CHARTA LEARNING · LESSON NOTES

Is it worth it? NPV, IRR and returns

Quantitative Methods

1 A story: pay me now or pay me later?

- $100 today, or $110 in a year?
- Depends on what you could earn elsewhere
- Money later is worth less than money now
- Every idea here grows from that

IN THE LESSON

- Imagine a friend offers you a choice.
- They can give you one hundred dollars today, or one hundred and ten dollars in a year.
- Which would you take?
- If a savings account paid five per cent, your hundred would become one hundred and five, so waiting for one hundred and ten wins.
- But at twelve per cent, take the money now.
- Money later is worth less than money now, and how much less depends on the rate you could earn elsewhere.
- Everything here grows from that.

2 Net present value: what is the future worth today?

- Discounting: shrink future money back to today
- Present value = what a future payment is worth now
- Add them all up, then subtract the cost
- What is left = net present value (NPV)

IN THE LESSON

- Suppose a project costs money today and pays you back over the next few years.
- To judge it, we put all the money on the same footing: today.
- Shrinking a future payment back to today is called discounting.
- The amount it shrinks to is its present value.
- The rate we shrink it by is the required return, the minimum yearly return you would accept to take the risk.
- Here is the recipe.
- Shrink every future payment to today, add them up, then take away what the project costs.
- What is left is the net present value, or N P V.
- It is the extra value the project creates, measured in money, today.

3 The N P V formula

$$NPV = CF_0 + CF_1 \div (1 + r)^1 + CF_2 \div (1 + r)^2 + \dots$$

Cash flow	Money in (+) or out (−)
Required return	Your minimum yearly return
Time	Years until it happens

IN THE LESSON

- Here is the formula.
- N P V equals the sum of each cash flow divided by one plus r, raised to the power t.
- Let's break it down.
- A cash flow is money moving in or out.
- The cash flow at time zero is the cost today, and we write it as a negative number, because money leaves your pocket.
- The later ones are positive money coming back.
- The letter r is the required return as a decimal, so eight per cent is zero point zero eight.
- The letter t is how many years away that cash flow is.
- Dividing by one plus r, t times over, is the shrinking.
- Further away means more shrinking.
- Add everything, including the negative cost, and you get the N P V.
- If it is positive, accept.

4 Example 1: is this project worth doing? (set-up)

Required return 8% a year

Year 0: cost	money out	**−500**
Year 1: inflow	money in	**+200**
Year 2: inflow	money in	**+200**
Year 3: inflow	money in	**+200**
Question	is the NPV above zero?	**?**

IN THE LESSON

- Our first example.
- A project costs five hundred dollars today.
- It then pays two hundred dollars at the end of each of the next three years.
- You require eight per cent a year.
- Should you do it?
- Three payments of two hundred add up to six hundred.
- Six hundred back for five hundred spent looks like a profit of a hundred.
- But that adds money from different dates as if it were all the same.
- That is the trap.
- We will discount each payment back to today, one year at a time, and see what the project is really worth.

5 Example 1, step 1: discount each payment

Step 1: present value of each inflow at 8%

Year 1	200 ÷ 1.08	**185.19**
Year 2	200 ÷ 1.08^2	**171.47**
Year 3	200 ÷ 1.08^3	**158.77**
Total present value	sum, unrounded	**515.42**

IN THE LESSON

- Step one.
- Shrink each payment to today.
- Year one: two hundred divided by one point zero eight equals one hundred and eighty-five dollars nineteen cents.
- Year two: two hundred divided by one point zero eight squared.
- Squared, because it has been two years of shrinking, not one.
- That gives one hundred and seventy-one dollars forty-seven cents.
- Year three: two hundred divided by one point zero eight cubed, which is one hundred and fifty-eight dollars seventy-seven cents.
- Same size payments, each worth less than the one before.
- That is waiting at work.
- Added together, the three come to five hundred and fifteen dollars forty-two cents.
- We keep the unrounded sum, so it differs from the rounded parts by a penny.

6 Example 1, step 2: subtract the cost

Step 2: NPV = what it is worth − what it costs

Present value of inflows	from step 1	**515.42**
Cost today		**−500.00**
NPV	515.42 − 500	**+15.42**
Decision	NPV above zero	**Accept**

IN THE LESSON

- Step two.
- The inflows are worth five hundred and fifteen dollars forty-two cents in today's money.
- The project costs five hundred dollars today.
- Subtract: five hundred and fifteen forty-two minus five hundred leaves fifteen dollars forty-two cents.
- That is the N P V.
- It is positive, so the project clears your eight per cent bar and adds fifteen dollars forty-two cents of value.
- Adding the payments suggested a hundred dollars of profit.
- After accounting for waiting, only about fifteen is real.
- The rest was just the price of waiting.
- Decision rule: positive N P V, accept.
- Negative, reject.

7 The internal rate of return

- IRR = the rate that makes NPV exactly zero
- The project's own built-in yearly return
- IRR above required return: accept
- No tidy formula: trial and error or calculator

IN THE LESSON

- N P V tells you how much value a project adds at one particular rate.
- But sometimes you want to ask a different question: what rate does this project actually earn?
- That is the internal rate of return, or I R R.
- It is the discount rate that makes the N P V exactly zero.
- Think of it as the project's own built-in yearly return.
- The rule is simple.
- If the I R R is above your required return, accept.
- If it is below, reject.
- There is no tidy formula for it.
- You find it by trial and error, or with the I R R button on a financial calculator.

8 Example 2: find the I R R by trial

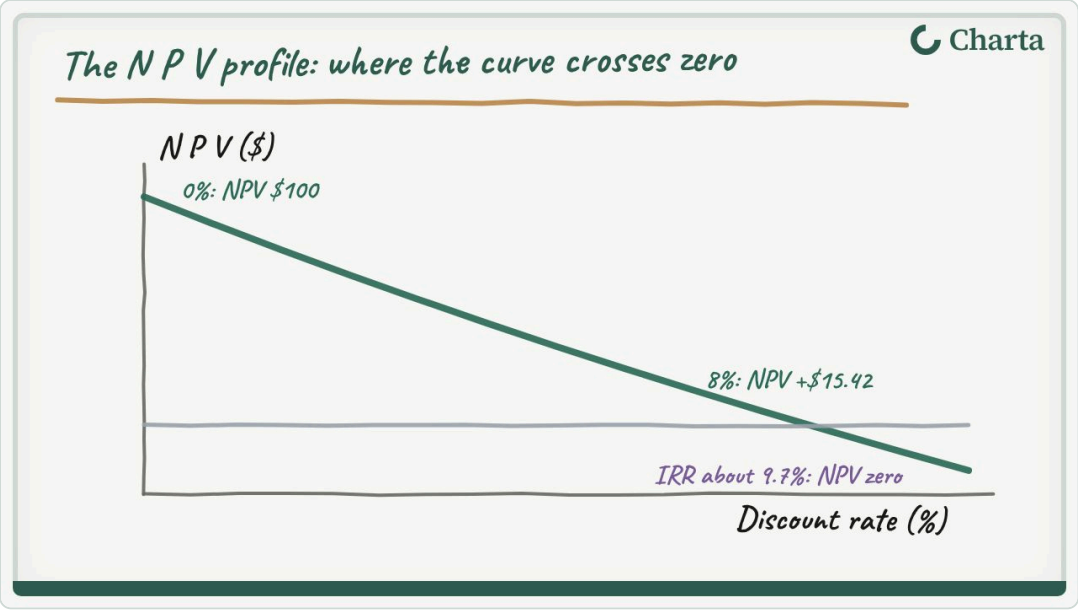
Same project: try different rates until NPV hits zero

NPV at 8%		**+15.42**
NPV at 9%		**+6.26**
NPV at 10%		**−2.63**
IRR (NPV = 0)	between 9% and 10%	**≈ 9.70%**
IRR vs required 8%	9.70 > 8	**Accept**

IN THE LESSON

- Example two uses the same project.
- The N P V is positive fifteen forty-two at eight per cent.
- Raise the rate to nine per cent and the inflows shrink more, so the N P V falls to six dollars twenty-six cents.
- Try ten per cent and it goes negative: minus two dollars sixty-three cents.
- So the N P V crosses zero somewhere between nine and ten per cent.
- Stepping in between, the crossing point is about nine point seven per cent.
- That is the I R R.
- Compare it with the eight per cent we require.
- Nine point seven is higher, so accept, which agrees with the positive N P V.
- The two answers agree, as they do for a single ordinary project.

9 The N P V profile: where the curve crosses zero



IN THE LESSON

- Here is the same project as a picture.
- The bottom axis is the discount rate, from zero up to twelve per cent.
- The side axis is the N P V in dollars.
- The curve slopes downward: the higher the rate, the more every future payment shrinks, so the N P V falls.
- At a rate of zero there is no shrinking at all, and the N P V is the full one hundred dollars of profit.
- At our eight per cent required return the N P V is fifteen dollars forty-two cents, above the line.
- The curve crosses the zero line at about nine point seven per cent.
- That crossing point is the I R R.
- Any required return to its left gives a positive N P V, and any to its right gives a negative one.

10 When N P V and I R R disagree

- Choosing one of two projects?
- NPV and IRR can rank them differently
- Follow NPV: it is measured in money
- IRR assumes reinvestment at the IRR itself

IN THE LESSON

- Here is a favourite point.
- Sometimes you must choose just one of two projects, and N P V and I R R rank them differently.
- One project might have a higher I R R, but be small.
- The other might have a lower I R R, but be much bigger.
- Which do you pick?
- Pick the one with the higher N P V.
- The reason is that N P V is measured in money, and money is what shareholders actually receive.
- There is a second reason.
- The I R R quietly assumes that cash received along the way can be reinvested at the I R R itself, which may be unrealistic.
- So when they conflict, follow N P V.

11 Holding period return: how much did you make?

- Total return over the time you held it
- Part 1: the change in price
- Part 2: income received (e.g. dividends)
- Leaving out income understates the return

IN THE LESSON

- Now a simpler question.
- You bought something, held it for a while, and now you want to know how well it did.
- The answer is the holding period return.
- It is the total return over whatever time you held it, whether that was a day, a year or a decade.
- And it counts two things.
- First, the change in price.
- Second, any income you received along the way, such as a dividend, which is a share of a company's profit paid to its owners.
- Many people forget the second part.
- A share whose price never moves but pays you five per cent a year still made you five per cent.

12 The holding period return formula

HPR = $(P_1 - P_0 + D_1) \div P_0$

Start value	What you paid at the start
End value	Price at the end
Income received	Dividends or interest paid

IN THE LESSON

- The formula is: holding period return equals end value, minus start value, plus income, all divided by start value.
- The start value is what you paid, written P zero.
- The end value is what it was worth when you sold, written P one.
- The income is any cash it paid you along the way, written D one.
- End minus start is the price gain, negative if the price fell.
- Adding the income gives your total gain in money.
- Dividing by the start value turns that gain into a percentage of what you put in.
- It says nothing about how long the period was.

13 Example 3: a share held for one year

Bought at $50, sold at $54, dividend $2

Price gain	54 − 50	**$4**
Plus dividend		**$2**
Total gain	4 + 2	**$6**
HPR	6 ÷ 50	**12%**

Price part 8% + dividend part 4% = 12%

IN THE LESSON

- Example three.
- You buy a share at fifty dollars.
- A year later you sell it for fifty-four.
- During the year it paid a dividend of two dollars.
- First, the price gain: fifty-four minus fifty is four dollars.
- Add the dividend of two, and your total gain is six dollars.
- Divide by the fifty you started with.
- Six divided by fifty is zero point one two, which is twelve per cent.
- Now split it.
- The price part is four over fifty, which is eight per cent.
- The dividend part is two over fifty, which is four per cent.
- A third of your return never showed up in the price.
- If you had looked only at the price, you would have said eight per cent, and been wrong.

14 Example 4: chaining several years together

Yearly returns +12%, −5%, +8%

Year 1	× 1.12	**+12%**
Year 2	× 0.95	**−5%**
Year 3	× 1.08	**+8%**
3-year return	1.12 × 0.95 × 1.08 − 1	**14.91%**
Steady yearly rate	1.14912^(1/3) − 1	**4.74%**

Adding gives 15%: close, but wrong

IN THE LESSON

- Example four.
- Suppose an investment returned twelve per cent in year one, then lost five per cent in year two, then gained eight per cent in year three.
- What was the return over the whole three years?
- The tempting answer is to add them, which gives fifteen.
- But that is wrong, because each year's return is earned on what the year before left you.
- So we multiply growth factors.
- Twelve per cent becomes one point one two.
- Minus five becomes zero point nine five.
- Plus eight becomes one point zero eight.
- Multiply: that gives one point one four nine one two.
- So the three-year return is fourteen point nine one per cent.
- The steady yearly return that gives the same result is the cube root of that, minus one: four point seven four per cent.

15 Two ways to measure a portfolio

- How good was the manager?
- How did I do, as the investor?
- Different questions, different answers
- Time-weighted vs money-weighted

IN THE LESSON

- Now for a puzzle.
- A fund manager looks after your money.
- Their investments do well in year one and badly in year two, and you added more money after the good year.
- At the end you ask two questions.
- How good was the manager?
- And how did I do?
- Surprisingly, these have different answers, because you decided when to put money in, not the manager.
- The time-weighted return judges the manager, by ignoring your deposits and withdrawals.
- The money-weighted return describes your own experience, because it cares about how much money you had invested, and when.

16 The two formulas

Time-weighted = $(1 + r_1) \times (1 + r_2) \times \ldots - 1$
Money-weighted = IRR of the client's cash flows

Sub-period return	HPR of each piece
Client's cash flows	Money in, money out, final value
Internal rate of return	Rate that makes their NPV zero

IN THE LESSON

- Here are the two measures.
- The time-weighted return cuts the period at every deposit or withdrawal.
- It takes the holding period return of each piece, adds one to each, multiplies them together, then takes one away.
- It is the chaining we just did.
- Because every piece is measured separately, your deposits do not matter.
- The money-weighted return is different.
- It is the internal rate of return of the client's own cash flows, which are the money they put in and the money they took out, plus the final value.
- Same I R R idea as before: the rate that makes the N P V of those flows zero.
- So deposits and their timing matter a great deal.

17 Example 5: the manager's view

Buy 1 share at $100. Year 1: $150, buy another. Year 2: $120

Year 1 return	150 ÷ 100 − 1	**+50%**
Year 2 return	120 ÷ 150 − 1	**−20%**
Time-weighted return	1.50 × 0.80 − 1	**20%**

IN THE LESSON

- Example five.
- You buy one share at one hundred dollars.
- A year later it has risen to one hundred and fifty, and you buy a second share at that price.
- In year two the price falls to one hundred and twenty, so your two shares are worth two hundred and forty dollars.
- First the time-weighted return, the manager's view.
- Year one: one hundred and fifty over one hundred, minus one, is fifty per cent.
- Year two: one hundred and twenty over one hundred and fifty, minus one, is minus twenty per cent.
- Chain them: one point five times zero point eight is one point two.
- So the time-weighted return is twenty per cent.
- Deposits played no part.
- On this measure, the manager did well.

18 Example 5, continued: the investor's view

The client's cash flows, found with a calculator

Today	first share bought	**−100**
End of year 1	second share bought	**−150**
End of year 2	holding is worth	**+240**
Money-weighted return	rate where NPV = 0	**−2.88%**
Gap to time-weighted	20 − (−2.88)	**22.88 pp**

IN THE LESSON

- Now the money-weighted return, your own experience.
- List your cash flows.
- At the start you paid one hundred, so minus one hundred.
- After one year you paid another one hundred and fifty, so minus one hundred and fifty.
- After two years your holding was worth two hundred and forty, so plus two hundred and forty.
- We need the rate that makes the N P V of these three flows zero.
- On a financial calculator, that rate is minus two point eight eight per cent.
- A loss!
- The manager earned twenty per cent, yet you lost money, because you bought the second share at the top.
- The gap is twenty-two point eight eight percentage points, all down to your timing.
- So judge a manager with the time-weighted return, since they did not choose your deposits.
- Describe the investor's experience with the money-weighted return.

19 Solving the pricing formulas backwards

Bond: Price = Face ÷ (1 + y)ⁿ → y = (Face ÷ Price)^(1/n) − 1
Stock: $P_0 = D_1 \div (r - g) \rightarrow g = r - D_1 \div P_0$

Market price	What the market charges today
Implied yield	Return the price promises
Next dividend	Dividend due next year
Required return	Return investors require
Implied growth	Steady yearly dividend growth

IN THE LESSON

- So far we picked a rate and found a price.
- Now flip it.
- We see the price, and ask what return it promises.
- Here are the two formulas.
- For a bond, a loan repaid with interest, with a single payment at the end, the price equals the face value divided by one plus the yield, raised to the number of years.
- Face value is what the bond repays at the end.
- Swap things around and the yield is face over price, to the power of one over the years, minus one.
- For a stock, the simplest model says the price equals next year's dividend divided by the required return minus the growth rate.
- Swap those around and the implied growth rate equals the required return, minus next year's dividend over the price.
- That last fraction is the dividend yield.
- A bond with several payments has no neat rearrangement, so we try yields until the price matches.

20 Example 6: the yield on a zero-coupon bond

Five-year zero-coupon bond, face $1,000, price $747.26

Total growth	1,000 ÷ 747.26	**1.3382**
Take the fifth root	1.3382^(1/5)	**1.0600**
Implied yield	1.0600 − 1	**6.00%**

IN THE LESSON

- Example six.
- A zero-coupon bond pays no regular interest.
- You just buy it cheaply and get the face value back at the end.
- This one has a face value of one thousand dollars, repays in five years, and costs seven hundred and forty-seven dollars twenty-six cents today.
- What yearly return is that?
- First, how much does the money grow?
- One thousand divided by seven hundred and forty-seven twenty-six is one point three three eight two.
- That growth took five years.
- So we need the fifth root, which a calculator does as raising to the power of one fifth.
- The answer is one point zero six zero zero.
- Take away one, and the implied yield is six per cent.

21 Example 7: a bond with coupons

3-year bond, 6% annual coupon, face 100, price 102.72

Try 6%	price comes to	**100.00**
Try 4%	price comes to	**105.55**
Try 5%	price comes to	**102.72**
Implied yield	price matches	**5.00%**

IN THE LESSON

- Example seven.
- Many bonds pay a regular payment called a coupon, so no neat rearrangement works.
- Here the bond has three years left, pays a coupon of six dollars a year, repays one hundred at the end, and the price is one hundred and two dollars seventy-two cents.
- We guess a yield and test it.
- Try six per cent.
- The price comes out at exactly one hundred, which is too low.
- Try four per cent.
- The price is one hundred and five fifty-five, too high.
- Try five per cent.
- The price is one hundred and two seventy-two.
- A match.
- So the implied yield is five per cent.
- One handy check: the price is above one hundred, called trading above par, so the yield must be below the coupon rate.
- Five is below six.

22 Example 8: the growth a stock price assumes

Price $50, required return 9%

Dividend yield	D_1 = 2.00: 2 ÷ 50	**4.00%**
Implied growth g	9.00% − 4.00%	**5.00%**
If 2.00 was just paid (D_0)	$D_1 = 2.00 \times (1 + g)$	**$D_1 \approx$ 2.10**
Solve for g	(4.50 − 2.00) ÷ 52	**4.81%**

IN THE LESSON

- Example eight, for a stock.
- The price is fifty dollars, next year's dividend is expected to be two dollars, and investors require nine per cent.
- The dividend yield is two over fifty, which is four per cent.
- Implied growth is the required return minus that: nine minus four, which is five per cent a year.
- Now the trap.
- Suppose the two dollars was the dividend just paid, not next year's.
- The top of the formula must be next year's, so we grow it first.
- Next year's dividend is two times one plus g, about two dollars ten.
- That puts g on both sides.
- Solving it, nine per cent of fifty is four fifty, take away two gives two fifty, divided by fifty-two.
- The growth is four point eight one per cent.

23 Cash flow additivity and forward rates

- Value of a package = sum of its parts
- If not, riskless profit (arbitrage)
- Forward rate: a future period's rate, fixed today
- Two routes, same end point, same result

IN THE LESSON

- Our last idea is cash flow additivity.
- It says the value of a set of cash flows is just the sum of the value of each one.
- A bond paying a coupon this year and principal next year is worth the same as two separate payments added together.
- If it were not true, you could buy the package, sell the pieces, and keep the difference with no risk.
- That is called arbitrage, and markets do not leave it lying around.
- From this one idea we can find a forward rate, which is the interest rate for a future period that is fixed today.
- The trick is to compare two routes that cover the same time and must end up in the same place.

24 The forward rate formula

$$(1 + z_2)^2 = (1 + z_1) \times (1 + f)$$
$$f = (1 + z_2)^2 \div (1 + z_1) - 1$$

One-year spot rate	Rate for money lent one year
Two-year spot rate	Rate for money lent two years
Forward rate	Year-two rate agreed today

IN THE LESSON

- Here is the idea in a formula.
- A spot rate is the rate for money lent starting today, for a given number of years.
- Take a one-year and a two-year spot rate.
- Route one: invest for two years at the two-year spot rate.
- You end up with one plus that rate, squared.
- Route two: invest for one year at the one-year spot rate, then roll the money over for a second year at the forward rate.
- You end up with one plus the one-year rate, times one plus the forward rate.
- The two routes must give the same amount, otherwise there would be arbitrage.
- So we set them equal.
- Rearrange to find the forward rate, which is the rate for year two, agreed today.

25 Example 9: the implied forward rate

One-year spot 4%, two-year spot 5%

Lock in two years	1.05^2	**1.1025**
Roll over	1.04 × (1 + f)	**must equal 1.1025**
Divide	1.1025 ÷ 1.04	**1.0601**
Forward rate f	1.0601 − 1	**6.01%**

IN THE LESSON

- Example nine.
- The one-year spot rate is four per cent.
- The two-year spot rate is five per cent.
- What is the one-year rate, one year from now, that is implied?
- Route one: lock in the two-year rate.
- One point zero five squared is one point one zero two five.
- Route two: one point zero four, times one plus the forward rate.
- These must match.
- So one plus f equals one point one zero two five divided by one point zero four, which is one point zero six zero one.
- The forward rate is six point zero one per cent.
- It is higher than both spot rates: the two-year rate is roughly an average of four and the second year's rate, so the second year must pay about six.

26 Example 10: pricing a bond with additivity

2-year bond, 6% coupon, face 100; spot rates 4% and 5%

Year-1 payment 6	6 ÷ 1.04	**5.77**
Year-2 payment 106	106 ÷ 1.05^2	**96.15**
Bond price	sum, unrounded	**101.91**

IN THE LESSON

- Example ten uses additivity to price a bond.
- A two-year bond pays a six dollar coupon after one year and six dollars plus a hundred of face value after two.
- The spot rates are four and five per cent as before.
- By additivity, the bond is just two zero-coupon payments.
- The first payment, six dollars in one year, is discounted at the one-year rate: six divided by one point zero four, which is five dollars seventy-seven cents.
- The second, one hundred and six in two years, is discounted at the two-year rate: one hundred and six divided by one point zero five squared, which is ninety-six dollars fifteen cents.
- Add them: the bond is worth one hundred and one dollars ninety-one cents.
- Using one rate for both would treat two different things as the same.

27 Mistakes that cost marks

- Adding cash flows without discounting
- Choosing higher IRR over higher NPV
- Adding yearly returns, or leaving out income
- Using TWR for the client, or MWR for the manager
- Last dividend for g; subtracting rates for forwards

IN THE LESSON

- Common mistakes.
- One.
- Adding up cash flows without discounting.
- Money from different dates cannot be added until it is brought to the same date.
- Two.
- When N P V and I R R disagree, picking the higher I R R.
- Pick N P V.
- Three.
- Leaving the income out of a holding period return, or adding yearly returns instead of multiplying them.
- Four.
- Using the money-weighted return to judge a manager, or the time-weighted return to describe what the client earned.
- Five.
- In implied growth, using the dividend just paid instead of next year's, and in forward rates, subtracting spot rates instead of dividing growth factors.

28 Recap: the five ideas

NPV = Σ CF ÷ (1 + r)^t HPR = (P_1 − P_0 + D_1) ÷ P_0
Implied g = r − D_1 ÷ P_0 (1 + z_2)^2 = (1 + z_1)(1 + f)

N P V	Discount, add, subtract cost
Holding period return	Price change plus income
Time-weighted	Manager: chain; client: IRR
Implied	Solve pricing backwards
Forward rate	Two routes must match

IN THE LESSON

- N P V discounts every cash flow, adds them, and subtracts the cost.
- Positive means accept, and if it disagrees with the I R R, follow N P V.
- The I R R is the rate that makes N P V zero.
- Holding period return is the end value minus the start value plus income, over the start value, and years are chained by multiplying.
- Time-weighted return judges the manager and ignores deposits.
- Money-weighted return is the I R R of the client's flows.
- Implied yields and growth rates come from solving the pricing formulas backwards.
- And a forward rate comes from setting two routes equal, because cash flow additivity rules out free money.

29 Try three quick questions

1. Project A has the higher IRR, project B the higher NPV. You can do only one. Which?
   Answer: B. Follow NPV: it is in money and its reinvestment assumption is more realistic.
2. Buy a share at $40, receive $1 dividend, sell at $42. Holding period return?
   Answer: (42 − 40 + 1) ÷ 40 = 7.5%
3. One-year spot 3%, two-year spot 4%. Implied one-year forward rate, one year ahead?
   Answer: $1.04^2 \div 1.03 - 1 = 5.01\%$

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- A project has a higher I R R but a lower N P V than another project, and you can only do one.
- Which do you choose?
- The answer is the one with the higher N P V.
- Question two.
- You buy a share for forty dollars, receive a dividend of one dollar, and sell for forty-two dollars.
- What is the holding period return?
- The gain is three dollars over forty, so seven and a half per cent.
- Question three.
- The one-year spot rate is three per cent and the two-year spot rate is four per cent.
- What is the implied forward rate?
- One point zero four squared over one point zero three, minus one, is about five point zero one per cent.