CHARTA LEARNING · LESSON NOTES

Bond valuation: prices, yields and matrix pricing

Fixed Income

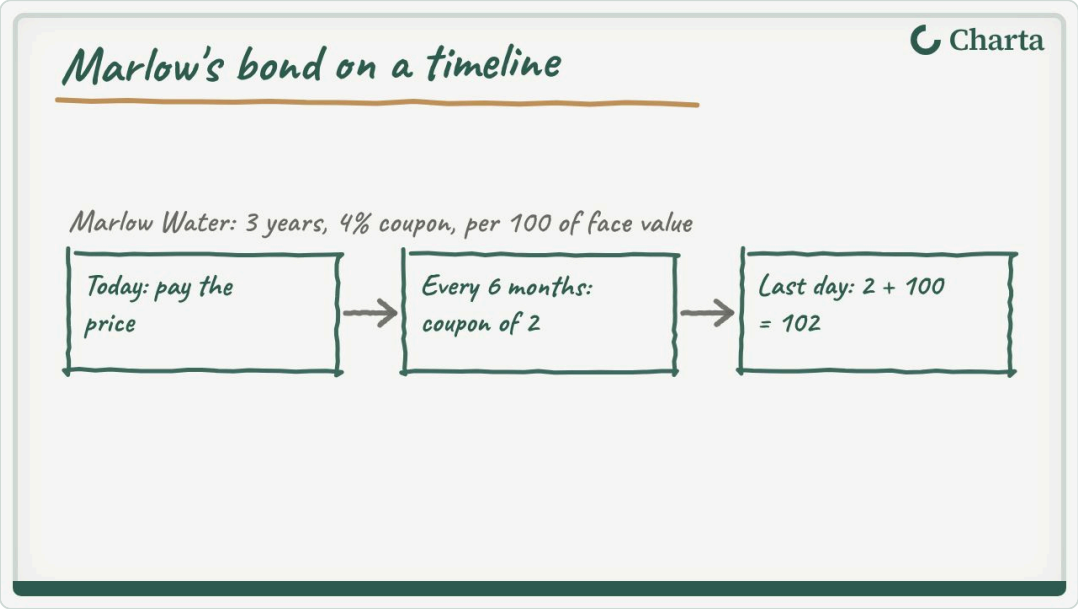
1 A story: a bundle of promises

- A bond is a loan you can buy and sell
- Promises: coupons, then the face value (par)
- Yield = the return buyers demand
- Price = present value of the promises

IN THE LESSON

- Picture a friend who lends a neighbour some money.
- The neighbour promises small payments every six months, and the whole sum back at the end.
- Now your friend wants to sell that loan to you.
- What would you pay?
- You would add up the promises, but count later ones for less, because money in your hand today is worth more.
- A bond is exactly this loan, packaged so that it can be traded.
- The borrower, here a firm called Marlow Water, is the issuer.
- The amount repaid at the end is the face value, also called par.
- The regular payments are coupons.
- The return buyers demand is the yield.
- A bond's price is the present value of its promised payments, discounted at that yield.

2 Marlow's bond on a timeline



IN THE LESSON

- Here is Marlow Water's bond drawn on a timeline.
- It lasts three years.
- Its face value is one hundred, and prices in this lesson are always quoted per one hundred of face value.
- Its coupon rate is four per cent a year, but coupons are paid twice a year, so each coupon is half of four, which is two.
- Coupons come every six months in this lesson, so three years make six periods.
- Read the boxes from left to right.
- Today you pay the price, which we are about to find.
- Every six months you receive two.
- On the last day you receive the final coupon of two together with the face value of one hundred, a payment of one hundred and two.

3 The bond price formula

PV = PMT/(1+r) + PMT/(1+r)² + … + (PMT + FV)/(1+r)^N

Bond price, PV	What you pay today
Coupon, PMT	Coupon per period
Face value, FV	Repaid at the end
Yield per period, r	Annual yield ÷ 2
Periods, N	Years × 2

IN THE LESSON

- Now the formula.
- The price is the present value of every payment.
- The coupon is the regular payment, written P M T.
- The face value, written F V, comes back with the last coupon.
- The letter N is the number of periods, here six half-years.
- And the letter r is the yield for one period.
- A yield is quoted per year, so with coupons twice a year you divide it by two: five per cent a year is two point five per cent a period.
- Each payment is divided by one plus r, raised to the number of periods it waits.

4 Example 1: set-up

3 years, 4% coupon, face value 100, buyers want a 5% yield

Coupon each period	4% of 100 ÷ 2	**2**
Yield each period	5% ÷ 2	**2.5%**
Number of periods	3 years × 2	**6**
Price per 100	what we want	**?**

IN THE LESSON

- Example one.
- Marlow Water's bond has a face value of one hundred, a coupon rate of four per cent, and three years to go.
- Buyers in the market want a yield of five per cent.
- What should the bond cost?
- First, convert everything to six-month periods.
- Half of the four per cent coupon on one hundred gives a coupon of two each period.
- Half of the five per cent yield gives two point five per cent per period, which is zero point zero two five as a decimal.
- Three years are six periods.
- The yield is higher than the coupon rate, so expect a price below one hundred.

5 Example 1: payment by payment

Each payment divided by 1.025 for every period it waits

Period 1: 2 ÷ 1.025		**1.95**
Period 2: 2 ÷ 1.025²		**1.90**
Periods 3 to 5	three more coupons	**5.44**
Period 6: 102 ÷ 1.025⁶	coupon + face value	**87.95**
Price per 100	sum, before rounding	**97.25**

IN THE LESSON

- Take the payments one by one.
- The first coupon of two is divided by one point oh two five, and is worth one point nine five today.
- The second coupon is divided by one point oh two five squared, worth one point nine.
- The third, fourth and fifth coupons together are worth five point four four.
- The last payment is the final coupon plus the face value, one hundred and two, divided by one point oh two five to the power six.
- That is worth eighty-seven point nine five.
- Add the pieces, keeping the unrounded figures behind them, and the price is ninety-seven point two five per one hundred of face value.

6 Example 1: the shortcut

Coupons as an annuity, plus the face value as a lump sum

PV of the six coupons	2 × 5.5081	**11.02**
PV of the face value	100 ÷ 1.025^{6}	**86.23**
Price per 100	11.02 + 86.23	**97.25**
Cash received in total	6 × 2 + 100	**112**

IN THE LESSON

- Adding six separate pieces is slow, so use the shortcut.
- The six coupons are an annuity, a level payment each period, which you met in the time value of money.
- Its present value is the coupon times one minus one point oh two five to the minus six, divided by point oh two five.
- That is two times five point five zero eight one, which is eleven point zero two.
- The face value is a single lump sum: one hundred divided by one point oh two five to the six, which is eighty-six point two three.
- The two parts add to ninety-seven point two five, the same answer.
- Look at the split.
- Almost nine tenths of the price comes from the face value at the end.
- Total cash received is one hundred and twelve, far above the price.

7 Example 2: on the calculator

Keys for the same bond, one entry for each piece

N	periods, not years	**6**
I/Y	per period, in percent	**2.5**
PMT	coupon per period	**2**
FV	face value	**100**
CPT PV	negative = cash paid	**−97.25**

IN THE LESSON

- In practice you will use a financial calculator, so let us enter the same bond.
- Press N and enter six, the number of periods.
- Press I Y and enter two point five, the yield per period as a percentage, not as a decimal.
- Press P M T and enter two.
- Press F V and enter one hundred.
- Now press C P T, then P V.
- The calculator shows minus ninety-seven point two five.
- The minus sign is not an error.
- It is the calculator's way of saying this is cash you pay out.
- The common mistake is keying the annual numbers.
- Always enter the rate, the count and the coupon per period.

8 A story: why a bond is not always worth one hundred

- Coupon rate: fixed in advance
- Yield: what the market demands today
- Yield above coupon rate: discount (below 100)
- Yield below coupon rate: premium (above 100)

IN THE LESSON

- Suppose Marlow Water sold its bond a year ago when buyers wanted four per cent, and the bond pays a coupon rate of four per cent.
- It was worth exactly one hundred.
- Now new bonds yield five per cent.
- Would you pay one hundred for Marlow's four per cent bond?
- No.
- The only way to tempt you is a lower price.
- This is the heart of bond pricing.
- The coupon rate is fixed in advance.
- The yield is whatever the market demands today.
- When they are equal, the bond sells at par.
- When the yield is above the coupon rate, it sells at a discount.
- When below, at a premium.

9 Marlow's bond at four different yields

Three years, 4% coupon, per 100 of face value

Required yield	Price per 100
3%: premium	102.85
4%: par	100.00
5%: discount	97.25
6%: discount	94.58

IN THE LESSON

- Let us put numbers on it.
- Same bond: three years, a coupon of four per cent, priced per one hundred of face value.
- Each row changes only the yield.
- At a yield of three per cent, below the coupon rate, the price is one hundred and two point eight five, a premium.
- At four per cent, where yield equals coupon rate, the price is exactly one hundred, par.
- At five per cent, the price is ninety-seven point two five, our example one.
- At six per cent, ninety-four point five eight.
- Read down the column: as the yield goes up, the price goes down, every time.
- Price and yield move in opposite directions, because the coupons are fixed and a higher discount rate shrinks their present value.

10 Example 3: a premium and par

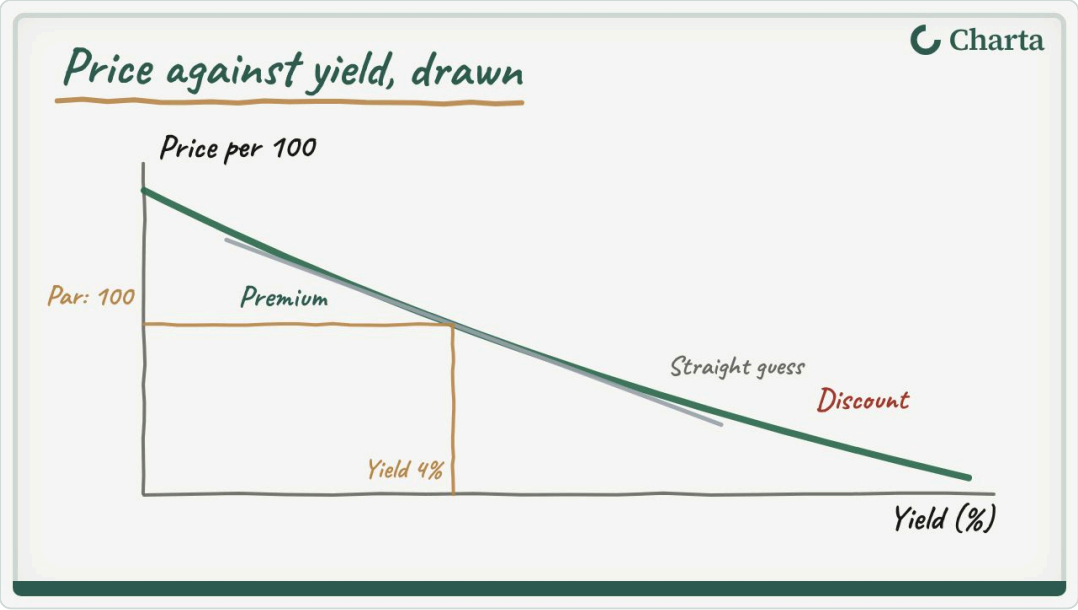
Same bond, yield falls to 3%: 1.5% per period

PV of the coupons	2 × 5.6972	**11.39**
PV of the face value	100 ÷ 1.015^{6}	**91.45**
Price at a 3% yield	sum, before rounding	**102.85**
Premium over par	102.85 − 100	**2.85**
Price at a 4% yield	coupon rate = yield	**100.00**

IN THE LESSON

- Example three, the premium case.
- The yield falls to three per cent, so the rate per period is one point five per cent.
- The six coupons of two are worth two times five point six nine seven two, which is eleven point three nine.
- The face value of one hundred, divided by one point oh one five to the six, is worth ninety-one point four five.
- The price is the sum, one hundred and two point eight five, allowing for rounding.
- That is two point eight five above par, a premium.
- Now the par case.
- If the yield equals the coupon rate, four per cent, each period is two per cent, and the price comes out at exactly one hundred.

11 Price against yield, drawn



IN THE LESSON

- Now the picture, for a longer bond so the bend is easy to see.
- Yield runs along the bottom, price up the side.
- The green curve is the price of a ten-year bond with a four per cent coupon.
- At low yields the price is high.
- As the yield rises the price falls.
- The gold lines mark par: at a yield of four per cent, equal to the coupon rate, the price is one hundred.
- To the left of that point the bond sells at a premium, to the right at a discount.
- Now look at the grey straight line, which touches the curve at par.
- The curve lies above it on both sides.
- The price does not fall in a straight line.
- It bends.

12 Example 4: gains bigger than losses

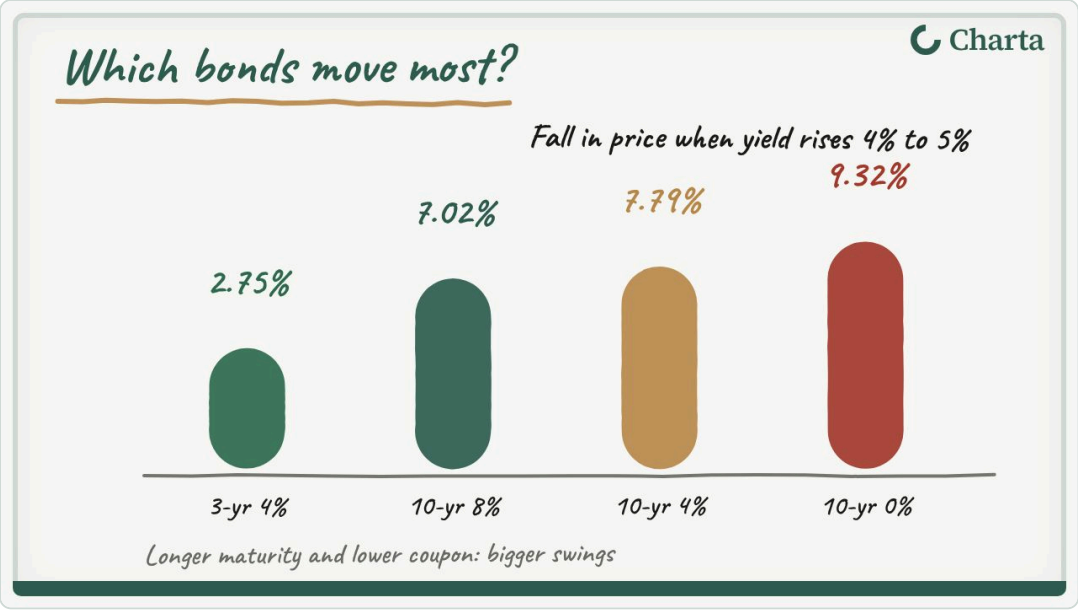
Ten-year bond, 4% coupon: move the yield one point each way

Price at a 4% yield	coupon rate = yield	**100.00**
Yield up to 5%	price falls	**92.21**
Fall in price	100 − 92.21	**−7.79**
Yield down to 3%	price rises	**108.58**
Rise in price	108.58 − 100	**+8.58**

IN THE LESSON

- Example four uses that ten-year bond with a four per cent coupon.
- At a four per cent yield it sells at one hundred.
- Now let the yield rise by one point to five per cent.
- The price drops to ninety-two point two one, a fall of seven point seven nine.
- Now run it the other way.
- Let the yield fall by one point to three per cent.
- The price climbs to one hundred and eight point five eight, a rise of eight point five eight.
- The rise is bigger than the fall, although the yield moved by the same amount.
- That is the bend in the curve.
- For the holder, it is good news: a bond gains more when yields fall than it loses when they rise.
- Later this bend is called convexity.

13 Which bonds move most?



IN THE LESSON

- Not every bond reacts to yields in the same way.
- Each bar shows the percentage fall in price when the yield rises from four to five per cent.
- On the left, a three-year bond with a four per cent coupon falls two point seven five per cent.
- Next, a ten-year bond with an eight per cent coupon falls seven point zero two.
- Then the ten-year bond with a four per cent coupon falls seven point seven nine.
- And a ten-year zero coupon bond, which pays nothing until the end, falls nine point three two.
- Two lessons.
- Longer maturity means a bigger swing, because discounting bites harder on distant money.
- Lower coupon means a bigger swing, because more value sits in the distant face value.

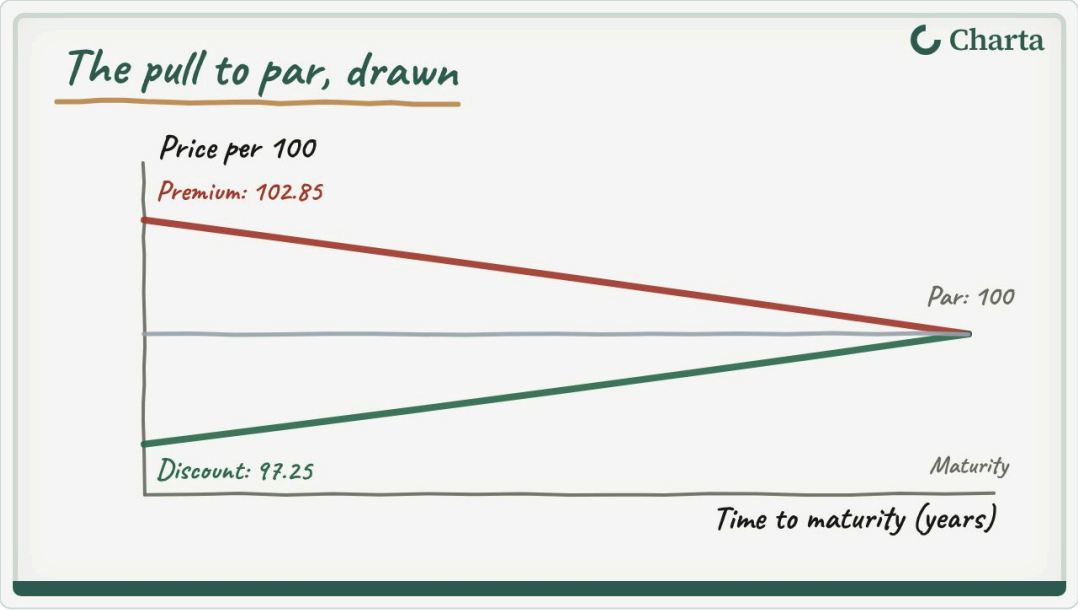
14 A story: the ticket that must end at face value

- Maturity: the issuer repays exactly par
- Premium bond: price drifts down to par
- Discount bond: price drifts up to par
- Even if the yield never changes

IN THE LESSON

- Think of a concert ticket with a face value of one hundred.
- Today it sells for one hundred and three, because the seats are good.
- Everyone knows that on the night the venue will only honour one hundred.
- So as the date nears, the price has to drift down.
- A bond is the same.
- At maturity the issuer repays exactly the face value, no more.
- So a bond trading above par must fall towards par, and a bond below par must rise towards par.
- This is called the pull to par.
- It happens even if the yield never moves.

15 The pull to par, drawn



IN THE LESSON

- Here is the pull to par as a picture.
- Time runs along the bottom, from today to maturity in three years.
- Price is up the side.
- The grey line is par, one hundred.
- The red curve is the same Marlow bond when buyers want three per cent, so it begins above par at a premium.
- The green curve is the bond when buyers want five per cent, so it begins below par at a discount.
- In both cases the yield never changes.
- Now follow them to the right.
- The red curve slides down, the green one climbs, and they close in on par together.
- On the last day, both are exactly one hundred.

16 Example 5: a discount bond, six months later

Yield stays at 5%; one coupon has been paid, 5 periods remain

Price now, 6 periods left	5% yield	**97.25**
Price in 6 months, 5 left	5% yield	**97.68**
Price gain	97.68 − 97.25	**+0.43**
Coupon received		**2.00**
Return over the half-year	2.43 ÷ 97.25	**2.50%**

IN THE LESSON

- Example five follows the discount bond, bought at ninety-seven point two five when the yield is five per cent.
- Six months pass and the first coupon of two is paid.
- The yield is still five per cent, but now five periods remain.
- Discount the remaining payments, and the price is ninety-seven point six eight.
- The price has risen by zero point four three, a pull towards par.
- Now add up what the investor earned in the half-year: the coupon of two plus the price gain of zero point four three is two point four three.
- Divide by the starting price of ninety-seven point two five, and the return is two point five per cent.
- That equals the yield per period, and it beats the coupon rate of two per cent a period, thanks to the price gain.

17 Example 6: a premium bond, six months later

Yield stays at 3%; one coupon has been paid, 5 periods remain

Price now, 6 periods left	3% yield	**102.85**
Price in 6 months, 5 left	3% yield	**102.39**
Price change	102.39 − 102.85	**−0.46**
Coupon received		**2.00**
Return over the half-year	1.54 ÷ 102.85	**1.50%**

IN THE LESSON

- Example six is the premium bond, bought at one hundred and two point eight five when the yield is three per cent.
- Six months later, with the yield unchanged and five periods left, the price is one hundred and two point three nine.
- The price has fallen by zero point four six.
- The investor received the coupon of two but lost zero point four six of price.
- Net, that is one point five four.
- Divide by one hundred and two point eight five, and the return is one point five per cent for the half-year.
- That is the yield per period again, three per cent a year, and it is less than the bond's coupon rate of two per cent per period.
- So a premium bond earns less than its coupon rate suggests, and a discount bond earns more.

18 Four mistakes to avoid so far

- 1. Annual yield with periodic coupons: divide by two
- 2. Judging by coupon alone, not coupon versus yield
- 3. Forgetting the bend: falls gain more than rises lose
- 4. Pull to par is not a loss, if yield is unchanged

IN THE LESSON

- Here are the traps that cost marks.
- One.
- Mixing years and periods.
- With semiannual coupons, divide the yield by two and double the years, and use the coupon per period.
- Two.
- Thinking a higher coupon always means a higher price.
- What matters is the coupon compared with the yield.
- Three.
- Forgetting the bend: a fall in yield lifts the price more than a rise lowers it.
- Four.
- Reading the pull to par as a loss: the premium bond paid a bigger coupon in the meantime.
- And it assumes the yield and credit quality stay the same.

19 A story: the rent paid halfway through the month

- Coupons are paid on set dates; interest builds daily
- Accrued interest = the seller's share of the coupon
- Flat (clean) price: quoted, without accrued
- Full (dirty) price: paid, with accrued

IN THE LESSON

- Imagine you rent out a flat, and the tenant pays rent on the first of each month.
- On the fifteenth you sell the flat to a new owner.
- The new owner will collect a whole month's rent on the first, but half of that month belonged to you, because you were the owner for those days.
- Bonds work the same way: interest builds up every day, though the coupon is paid on set dates.
- The part of the coupon the seller has earned is accrued interest.
- The price quoted on screens, which leaves it out, is the flat price, also called the clean price.
- The price the buyer really pays, with it added, is the full price, also called the dirty or invoice price.

20 Full price, flat price and accrued interest

Full = Flat + AI
AI = PMT × t ÷ T

Full price	Cheque the buyer writes
Flat price	The quoted price
Accrued interest	Seller's share of the coupon
Days since coupon	Count since the last coupon
Days in period	Whole coupon period, 180

IN THE LESSON

- Two short formulas.
- First, the full price equals the flat price plus accrued interest.
- The flat price is the quote you see.
- Accrued interest is added on top, and the total is the cheque the buyer writes.
- Second, accrued interest equals the coupon for the period times the share of the period that has passed.
- We count the days since the last coupon, and divide by the days in the coupon period.
- In this lesson we use the thirty over three hundred and sixty day count, which treats every month as thirty days, so a half-year is one hundred and eighty days.
- Government bonds often use actual days.
- Accrued interest grows in a straight line from zero just after a coupon to nearly the whole coupon just before the next.

21 Example 7: accrued interest

Coupons 15 March and 15 September; bought 15 June; 30/360

Coupon per period	4% ÷ 2 of 100	**2.00**
Days since last coupon	March to June, 30/360	**90**
Days in the coupon period	6 × 30	**180**
Accrued interest	2 × 90 ÷ 180	**1.00**

IN THE LESSON

- Example seven.
- Marlow's bond pays its coupon of two on the fifteenth of March and the fifteenth of September.
- You buy it on the fifteenth of June.
- Using the thirty over three hundred and sixty convention, March to June is three months of thirty days, which is ninety days.
- The coupon period is one hundred and eighty days.
- So the share of the coupon that the seller has earned is ninety over one hundred and eighty, one half.
- Accrued interest is two times one half, which is one.
- You pay the seller this on top of the quoted price, and on the fifteenth of September you receive the whole coupon.

22 Example 8: from the full price to the flat price

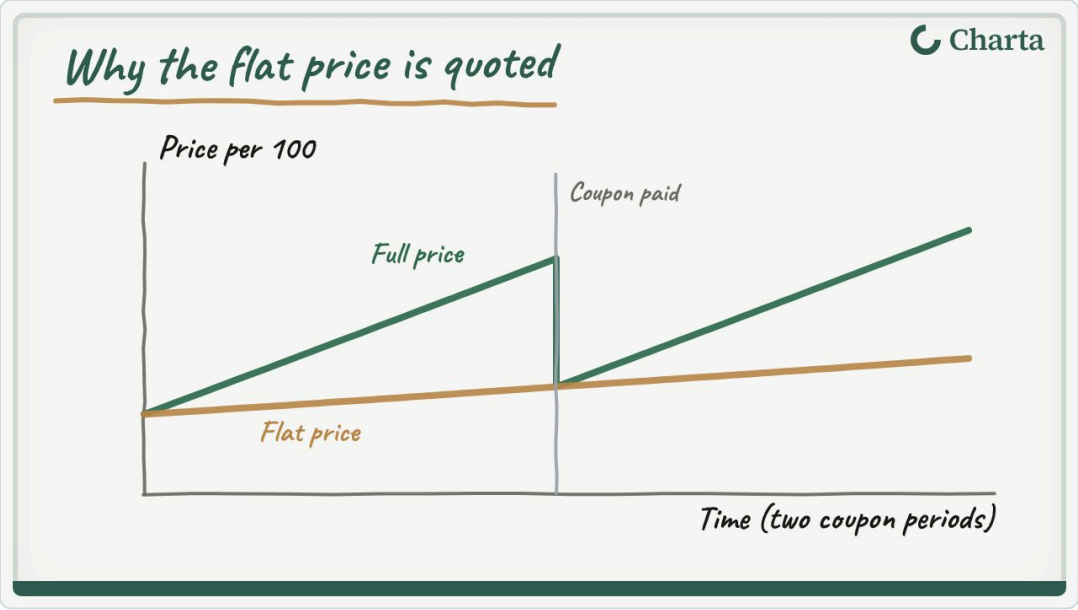
Yield 5%; bought 15 June, half a coupon period after the coupon

Price at the 15 March coupon	6 periods left	**97.25**
Full price on 15 June	price × 1.025^0.5	**98.45**
Less accrued interest	from example 7	**−1.00**
Flat price: the quote	98.45 − 1.00	**97.45**

IN THE LESSON

- Now find the price.
- On the fifteenth of March, just after a coupon, the bond had six periods to go and was worth ninety-seven point two five at a five per cent yield, as we found in example one.
- Three months later, half a period has passed and every payment is half a period nearer.
- So the full price is that price times one point oh two five to the power one half, which comes to ninety-eight point four five, working from the unrounded price.
- You can check this by discounting the six remaining payments.
- The first arrives in half a period, and the answer is the same.
- Now the flat price: take away the accrued interest of one, and the flat price is ninety-seven point four five.
- The screen shows ninety-seven point four five, the buyer pays ninety-eight point four five.

23 Why the flat price is quoted



IN THE LESSON

- Here is why screens quote the flat price.
- Time runs along the bottom, across two coupon periods, with the coupon date in the middle.
- Price is up the side.
- The green line is the full price, what a buyer really pays.
- Follow it.
- All through the first period it climbs, because accrued interest keeps building.
- Then the coupon is paid and, in one step, the full price drops by about the coupon.
- Nothing has gone wrong.
- The seller has simply been paid.
- The line then climbs again, a sawtooth.
- Now the gold line, the flat price.
- It strips the accrued interest out, so it has no sawtooth at all, just a gentle drift.
- It shows whether the market has really moved, so it is quoted.

24 Example 9: the day before and the day after the coupon

Same bond and yield; coupon paid on 15 September

14 Sep: accrued interest	179 days of 180	**1.99**
14 Sep: full price	flat price is 97.67	**99.66**
16 Sep: accrued interest	1 day of 180	**0.01**
16 Sep: full price	flat price is 97.68	**97.69**
Fall in the full price	99.66 − 97.69	**1.97**

IN THE LESSON

- Example nine shows the effect in numbers.
- Take the same bond, the same five per cent yield.
- On the fourteenth of September, one day before the coupon, one hundred and seventy-nine days have passed.
- Accrued interest is almost the whole coupon, one point nine nine, the full price is ninety-nine point six six, and the flat price is ninety-seven point six seven.
- Two days later, on the sixteenth of September, the coupon has been paid.
- Only one day has passed in the new period, so accrued interest is zero point zero one, the full price is ninety-seven point six nine, and the flat price is ninety-seven point six eight.
- The full price fell by one point nine seven in two days, but the flat price was ninety-seven point six seven on the first day and ninety-seven point six eight on the second, hardly any change.

25 A story: pricing a house nobody has sold

- Many bonds rarely trade: no recent price
- Use comparables: same credit quality and sector
- Estimate the missing yield from their yields
- Then discount the bond's own cash flows

IN THE LESSON

- How would you value a house on a street where nothing has sold for years?
- You would look at similar houses nearby that did sell, a little smaller and a little bigger than yours, and place yours between them.
- Bonds have the same problem.
- Many bonds, especially corporate and municipal ones, trade rarely.
- Matrix pricing solves this.
- You find comparable bonds that did trade, with the same credit quality and from the same sector.
- You look at their yields.
- You estimate the missing yield for your bond, and then you discount your bond's own cash flows at that estimated yield.
- The comparables sit in a grid of maturity against rating, hence the name.

26 The comparables on one grid

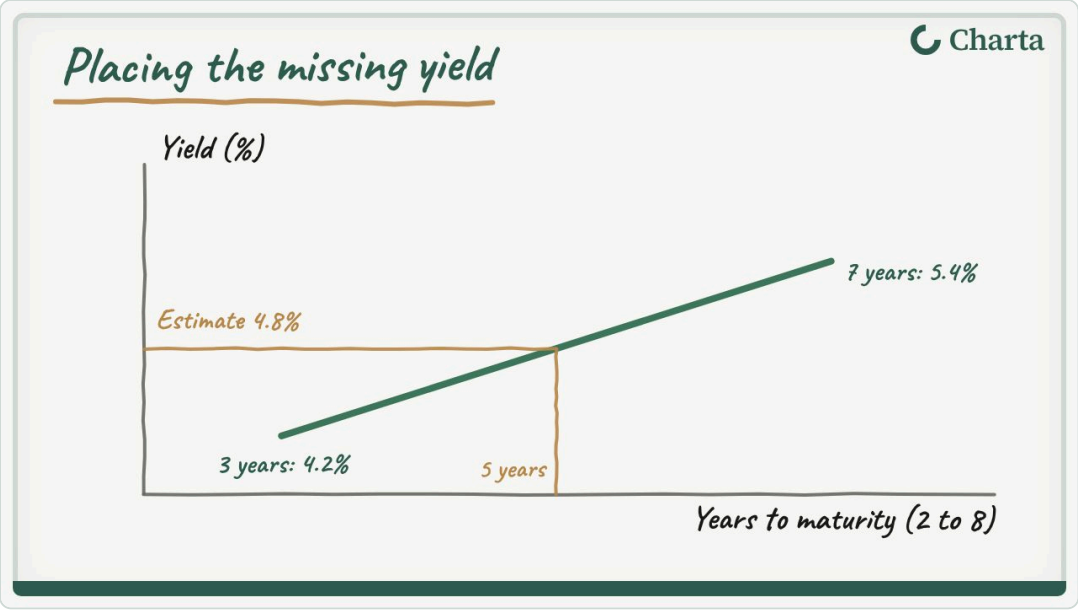
Triple B utilities, yields quoted per year, twice-a-year basis

Bond	Yield
BBB utility, 3 years: traded	4.2%
Marlow Water, 5 years: no trade	?
BBB utility, 7 years: traded	5.4%

IN THE LESSON

- Here is Marlow Water's problem.
- Its five-year bond, with a coupon of four point five per cent paid twice a year, has not traded for weeks.
- We look for bonds from similar firms with the same rating, say triple B, in the same utility sector.
- Two traded today.
- A three-year bond yields four point two per cent.
- A seven-year bond yields five point four per cent.
- Our bond, at five years, falls between them.
- We need a yield for the middle row, and that is what we will estimate.
- The choice of comparables matters a great deal: a different sector or rating gives a different answer.

27 Placing the missing yield



IN THE LESSON

- Let us draw it.
- Maturity in years runs along the bottom, yield up the side.
- Plot the two traded bonds first: the three-year bond at four point two per cent, and the seven-year bond at five point four.
- Now join them with a straight line.
- That line is our estimate of what yields look like for bonds in between.
- Mark five years on the bottom, go straight up to the line, and read across.
- The point lands at four point eight per cent.
- That is the estimated yield for Marlow's five-year bond.
- The straight line is an assumption: the real curve bends.

28 Straight-line interpolation

Yield = Ys + (x ÷ y) × (Yl − Ys)

Estimated yield	Yield for your bond
Shorter bond's yield	Yield, shorter maturity
Longer bond's yield	Yield, longer maturity
Years from short to target	Years: short to your bond
Years from short to long	Years: short to long bond

IN THE LESSON

- Here is the arithmetic behind that line, called linear interpolation.
- The estimated yield equals the shorter bond's yield, plus a share of the gap between the two yields.
- The share is how far along the maturity range your bond sits.
- Take each piece.
- The shorter bond's yield is the starting point.
- The longer bond's yield, minus the shorter yield, is the gap.
- The top of the fraction is the years from the short bond to your bond, and the bottom is the years from the short bond to the long bond.
- Use the same quoting basis for all yields.

29 Example 10: the yield for a five-year bond

Comparables: 3 years at 4.2%, 7 years at 5.4%; target 5 years

Weight: years along	(5 − 3) ÷ (7 − 3)	**0.5**
Gap between the yields	5.4% − 4.2%	**1.2%**
Share of the gap	0.5 × 1.2%	**0.6%**
Estimated yield	4.2% + 0.6%	**4.8%**

IN THE LESSON

- Example ten.
- The short bond is the three-year bond at four point two per cent.
- The long bond is the seven-year bond at five point four.
- Marlow's bond has five years.
- The years from short to target are five minus three, which is two.
- The years from short to long are seven minus three, which is four.
- So the fraction is two over four, one half.
- The gap between the yields is five point four minus four point two, which is one point two.
- Half of the gap is point six.
- Add it to the starting yield: four point two plus point six is four point eight per cent.
- Now we use it to discount.

30 Example 11: from the yield to a price

5 years, 4.5% coupon (2.25 each half-year), yield 4.8% (2.4% each)

Periods and rate per period	5 × 2; 4.8% ÷ 2	**10; 2.4%**
PV of the ten coupons	2.25 × 8.7975	**19.79**
PV of the face value	$100 \div 1.024^{10}$	**78.89**
Estimated price per 100	sum of the two	**98.68**

IN THE LESSON

- Now the price.
- Marlow's bond pays four point five per cent a year, twice a year, so each coupon is two point two five.
- Five years give ten periods.
- The estimated yield of four point eight per cent a year is two point four per cent for each period.
- The ten coupons of two point two five are worth two point two five times eight point seven nine seven five, which is nineteen point seven nine.
- The face value of one hundred, divided by one point oh two four to the power ten, is worth seventy-eight point eight nine.
- The sum is ninety-eight point six eight.
- The coupon rate is below the estimated yield, so a price below par makes sense.
- Two traded yields have become a price for a bond that did not trade.

31 Where matrix pricing helps, and where it misleads

- Use 1: value a bond that rarely trades
- Use 2: set the coupon on a new issue near par
- Risk: weak comparables (rating, sector, liquidity)
- Straight line ignores the bend: an estimate only

IN THE LESSON

- Matrix pricing has two main uses.
- The first is to value a thin bond, as we did.
- The second is to set the coupon on a new bond: if a five-year bond of this kind should yield four point eight per cent, then a coupon of four point eight per cent makes the new bond price at one hundred, close to par.
- Now the limits.
- Using the wrong comparables is the biggest risk, such as a different rating, sector or liquidity.
- A straight line ignores the bend of the curve.
- And it assumes issuers sharing a rating are interchangeable, which credit analysis exists to question.

32 Recap: the formulas

Price = PV of CFs at r
Full = Flat + AI
Yield = Ys + (x ÷ y)(Yl − Ys)

Bond price, PV	Coupons + face value, at r
Yield per period, r	Annual yield ÷ 2 if semiannual
Full price	Flat + accrued interest
Accrued interest	PMT × t ÷ T
Estimated yield	Interpolate between comparables

IN THE LESSON

- Let us pull it together.
- The price of a bond is the present value of its coupons and its face value, discounted at the yield per period, so halve the annual yield and double the years when coupons are twice a year.
- Price and yield move in opposite directions.
- A bond is at par when the coupon rate equals the yield, at a premium when the yield is below, and at a discount when above.
- With the yield unchanged, a premium falls to par and a discount rises to par.
- The full price is the flat price plus accrued interest, and the flat price is the one quoted.
- And matrix pricing interpolates a yield from comparable bonds, then discounts the cash flows at it.

33 Try three quick questions

1. Two years, 6% coupon twice a year, yield 8%. Premium or discount? Price?
   Answer: Discount. $3 \times 3.6299 + 100 \div 1.04^{4} = 96.37$
2. Flat price 98.50, coupon 4 a period, 45 of 180 days passed. Full price?
   Answer: Accrued = 4 × 45 ÷ 180 = 1.00. Full = 99.50
3. Comparables: 3.0% at 2 years, 4.2% at 6 years. Yield at 4 years?
   Answer: 3.0% + (2 ÷ 4) × 1.2% = 3.6%

IN THE LESSON

- Now try three quick questions.
- Pause after each one.
- Question one.
- A two-year bond pays a coupon of six per cent twice a year, and buyers want a yield of eight per cent.
- Is it a premium or a discount, and what is the price?
- The yield is above the coupon rate, so it is a discount.
- Each coupon is three, there are four periods, and the rate per period is four per cent, so the price is ninety-six point three seven.
- Question two.
- A bond has a flat price of ninety-eight point five.
- The coupon per period is four, and forty-five of a hundred and eighty days have passed.
- Accrued interest is four times a quarter, which is one, so the full price is ninety-nine point five.
- Question three.
- Comparable bonds yield three per cent at two years and four point two at six years.
- What is the yield at four years?
- Halfway, so three plus half of one point two: three point six per cent.