Key terms: derivatives

Key terminology

1 How to use these terms

- Each term: a story, a rule, a small example
- Jump to any word a topic leaves unexplained
- Every number is made up and round
- Mistakes slide halfway, quiz at the end

IN THE LESSON

- Here is how to use this lesson.
- Think of it as a small dictionary that explains itself.
- Each term comes with a short story, a plain definition, and a worked example you can redo on paper.
- If a word in another topic stops you, jump to its slide.
- The words build on each other, so watch in order if you can.
- Arbitrage comes first, because forwards and options both lean on it.
- Halfway through is a slide of look alike mistakes, and at the end a recap and three questions.
- Ready?

2 Arbitrage: a free lunch that vanishes

- Arbitrage: a profit with no risk
- Buy the cheap one, sell the dear one
- Same thing, two prices: the gap cannot last
- It forces prices into line

IN THE LESSON

- Imagine two stalls on one street selling the very same loaf, one cheaper than the other.
- A quick thinker buys at the cheap stall, sells at the dear one, and pockets the difference.
- Arbitrage means buying and selling equivalent things at different prices for a profit that carries no risk.
- The profit is certain, and you need not guess the future.
- Because everyone would do it, the gap does not last.
- Buyers push the cheap price up, sellers push the dear price down, until the two prices meet.
- That is why we say arbitrage forces prices into line.
- You will meet this word in almost every topic: Quantitative Methods, Economics, Equity Investments, Fixed Income, Derivatives and Alternative Investments.

3 Example 1: arbitrage between two shops

One-ounce gold coin, same street, same day. Two prices

Buy a coin at shop A	pay	**$100**
Sell it at shop B	receive	**$103**
Profit on one coin	103 − 100	**$3**
Profit on 50 coins	50 × $3	**$150**

IN THE LESSON

- A small example.
- A one ounce gold coin sells for one hundred dollars at shop A and one hundred and three dollars at shop B.
- Step one, buy a coin at shop A for one hundred.
- Step two, carry it across and sell it at shop B for one hundred and three.
- You are left with three dollars, and you never carried any risk, because both prices were known when you acted.
- Do it fifty times and the profit is one hundred and fifty dollars.
- But fifty buyers at shop A push its price up, and fifty sellers at shop B push its price down.
- Within minutes the three dollar gap is gone.

4 Forward price: today's price for later

- Forward: a price fixed today, delivery later
- Forward price: makes the contract worth 0 at the start
- Spot price: the price for delivery today
- It comes from the cost of carry, not a forecast

IN THE LESSON

- Suppose you promise today to buy a bicycle later, at a price you and the seller agree right now.
- That is a forward contract: an agreement made today to trade something at a future date, at a price fixed today.
- The fixed price is the forward price.
- It is set so the contract is worth nothing to either side at the start.
- The spot price is the price for delivery today.
- Here is the surprise.
- The forward price is not a guess about where the spot price will go.
- It comes from the cost of carrying the asset until delivery.
- Buying the asset now and holding it until delivery costs interest on the money.
- The forward price must match that.
- You will meet this in Equity Investments and Derivatives.

5 The simplest forward price

Forward price F0 = S0 × (1 + r)^T

Spot price (S0)	Price for delivery today
Safe rate (r)	Safe yearly rate, 4% = 0.04
Years (T)	Years until delivery
Forward price (F0)	Price agreed today

IN THE LESSON

- Here is the rule for an asset that pays nothing and costs nothing to hold.
- The forward price equals the spot price compounded at the risk free rate for the life of the contract.
- The risk free rate is what a completely safe borrower pays or a safe lender earns.
- In symbols, F zero equals S zero times one plus r, all raised to the power T.
- S zero is today's spot price.
- The letter r is the safe rate for one year, written as a decimal, so four per cent is zero point zero four.
- T is the number of years until delivery.
- The answer, F zero, is the forward price.
- We compound once a year, as the Derivatives lessons do.
- There is no forecast anywhere in the formula.

6 Example 2: the fair forward price

Selwyn Tools share, no dividend, safe rate 4% a year

Spot price	today	**$200**
One-year forward	200 × 1.04	**$208**
Two-year forward	200 × 1.04²	**$216.32**

IN THE LESSON

- Example two.
- A share of Selwyn Tools pays no dividend, and costs two hundred dollars today.
- The safe rate is four per cent a year.
- What is the one year forward price?
- Two hundred times one point zero four is two hundred and eight dollars.
- For two years, compound twice.
- Two hundred times one point zero four squared is two hundred and sixteen dollars and thirty two cents.
- A longer contract has a higher forward price, because there is more waiting to pay for.
- And nobody asked whether the share will rise or fall.
- The forward price is the same whether you expect good news or bad.

7 Example 3: arbitrage holds the price

Fair forward $208, but a dealer quotes $215

Today: borrow, buy one share	loan of	**$200**
Today: agree to sell at	quoted forward	**$215**
Year end: repay the loan	200 × 1.04	**$208**
Year end: deliver the share	receive	**$215**
Risk-free profit	215 − 208	**$7**

IN THE LESSON

- Now watch arbitrage keep the forward price in place.
- Suppose a dealer quotes the one year forward at two hundred and fifteen dollars, but the fair price is two hundred and eight.
- The quote is too high, so you sell the forward.
- Today, borrow two hundred dollars, buy one share, and agree to sell it in a year for two hundred and fifteen.
- A year later the loan has grown to two hundred and eight, because two hundred times one point zero four is two hundred and eight.
- You hand over the share, receive two hundred and fifteen, and repay two hundred and eight.
- You keep seven dollars, with no risk and none of your own money.
- If the quote were too low, you would do the reverse.

8 Convenience yield: holding the real thing

- Convenience yield: the benefit of holding the real thing
- Example: a bakery that must never run out of flour
- Largest when stocks are tight
- It lowers the forward price relative to spot

IN THE LESSON

- Imagine you run a bakery.
- Sacks of flour in your cupboard are not the same as a contract promising flour later.
- In a shortage, the sacks keep your ovens running.
- The benefit of holding the physical commodity rather than a futures contract is called the convenience yield.
- It is largest when inventories are tight.
- Because owners enjoy this benefit, the price for later delivery drops relative to today's price.
- We treat it like a dividend: a benefit of ownership that the forward buyer misses.
- You will meet it in Derivatives and Alternative Investments.

9 Forward price with benefits and costs

F0 = (S0 − PV benefits + PV costs) × (1 + r)^T

Spot price (S0)	Price for delivery today
Benefits (PV)	Dividends, convenience yield
Costs (PV)	Storage and insurance
Forward price (F0)	Price agreed today

IN THE LESSON

- Here is the forward price with every cost and benefit included.
- Start with the spot price.
- Subtract the present value of the benefits of owning, such as dividends, or the convenience yield.
- Add the present value of the costs of owning, such as storage and insurance.
- Then compound the result at the safe rate for the life of the contract.
- Present value means a future amount in today's money.
- Benefits make owning more attractive, so the forward buyer, who does not own the asset, should pay less.
- Costs do the opposite.
- Still no forecast.

10 Example 4: oil and convenience yield

Oil spot $60, rate 5%, storage 2, convenience yield 5 (today's money)

No convenience yield	(60 + 2) × 1.05	**$65.10**
Convenience yield	worth 5 today	**$5**
With it	(60 − 5 + 2) × 1.05	**$59.85**
Forward below spot, called	59.85 is under 60	**backwardation**

IN THE LESSON

- Example four, a barrel of oil.
- The spot price is sixty dollars, the safe rate is five per cent, and the contract runs one year.
- Storage and insurance cost two dollars in today's money.
- First, ignore the convenience yield.
- Sixty plus two is sixty two, and times one point zero five is sixty five dollars and ten cents.
- Now add the fact that refiners value having oil on hand.
- The present value of that convenience yield is five dollars.
- Take sixty, subtract five, add two, which gives fifty seven, and compound by one point zero five.
- The forward price is fifty nine dollars and eighty five cents.
- That is below the spot price of sixty.
- A forward below spot has a name, backwardation, and the next slide draws it.

11 Contango and backwardation, drawn

Chart: Forward price ($) vs Years to delivery (0 to 2)

- Spot 100
- Contango: about 112
- Backwardation: about 94

IN THE LESSON

- Time to delivery runs along the bottom, from today out to two years.
- The forward price is up the side.
- The grey line is the spot price, one hundred dollars.
- The upper curve starts at spot and climbs, reaching about one hundred and twelve at two years.
- Costs outweigh benefits, so a longer wait means a higher price.
- A curve that slopes up like this is in contango.
- The lower curve falls, reaching about ninety four at two years.
- Benefits win, so a longer wait means a lower price.
- A curve that slopes down is in backwardation.
- Futures prices have the same shapes.
- You will meet both words in Derivatives and Alternative Investments.

12 Example 5: why the shape matters

Near contract $50. Long position rolled 4 times a year

Contango: next contract $51	(50 − 51) ÷ 50	**−2%**
Four rolls a year	4 × (−2%)	**−8%**
Backwardation: next at $49	(50 − 49) ÷ 50	**+2%**
Four rolls a year	4 × 2%	**+8%**

Same spot price, opposite results: the curve shape did it

IN THE LESSON

- Why does the shape matter?
- Because of rolling.
- A futures contract expires, so an investor who wants to stay long must sell the expiring contract and buy the next one.
- That is called rolling.
- The roll return is the near price minus the next price, over the near price.
- Take a near contract at fifty dollars.
- In contango, the next contract is fifty one.
- The roll return is fifty minus fifty one, over fifty, which is minus two per cent.
- Four rolls a year cost eight per cent.
- In backwardation, the next contract is forty nine, giving plus two per cent a roll, and plus eight per cent over four rolls.
- The spot price never moved.
- Contango costs a long holder money.
- Backwardation pays.

13 Options and the exercise price

- Option: a right, not a duty. Underlying: the asset
- Exercise price (strike) X: the price fixed in it
- Premium: the fee the buyer pays up front
- Voucher: buy a bicycle at X = $50, premium $3
- Bicycle at $65: use it. At $40: skip it

IN THE LESSON

- Now options.
- Imagine a shop sells you a voucher that lets you buy a bicycle for fifty dollars any time this month.
- You pay three dollars for the voucher.
- If bicycles jump to sixty five dollars, the voucher is gold.
- If they fall to forty, you simply do not use it, and you lose only the three dollars.
- That voucher is an option.
- An option is a contract giving its buyer a right, but not a duty, to trade an asset at a fixed price.
- The asset is the underlying, such as a share.
- The fixed price, fifty dollars here, is the exercise price, also called the strike, and written capital X.
- The fee, three dollars, is the premium.
- You will meet the exercise price in Derivatives, and in Financial Statement Analysis, where employee share options are valued.

14 Call option: the right to buy

Call payoff = max(0, S − X)

Share price (S)	Price of the share on the day
Exercise price (X)	Price you may buy at
Call payoff	Never below zero

IN THE LESSON

- A call option is the right, but not the obligation, to buy the underlying at the exercise price.
- The payoff of a call at expiry is the larger of zero and S minus X.
- S is the price of the underlying on the day the option ends.
- X is the exercise price.
- If the share is above X, you buy at X something worth S, and gain the difference.
- If the share is below X, buying at X would be a bad deal, so you walk away and the payoff is zero.
- Nobody can force you to buy, so a call payoff is never negative.
- Profit is different: it takes off the premium paid at the start.
- You will meet calls in Corporate Issuers, Equity Investments, Fixed Income and Derivatives.

15 Example 6: a call at four prices

Call, exercise price X = $50, premium $3. Profit per share

Share at $40	payoff 0, less premium 3	**−$3**
Share at $50	payoff 0, less premium 3	**−$3**
Share at $58	payoff 8, less premium 3	**+$5**
Share at $65	payoff 15, less premium 3	**+$12**
Worst case for the buyer	the premium	**−$3**

IN THE LESSON

- Take a share of Ravenscroft Tools and a call with an exercise price of fifty dollars.
- The premium was three dollars.
- At expiry, test four share prices.
- At forty, buying at fifty would be silly, so the payoff is zero and the profit is minus three, the premium.
- At fifty, the payoff is still zero, and the profit is minus three.
- At fifty eight, the payoff is eight dollars, and after the premium the profit is five.
- At sixty five, the payoff is fifteen, and the profit is twelve.
- So the buyer's worst case is the premium, three dollars.
- The upside has no ceiling.
- The seller of a call, the writer, keeps the premium but loses more and more as the share rises.

16 Put option: the right to sell

Put payoff = max(0, X − S)

Exercise price (X)	Price you may sell at
Share price (S)	Price of the share on the day
Put payoff	Never below zero

IN THE LESSON

- Now the put option: the right, but not the obligation, to sell the underlying at the exercise price.
- Think of car insurance, which lets you hand over a wrecked car to the insurer at an agreed value.
- The payoff of a put at expiry is the larger of zero and X minus S.
- Notice that the letters swap places compared with the call.
- For a put it is X minus S, because you gain when the share ends below the exercise price.
- You sell at X something worth only S.
- If the share ends above X, selling at X would mean selling below the market, so you do nothing and the payoff is zero.
- You will meet puts in Derivatives and Portfolio Management.

17 Example 7: a put at four prices

Put, exercise price X = $50, premium $4. Profit per share

Share at $35	payoff 15, less premium 4	**+$11**
Share at $50	payoff 0, less premium 4	**−$4**
Share at $58	payoff 0, less premium 4	**−$4**
Share at $0 (cannot fall further)	payoff 50, less premium 4	**+$46**

IN THE LESSON

- Same share, now a put with an exercise price of fifty dollars and a premium of four dollars.
- At expiry the share is at thirty five.
- You sell at fifty something worth thirty five, a payoff of fifteen.
- Take off the premium and the profit is eleven.
- At fifty or at fifty eight, the put is abandoned and the profit is minus four.
- Now the extreme case.
- The most a share can fall is to zero.
- Then the payoff is fifty, and the profit is forty six.
- That is why the holder's gain is capped at the exercise price less the premium: the share cannot fall below zero.
- The loss is capped too, at the premium.

18 Call and put payoffs, drawn

Chart: Payoff ($) vs Share price at expiry ($)

- Exercise price 50
- Call pays 50 at 100
- Put pays 50 at 0

IN THE LESSON

- Here are the two payoffs drawn.
- The share price at expiry runs along the bottom, from zero to one hundred.
- The payoff is up the side.
- The vertical grey line marks the exercise price, fifty.
- The call is flat at zero until the share reaches fifty, then rises dollar for dollar, reaching fifty when the share is at one hundred.
- The put is the mirror image.
- It starts high, at fifty when the share is worth nothing, falls as the share rises toward fifty, and stays flat at zero beyond.
- Both lines bend at the exercise price.

19 Who risks what in an option

Calls and puts: buyer and writer

Position	Limit
Call buyer: most it can lose	the premium
Call writer: loss as price rises	no ceiling
Put buyer: most it can gain	X − premium
Put writer: most it can lose	X − premium

IN THE LESSON

- Let us collect who risks what.
- Every option has a buyer, who holds the right, and a writer, who takes on the duty.
- For a call, the buyer can lose no more than the premium, but can gain without limit as the underlying rises.
- The writer of a call is the mirror image.
- At best the writer keeps the premium, but the loss has no ceiling as the underlying rises.
- For a put, the buyer's gain is capped, because the underlying cannot fall below zero.
- The most the buyer can gain is the exercise price less the premium.
- The writer of a put has the matching limit on the loss.
- Exams love to ask who has the unlimited loss.

20 European and American options

- European: exercise only on the expiry date
- American: exercise on any day up to expiry
- The names are about rules, not places
- American is worth at least as much as European
- Most pricing formulas value European options

IN THE LESSON

- Two more words describe when an option may be used.
- Think of a coupon for a free coffee.
- A European option is like a coupon valid on one day only, the expiry date.
- An American option is like a coupon valid on any day up to that date.
- The names are about the rules, not about where the option trades.
- Because the American holder has every choice the European holder has, plus more, an American option is never worth less than the equivalent European one.
- Most pricing formulas value the European kind directly.
- You will meet both words in Fixed Income, for callable bonds, and in Derivatives.

21 Example 8: the extra freedom

Two calls, exercise price X = $50. Payoffs only

Share on day 30	jumps up	**$58**
American: exercise then	58 − 50	**$8**
Share at expiry	slips back	**$52**
European: payoff at expiry	52 − 50	**$2**
American holder is ahead by	8 − 2	**$6**

IN THE LESSON

- A small case shows the extra freedom.
- Both options are calls with an exercise price of fifty.
- On day thirty, the share jumps to fifty eight.
- The American holder can exercise now, buying at fifty something worth fifty eight, and lock in eight dollars.
- The European holder cannot touch the option until expiry.
- By expiry the share has slipped back to fifty two, so the European payoff is only two dollars.
- The American holder came out six dollars ahead.
- The American holder could also have waited until expiry, so the extra freedom is never a disadvantage.

22 Mistakes and look-alikes

- Forward price: cost of carry, not a forecast
- Contango: later dearer. Backwardation: cheaper
- Call: S − X. Put: X − S
- Exercise price is the deal. Premium is the fee
- European or American: timing, not place

IN THE LESSON

- Time for the look alikes.
- First, the forward price is not a forecast.
- It is the spot price carried forward at the safe rate, with costs and benefits, whatever you expect the market to do.
- Second, contango and backwardation.
- Contango has later delivery dearer than spot.
- Backwardation has later delivery cheaper.
- Third, call and put payoffs.
- For a call it is S minus X.
- For a put it is X minus S.
- Fourth, the exercise price and the premium are different things.
- The exercise price is the fixed price of the trade.
- The premium is what you pay for the option.
- Fifth, European and American describe timing, not geography.
- Now, on to insurance.

23 Protective put: insurance for a share

- Protective put: own the share, buy a put on it
- Floor: worth at least X at expiry
- Upside stays open, less the premium
- Premium: the insurance cost, paid whatever happens

IN THE LESSON

- Now a strategy built from pieces you know.
- Think of insuring your house: you still own it, and you pay a premium so a disaster cannot ruin you.
- A protective put is the same for a share.
- You own the share, and you buy a put on it.
- If the share falls below the exercise price, the put pays the gap, so your losses below that level are covered.
- If the share rises, you keep the gain, less the premium.
- The premium is the cost of the insurance, paid whatever happens.
- You will meet this in Derivatives and Portfolio Management.

24 Example 9: the protective put

Buy share at $80 and a put, X = $75, premium $3

Share falls to $60	−20 + 15 − 3	**−$8**
Share stays at $80	0 + 0 − 3	**−$3**
Share rises to $100	+20 + 0 − 3	**+$17**
Worst case, any price ≤ $75	80 − 75 + 3	**−$8**

Without the put, the share at $60 would lose $20

IN THE LESSON

- Here are the numbers.
- You buy a share at eighty dollars and a put with an exercise price of seventy five, for a premium of three dollars.
- Profit is per share.
- If the share falls to sixty, you lose twenty on the share, the put pays fifteen, and you paid three for the put.
- That is minus eight in all.
- If the share stays at eighty, the put is useless and you are down the premium, minus three.
- If it rises to one hundred, you gain twenty on the share, less three, which is seventeen.
- The worst case is the same whether the share falls to sixty or to zero.
- It is eighty minus seventy five, plus three, which is eight dollars.
- That fixed worst case is what the premium buys.

25 Reference rate and the floating coupon

Coupon rate = reference rate + margin. Margin 1%

Reference rate 3%	3% + 1%	**4%**
Reference rate 4%	4% + 1%	**5%**
Reference rate 2%	2% + 1%	**3%**
Interest on $1m at 5%, a year	$1m × 5%	**$50,000**

IN THE LESSON

- Next, rates.
- A floating rate loan is a loan whose interest rate follows the market instead of staying fixed.
- What does it follow?
- A published benchmark called the reference rate, such as a standard short term interbank rate.
- At the start of each period, the reference rate is read, and the coupon, which is the interest payment, is set from it.
- The floating coupon rate equals the reference rate plus a margin, a fixed extra amount agreed when the loan is made.
- Let the margin be one per cent.
- When the reference rate is three per cent, the coupon rate is four.
- At four, it is five.
- At two, it is three.
- On one million dollars at five per cent for the year, the interest is fifty thousand dollars.
- You will meet this in Fixed Income and Derivatives.

26 Interest rate swap

- Swap: trade fixed interest for floating interest
- Notional: the amount the interest is worked out on
- The notional itself never changes hands
- Only the net difference is paid each date
- Turns debt from one type into the other

IN THE LESSON

- Now suppose a company has a loan whose rate floats, and it would sleep better with a fixed rate.
- Repaying and borrowing again is costly.
- Instead it can use an interest rate swap.
- A swap is an agreement to exchange fixed rate interest payments for floating rate interest payments, worked out on an amount called the notional.
- The notional is only a yardstick.
- It never changes hands.
- On each payment date, the two amounts are netted, so only the difference is paid.
- The party paying fixed is the fixed payer, who receives floating.
- The other party does the reverse.
- So a swap converts debt from one type to the other without refinancing it.
- You will meet swaps in Fixed Income and Derivatives.

27 Example 10: one payment date

Notional $5m. Yarrow pays fixed 4%, receives the reference rate

Fixed amount Yarrow pays	4% × $5m	**$200,000**
Reference rate 3%: net	gets 150,000, pays 200,000	**pays $50,000**
Reference rate 4%: net	gets 200,000, pays 200,000	**$0**
Reference rate 5%: net	gets 250,000, pays 200,000	**gets $50,000**

IN THE LESSON

- A concrete case.
- Yarrow Mills has a swap on a notional of five million dollars.
- It pays a fixed rate of four per cent and receives the reference rate, once a year.
- The fixed amount is two hundred thousand dollars.
- If the reference rate is three per cent, Yarrow receives one hundred and fifty thousand and pays two hundred thousand, so the net is that Yarrow pays fifty thousand.
- If the reference rate is four per cent, the two amounts are equal, and nothing moves.
- If it is five per cent, Yarrow receives two hundred and fifty thousand, pays two hundred thousand, and gets fifty thousand.
- So the fixed payer gains when rates rise and loses when they fall.

28 How a swap fixes a floating loan

Yarrow Mills: a floating loan plus a swap

Loan costs reference rate + 1% →

Swap pays reference rate, takes 4% →

Net cost: 4% + 1% = 5%, fixed

IN THE LESSON

- Now see why this helps.
- Yarrow Mills has a floating rate loan.
- It pays its lender the reference rate plus a margin of one per cent, so its interest bill changes whenever the market moves.
- Add the swap.
- Under the swap, Yarrow receives the reference rate and pays four per cent.
- Follow the reference rate through the three boxes.
- It goes out to the lender, and it comes back in from the swap, so it cancels.
- What is left is the margin of one per cent and the fixed four per cent, a total of five per cent, whatever the market does.
- The swap turned its cost from floating into fixed, with no new borrowing.

29 The swap rate

Discount factor = 1 ÷ [(1 + f1)…(1 + ft)]
Swap rate = (1 − final factor) ÷ sum of factors

Forward rates	Expected rate for one year
Discount factor	Today's value of $1 on that date
Last factor	The factor for the last date
Sum of factors	Add up every payment date
Swap rate	The fixed rate, the price

IN THE LESSON

- What fixed rate makes a swap fair?
- The swap rate is the fixed rate that makes the swap worth zero at the start, so neither side pays to enter.
- Start from the expected floating rates for each year, called forward rates.
- First, build a discount factor for each date.
- That is what one dollar received on that date is worth today.
- Divide one by the product of one plus each forward rate up to that date.
- Second, the swap rate equals one minus the final discount factor, divided by the sum of all the discount factors.
- The top is one minus the last factor because floating payments, valued today, add up to one dollar now less one dollar at the end.
- The swap rate is also the par yield implied by the discount factors, and the curve of swap rates across maturities is a market benchmark.

30 Example 11: a three-year swap rate

Forward rates for years 1, 2, 3: 3%, 4%, 5%

Discount factor, year 1	1 ÷ 1.03	**0.9709**
Discount factor, year 2	1 ÷ (1.03 × 1.04)	**0.9335**
Discount factor, year 3	1 ÷ (1.03 × 1.04 × 1.05)	**0.8891**
Sum of the three factors	0.9709 + 0.9335 + 0.8891	**2.7935**
Swap rate	(1 − 0.8891) ÷ 2.7935	**3.97%**

A plain average of 3%, 4%, 5% would give 4%

IN THE LESSON

- Example.
- Expected floating rates for the three years are three per cent, four per cent and five per cent.
- The discount factor for year one is one divided by one point zero three, which is zero point nine seven zero nine.
- For year two, divide again by one point zero four, giving zero point nine three three five.
- For year three, divide again by one point zero five, giving zero point eight eight nine one.
- Add the three factors to get two point seven nine three five.
- One minus the last factor is zero point one one zero nine.
- Divide, and the swap rate is three point nine seven per cent.
- It is not the plain average of the three rates, which would be four per cent, because earlier payments are worth more today.
- At this rate, the swap starts worth nothing to either side.

31 Participation rate: two meanings

- Economics: labour force ÷ working-age population
- Labour force = employed + unemployed and looking
- Equities: your share of the day's trading volume
- Same words, different share: check the topic

IN THE LESSON

- The last term travels between topics, and it means something a little different in each.
- Picture a village fair.
- What share of the villagers old enough to take part turn up?
- That is a participation rate.
- In Economics, the participation rate is the labour force divided by the working age population.
- The labour force is everyone who either has a job or is actively looking for one.
- In Equity Investments, a trader's participation rate is the share of the day's market volume that the trader's own orders make up.
- Same words, two different shares, so check which topic you are in.

32 Example 12: participation and unemployment

Working-age 100,000. Employed 62,000. Unemployed, looking 4,000

Labour force	62,000 + 4,000	**66,000**
Participation rate	66,000 ÷ 100,000	**66%**
Unemployment rate	4,000 ÷ 66,000	**6.06%**
1,000 give up: participation	65,000 ÷ 100,000	**65%**
Unemployment rate now	3,000 ÷ 65,000	**4.62%**

Trading: 10% of 1,000,000 shares a day = 100,000 a day

IN THE LESSON

- Here is a small country with a working age population of one hundred thousand people.
- Sixty two thousand have jobs, and four thousand have none but are looking.
- The labour force is sixty six thousand.
- The participation rate is sixty six thousand over one hundred thousand, sixty six per cent.
- The unemployment rate is four thousand over sixty six thousand, six point zero six per cent.
- Now one thousand of the unemployed lose hope and stop looking.
- The labour force drops to sixty five thousand, so participation falls to sixty five per cent, and the unemployed fall to three thousand, so the unemployment rate drops to four point six two per cent.
- Nobody got a job, yet the rate looks better: falling participation can flatter unemployment.
- For a trader, ten per cent of a day's volume of one million shares means trading one hundred thousand shares a day.

33 Recap: the key relationships

Forward F0 = S0 × (1 + r)^T; Swap rate = (1 − last factor) ÷ sum
Call max(0, S − X); Put max(0, X − S)

Arbitrage	Free profit forces prices level
Curve shape	Up: contango. Down: backwardation
Protective put	Share plus put: floor at X
Option style	American any day, European at end
Participation	Labour force ÷ working-age people

IN THE LESSON

- Arbitrage is a profit with no risk, and it is what forces prices into line.
- The forward price is the spot price carried forward at the safe rate, with benefits taken off and costs added.
- It is not a forecast.
- When the curve slopes up it is contango, and when it slopes down it is backwardation, which rewards a long position that rolls.
- A call pays the larger of zero and S minus X.
- A put pays the larger of zero and X minus S.
- American options may be used any day, European ones only at expiry.
- A protective put is a share plus a put, with a floor at X.
- A swap trades fixed for floating on a notional, at the swap rate.
- Participation rate is a share of people or of trading volume.

34 Try three quick questions

1. Share $150, safe rate 4%, no income. One-year forward price?
   Answer: 150 × 1.04 = $156. It is carry, not a forecast.
2. Call, X = $40, premium $2, share ends at $47. Payoff and profit?
   Answer: Payoff 47 − 40 = $7. Profit 7 − 2 = $5.
3. Later futures prices are below spot. Contango or backwardation?
   Answer: Backwardation: rolling a long position earns money.

IN THE LESSON

- Three questions, then the answers.
- Question one.
- A share costs one hundred and fifty dollars, the safe rate is four per cent a year, and the share pays nothing.
- What is the one year forward price?
- The answer is one hundred and fifty six dollars, not a forecast.
- Question two.
- A call has an exercise price of forty dollars and a premium of two dollars.
- The share ends at forty seven dollars.
- What are the payoff and the profit?
- The payoff is seven dollars, forty seven minus forty, and the profit is five dollars after the premium.
- Question three.
- Futures prices for later delivery are cheaper than spot.
- Is that contango or backwardation, and what does rolling a long position do?
- It is backwardation, and rolling a long position earns money each time.
- Thank you for listening.