?
   Answer: 4 ÷ 0.08 = $50

IN THE LESSON

- Now try three quick questions.
- Pause after each.
- Question one.
- Next year's dividend is two dollars ten, the required return is ten per cent, and growth is four per cent.
- What is the value of the share?
- Answer.
- Two ten divided by zero point zero six is thirty-five dollars.
- Question two.
- Cash flow from operations is eighty million, fixed capital investment is fifty million, and net borrowing is minus ten million.
- What is free cash flow to equity?
- Answer.
- Eighty minus fifty minus ten is twenty million.
- Question three.
- A plain preferred share pays four dollars a year, and investors require eight per cent.
- What is its value?
- Answer.
- Four divided by zero point zero eight is fifty dollars.
- Well done.