# Key terms: money, time and returns

Key terminology

## 1 How to use these terms

- A story, a plain meaning, a tiny example
- Round, made-up numbers you can follow
- Each term builds on the ones before
- A look-alikes slide, a recap and three questions

IN THE LESSON

- Here is how to use this lesson.
- Every term gets a short story, a plain meaning, and a tiny worked example with round numbers.
- Each term also tells you where the course uses it, so you know why it matters.
- You will meet them in the topic lessons on quantitative methods, corporate issuers, fixed income, equity, derivatives and others.
- The order matters, because each idea leans on the one before.
- And near the middle there is a slide of look-alikes, because several of these terms sound alike and cost marks.

## 2 Basis point: a tiny step in a rate

*1 basis point = 0.01 percentage point = 0.0001*

| | | |
|---|---|---|
| **Rate moves 4.00% to 4.25%** | 0.25 × 100 | **25 bp** |
| **Yield 5.10% to 5.50%** | 0.40 × 100 | **40 bp** |
| **100 bp equals** | one percentage point | **1.00%** |
| **A margin of 150 bp equals** | 150 ÷ 100 | **1.5%** |

IN THE LESSON

- Start with the smallest unit.
- Rates move in small steps, so finance uses a small unit: the basis point.
- One basis point is one hundredth of one percentage point.
- So a hundred basis points make one percentage point.
- If a rate moves from four per cent to four and a quarter per cent, it has moved by twenty five basis points.
- A bond yield that rises from five point one zero per cent to five point five zero has risen forty basis points.
- You will meet this in fixed income and in derivatives, wherever rates are quoted.

## 3 Compounding: interest on interest

*$100 at 10% a year, interest added to the balance*

| | | |
|---|---|---|
| **End of year 1** | 100 × 1.10 | **$110.00** |
| **End of year 2** | 110 × 1.10 | **$121.00** |
| **End of year 3** | 121 × 1.10 | **$133.10** |
| **Simple interest would give** | 100 + 3 × 10 | **$130.00** |
| **Interest on interest** | 133.10 − 130 | **$3.10** |

IN THE LESSON

- Now money growing.
- Picture a snowball rolling downhill.
- It picks up snow, and the bigger it gets, the more snow it picks up.
- That is compounding: interest is added to the balance, so later interest is earned on interest too.
- Put one hundred dollars in an account paying ten per cent a year.
- After year one you have one hundred ten.
- In year two the ten per cent applies to one hundred ten, so you gain eleven and have one hundred twenty one.
- In year three you reach one hundred thirty three dollars ten cents.
- Simple interest, paid only on the original hundred, would give one hundred thirty.
- The extra three dollars ten is interest on interest.
- You will meet compounding in nearly every topic.

## 4 Annualising a return

**Annual equivalent = (1 + r)^m − 1**
**Example: 2% a quarter, m = 4: $1.02^4 - 1 = 8.24\%$**

| | |
|---|---|
| **Rate per period, r** | Return earned in one period |
| **Periods in a year, m** | How many fit in a year |
| **Annual equivalent** | Yearly rate that matches it |

IN THE LESSON

- Returns come over different lengths of time: a week, a month, a quarter.
- To compare them we need one common clock, the year.
- Restating a return as the equivalent yearly rate is called annualising.
- We do not multiply by the number of periods.
- Instead we add one to the period return, raise it to the number of periods in a year, and take one away.
- Here r is the return for one period, and m is how many such periods fit in a year.
- Two per cent a quarter becomes eight point two four per cent a year, not eight.

## 5 Annualising magnifies noise

*Annualised return (%)*

| | |
|---|---|
| **1% a quarter** | **4.06%** |
| **1% a month** | **12.68%** |
| **1% a week** | **67.77%** |

*Same 1%, three clocks: short periods give shaky headlines*

IN THE LESSON

- Here is the warning that goes with annualising, as a picture.
- The bottom line carries three bars, and the side axis shows the annualised return in per cent.
- In every bar the same fund earned one per cent.
- The only difference is the length of the period.
- One per cent in a quarter is about four point oh six per cent a year.
- One per cent in a month is twelve point six eight.
- One per cent in a single week is sixty seven point seven seven.
- Same return, shorter clock, bigger headline.
- Annualising changes the units, not the evidence.

## 6 Opportunity cost: the road not taken

- Opportunity cost = best alternative given up
- $1,000 spent now could have earned 5%: $50 a year
- So money has a cost for every year you wait
- This price of waiting becomes the discount rate

IN THE LESSON

- Before we shrink future money, we need the idea that explains why we do it.
- It is opportunity cost.
- Imagine a free Saturday: mow lawns for pay, or go to the beach.
- The real cost of the beach is the pay you gave up.
- Opportunity cost is the value of the best alternative given up by a choice.
- With money, spending a thousand dollars now means giving up what it could earn elsewhere.
- At five per cent that is fifty dollars a year.
- So waiting is never free.
- You will meet this in economics, where it explains trade, and in corporate issuers, where it sets the price of capital.

## 7 Present value and the discount rate

**$PV = FV \div (1 + r)^n$**

| | |
|---|---|
| **Present value, PV** | Worth today |
| **Future amount, FV** | Amount arriving later |
| **Rate per period, r** | Discount rate for ONE period |
| **Number of periods, n** | Periods until it arrives |

IN THE LESSON

- Now we can shrink the future.
- Present value is what an amount arriving later is worth today, once the cost of waiting is taken off.
- Think of a gift voucher that cannot be used for three years.
- It is worth less to you now than a voucher you could spend today.
- The rate we use to shrink it is called the discount rate.
- It is the return available elsewhere on something equally risky, which is exactly the opportunity cost from the last slide.
- The formula divides the future amount by one plus the rate, once for every period.
- Present value is the starting point of valuation, and you will meet it in almost every topic in the course.

## 8 A higher discount rate shrinks more

*$1,000 due in 3 years: its value today*

| Discount rate | Present value |
|---|---|
| 4% | $889.00 |
| 8% | $793.83 |
| 12% | $711.78 |

IN THE LESSON

- Let us watch the discount rate at work.
- A thousand dollars is promised in three years.
- At a discount rate of four per cent, divide by one point oh four three times, and it is worth eight hundred eighty nine dollars today.
- At eight per cent it is worth seven hundred ninety three dollars eighty three.
- At twelve per cent it is worth seven hundred eleven dollars seventy eight.
- The promise is the same each time.
- Only the discount rate changed.
- A bigger discount rate means a bigger cost of waiting, so the same future money is worth less today.
- This is why so many valuations hinge on one estimate, the rate used to discount.

## 9 Discount factor: the shrink multiplier

*Discount factor $DF = 1 \div (1 + r)^t$, here with r = 5%*

| | | |
|---|---|---|
| **Year 1** | $1 \div 1.05$ | **0.9524** |
| **Year 2** | $1 \div 1.05^2$ | **0.9070** |
| **Year 3** | $1 \div 1.05^3$ | **0.8638** |
| **PV of $500 due in year 3** | 500 × 0.86384 | **$431.92** |

IN THE LESSON

- Often we discount many payments at the same rate, so it helps to do the division once.
- The discount factor is the present value of one dollar paid on a given date.
- It equals one divided by one plus the rate, raised to the number of periods.
- At five per cent, one dollar in a year has a factor of zero point nine five two four.
- In two years the factor is zero point nine zero seven oh.
- In three years it is zero point eight six three eight.
- Now any payment is just multiplied by the factor for its date.
- Five hundred dollars due in year three is worth five hundred times that factor, about four hundred thirty one dollars ninety two.
- You will meet this in fixed income and derivatives.

## 10 Required return rises with risk

- Bank deposit 2%
- Government bond 4%
- Share 9%
- New venture 15%

IN THE LESSON

- Where does the discount rate come from?
- It comes from the required return, the minimum return investors demand for holding an asset, given its risk.
- Think of lending a friend money.
- You would ask for little if the friend is dependable, and much more if the friend is a gambler.
- The picture shows this.
- The line runs from low risk on the left to high risk on the right.
- A bank deposit might need only two per cent.
- A government bond, four per cent.
- A share, nine per cent.
- A new venture, fifteen per cent.
- These are illustrations, not market facts.
- In a valuation, the required return is the discount rate, so every value rests on this estimate.
- You will meet it across equity, portfolio management and alternatives.

## 11 Face value and the zero-coupon bond

*Zero-coupon bond: face $1,000, 5 years, discount rate 6%*

| | | |
|---|---|---|
| **Face value** | repaid at maturity | **$1,000** |
| **Coupons along the way** | none | **$0** |
| **Discount factor, 5 years** | $1 \div 1.06^5$ | **0.7473** |
| **Price today** | 1,000 × 0.7473 | **$747.26** |
| **A 5% coupon bond pays** | 5% × 1,000, yearly | **$50** |

IN THE LESSON

- A bond is a loan you can trade.
- Its face value is the amount repaid at maturity, and the figure the coupon rate is applied to.
- It is almost never the price you pay.
- A five per cent coupon bond with a thousand dollars of face value pays fifty dollars every year.
- A zero-coupon bond is the simplest bond of all.
- It pays nothing along the way, only the face value at the end.
- So you buy it at a discount.
- Take a five year zero with a thousand dollars of face value, discounted at six per cent.
- The discount factor is zero point seven four seven three, so the price today is seven hundred forty seven dollars twenty six.
- You will meet both ideas in fixed income.

## 12 Annuity: a level payment each period

**PV of annuity = PMT × AF**
**AF = (1 − (1 + r)$^{-n}$) ÷ r**

| | |
|---|---|
| **Payment, PMT** | Same payment each period |
| **Annuity factor, AF** | Value of $1 paid each period |
| **Rate per period, r** | Rate per period |
| **Number of periods, n** | Number of payments |

IN THE LESSON

- Many things pay the same amount again and again.
- A rent cheque, a loan repayment, a pension.
- An annuity is a level payment made every period for a fixed number of periods, and in its ordinary form each payment comes at the end of the period.
- Valuing it one payment at a time would be slow.
- Instead we use a shortcut.
- The present value of an annuity is the payment multiplied by a number called the annuity factor.
- The annuity factor depends only on the rate and the number of periods.
- It equals one minus one over one plus r to the power n, all divided by r.
- You will meet annuities in quantitative methods, in equities and in fixed income.

## 13 Annuity factor: the sum of discount factors

*Five-year annuity at 6%, payment $2,000 a year*

| | | |
|---|---|---|
| **Sum of five discount factors** | at 6% | **4.2124** |
| **Formula check** | (1 − 1.06$^{-5}$) ÷ 0.06 | **4.2124** |
| **Payment each year** | | **$2,000** |
| **Present value** | 2,000 × 4.2124 | **$8,424.73** |

IN THE LESSON

- Where does the annuity factor come from?
- It is simply the discount factors added together.
- The annuity factor is the present value of one unit paid on each payment date.
- Take a five year annuity at six per cent.
- Each year has its own discount factor, each a little smaller than the last.
- Add the five of them and you get four point two one two four.
- The formula gives the same answer, which is a good check.
- Now a payment of two thousand dollars a year is worth two thousand times four point two one two four, or eight thousand four hundred twenty four dollars seventy three.
- One multiplication replaces five.

## 14 Perpetuity: a payment that never ends

**PV of perpetuity = PMT ÷ r**
**Example: $300 ÷ 0.06 = $5,000**

| | |
|---|---|
| **Payment, PMT** | Same payment, for ever |
| **Rate per period, r** | Discount rate per period |
| **Present value** | Worth today |

IN THE LESSON

- Now stretch the annuity until it never stops.
- A perpetuity is a level payment that continues forever.
- It sounds impossible to value, since there are endless payments to add up.
- But payments far enough away are worth almost nothing today, so the total is finite.
- In fact the answer is delightfully simple.
- The present value is the payment divided by the discount rate.
- A payment of three hundred dollars a year at six per cent needs five thousand dollars in the bank.
- You will meet this in quantitative methods, in corporate issuers for preferred shares, and in equity valuation.

## 15 An annuity is a perpetuity with the end cut off

*$1,000 a year for 10 years at 5%*

| | | |
|---|---|---|
| **Perpetuity starting now** | 1,000 ÷ 0.05 | **$20,000.00** |
| **Perpetuity starting after year 10** | 20,000 ÷ $1.05^{10}$ | **$12,278.27** |
| **Annuity = the difference** | 20,000 − 12,278.27 | **$7,721.73** |
| **Check: payment × factor** | 1,000 × 7.7217 | **$7,721.73** |

IN THE LESSON

- Here is a neat link between the last two terms.
- A ten year annuity of one thousand dollars at five per cent can be built from perpetuities.
- Start a perpetuity today, paying a thousand a year.
- It is worth a thousand divided by zero point oh five, which is twenty thousand dollars.
- Now start a second perpetuity, but only after year ten.
- Its value at year ten is also twenty thousand, so today it is twenty thousand divided by one point oh five to the tenth, or twelve thousand two hundred seventy eight dollars twenty seven.
- The annuity is the first perpetuity minus the second.
- That is seven thousand seven hundred twenty one dollars seventy three.
- The annuity factor agrees.
- Same answer, two routes.

## 16 Terminal value: everything after the forecast

*Yarrow Foods: cash flows 100, 110, 120; then 3% growth; r = 9%*

| | | |
|---|---|---|
| **Cash flow in year 4** | 120 × 1.03 | **123.60** |
| **Terminal value, end of year 3** | 123.60 ÷ (0.09 − 0.03) | **2,060.00** |
| **PV of terminal value** | 2,060 ÷ $1.09^{3}$ | **1,590.70** |
| **PV of the three forecast years** | added up | **276.99** |
| **Value today** | 1,590.70 + 276.99 | **1,867.69** |

IN THE LESSON

- You cannot forecast a company for ever.
- So analysts forecast a few years in detail, then use one number for everything after.
- That number is the terminal value: the value, at the end of the explicit forecast, of everything that follows.
- It usually comes from a constant growth formula, which is a perpetuity that grows.
- Take Yarrow Foods.
- Its cash flows are one hundred, one hundred ten and one hundred twenty in the next three years.
- After that they grow three per cent for ever, and the discount rate is nine per cent.
- Year four brings one hundred twenty three sixty.
- Divide by nine minus three per cent and the terminal value at year three is two thousand sixty.
- Discounted three years, it is worth one thousand five hundred ninety dollars seventy.

## 17 The terminal value dominates

*Value today ($)*

| | |
|---|---|
| **PV year 1** | **91.74** |
| **PV year 2** | **92.58** |
| **PV year 3** | **92.66** |
| **PV terminal** | **1,590.70** |

*The terminal value is about 85% of the $1,867.69 total*

IN THE LESSON

- Here are the pieces of Yarrow Foods as bars.
- The bottom line carries four bars, and the side axis shows the value today in dollars.
- The first three bars are the present values of the forecast cash flows, year one, year two and year three.
- They are each about ninety two dollars, so small they hardly show.
- The tall bar on the right is the present value of the terminal value, one thousand five hundred ninety dollars seventy.
- Out of the total of one thousand eight hundred sixty seven sixty nine, that is about eighty five per cent.
- The lesson is where to look.
- Most of the value rests on the long run growth assumption, so that assumption deserves the most scrutiny.
- You will meet this in equity valuation and financial modelling.

## 18 Net present value

**NPV = −Outlay + $CF_1 \div (1 + r)^1 + CF_2 \div (1 + r)^2 + \dots$**

| | |
|---|---|
| **Outlay today** | Cost paid today |
| **Cash flows, CF** | Cash coming back, by date |
| **Rate per period, r** | Required return |
| **Net present value** | Value added: accept if > 0 |

IN THE LESSON

- Now we can judge a project.
- Net present value, or N P V, is the value a project adds, in money, once every cash flow has been discounted and the outlay subtracted.
- So we discount each cash flow, add them, and subtract what we pay.
- The discount rate here is the required return.
- If the answer is positive, the project earns more than the required return and adds value, so accept.
- If negative, it destroys value.
- Because N P V is in dollars, it is denominated in what shareholders actually receive.
- You will meet it in quantitative methods and corporate issuers.

## 19 NPV example: a machine for $900

*Outlay $900 today, then $400 a year for 3 years, r = 10%*

| | | |
|---|---|---|
| **PV of three $400 payments** | 400 × 2.4869 | **$994.74** |
| **Outlay today** | paid now | **−$900.00** |
| **Net present value** | 994.74 − 900 | **+$94.74** |
| **Decision** | NPV above zero | **Accept** |

IN THE LESSON

- Let us run it.
- The machine costs nine hundred dollars today.
- It then returns four hundred dollars at the end of each of the next three years.
- The required return is ten per cent.
- The cash flows are an annuity, so use the annuity factor.
- At ten per cent for three years the factor is two point four eight six nine.
- Four hundred times that is nine hundred ninety four dollars seventy four.
- That is what the three payments are worth today.
- Subtract the nine hundred outlay and the net present value is ninety four dollars seventy four.
- It is positive, so the project adds value and we accept.

## 20 Internal rate of return

*Same machine: outlay $900, then $400 a year for 3 years*

| | | |
|---|---|---|
| **NPV at a rate of 10%** | above zero | **+$94.74** |
| **NPV at a rate of 20%** | below zero | **−$57.41** |
| **IRR: NPV is exactly zero** | found by trial | **15.89%** |
| **Required return** | 10% | **10%** |
| **IRR above required return** | so accept | **Accept** |

IN THE LESSON

- The internal rate of return, or I R R, asks the question the other way round.
- Instead of choosing a discount rate and finding the value, it finds the discount rate at which the net present value is exactly zero.
- For our machine, at ten per cent the N P V is plus ninety four seventy four.
- At twenty per cent it is minus fifty seven forty one.
- The rate that gives zero is fifteen point eight nine per cent.
- That is the I R R.
- It tells you the rate at which the project pays, not how much value it adds.
- The rule is to accept when the I R R is above the required return.
- Here fifteen point eight nine beats ten, so accept.

## 21 Hurdle rate: the bar to clear

- Hurdle rate = return that must be cleared first
- Project: IRR must beat the hurdle (machine: 10%)
- Fund: 8% on $100m, hurdle 5%: only 3% = $3m counts
- 20% of the $3m above the bar = $0.6m fee

IN THE LESSON

- A hurdle in a race is a bar you must clear.
- A hurdle rate in finance is a return that must be cleared before something is allowed.
- It appears in two places.
- In corporate finance, it is the minimum rate a project must beat before the firm will do it.
- Our machine clears a ten per cent hurdle.
- In investment funds, it is the return a fund must achieve before any incentive fee is earned.
- Say a fund makes eight per cent on a hundred million dollars, and the hurdle is five per cent.
- Only the three per cent above the bar, three million dollars, can attract a fee.
- At a twenty per cent share, the fee is six hundred thousand dollars.

## 22 Mistakes and look-alikes

- Annualise by compounding, never by multiplying
- Discount rate, required return, opportunity cost: one idea
- Face value is repaid; price is paid; they differ
- IRR is a rate; NPV is the money value added
- Annuity factor = the sum of the discount factors

IN THE LESSON

- Time for a pause, because several of these terms sound alike.
- Here are the mix-ups that cost marks.
- One.
- Annualising by multiplying.
- Compounding means you raise one plus the return to a power.
- Two.
- Treating the discount rate, the required return and the opportunity cost as three different numbers.
- They are three names for one idea, the return you give up by waiting, matched to the risk.
- Three.
- Confusing the face value with the price.
- One is repaid, the other is paid.
- Four.
- Reading the internal rate of return as the value added.
- It is a rate, while net present value is the money added.
- Five.
- Mixing up the annuity factor and the discount factor.
- The first is the sum of the second.

## 23 Weighted average cost of capital

**WACC = wd × rd × (1 − t) + we × re**

| | |
|---|---|
| **Debt weight** | Share funded by debt |
| **Cost of debt, rd** | Interest rate on debt |
| **Tax rate, t** | Interest saves this tax |
| **Equity weight** | Share funded by owners |
| **Cost of equity, re** | Return owners require |

IN THE LESSON

- A firm funds itself with a mix of borrowed money and owners' money, and each source expects a different return.
- The weighted average cost of capital, or W A C C, is the blended return a firm must earn to satisfy everyone who funded it.
- Each cost is multiplied by its weight, which is that source's share of total funding.
- Borrowing costs are cut by the tax rate, because interest is tax deductible.
- Firms use it as the discount rate for projects of average risk.
- You will meet it in corporate issuers and in equity valuation.

## 24 WACC example: forty and sixty

*40% debt at 6%, tax 25%; 60% equity at 10%*

| | | |
|---|---|---|
| **Debt cost after tax** | 6% × (1 − 0.25) | **4.5%** |
| **Debt contribution** | 40% × 4.5% | **1.8%** |
| **Equity contribution** | 60% × 10% | **6.0%** |
| **WACC** | 1.8% + 6.0% | **7.8%** |

*One rate for every project subsidises the risky ones*

IN THE LESSON

- Take a firm funded forty per cent by debt and sixty per cent by equity.
- Debt costs six per cent, and the tax rate is twenty five per cent.
- First, debt after tax: six times one minus zero point two five is four and a half per cent.
- Debt contributes forty per cent of that, which is one point eight per cent.
- Equity costs ten per cent, and contributes sixty per cent of it, which is six per cent.
- Add them, and the W A C C is seven point eight per cent.
- Now the warning.
- It is a single rate, and using it for every project subsidises the risky ones.

## 25 Economic profit: profit after charging for capital

*Ashgrove Bakery: profit after tax $240, capital $2,000*

| | | |
|---|---|---|
| **Charge for capital at 9%** | 9% × 2,000 | **$180** |
| **Economic profit** | 240 − 180 | **$60** |
| **Charge for capital at 14%** | 14% × 2,000 | **$280** |
| **Economic profit at 14%** | 240 − 280 | **−$40** |

IN THE LESSON

- Accounting profit charges for borrowed money, but not for the owners' money.
- Economic profit fixes that.
- It is revenue less all costs, including the opportunity cost of the owners' capital.
- Zero means a normal return: enough to stay in the industry, not enough to attract entrants.
- Take Ashgrove Bakery.
- After tax, its operations earn two hundred forty dollars on two thousand dollars of capital.
- The cost of capital is nine per cent, so the charge for capital is one hundred eighty.
- Economic profit is two hundred forty minus one hundred eighty, which is sixty dollars.
- The bakery created real value.
- If the cost of capital were fourteen per cent, the charge would be two hundred eighty and the economic profit minus forty.

## 26 Weighted average cost: a look-alike in inventory

*Open: 200 units at $10; buy 300 at $12; sell 400*

| | | |
|---|---|---|
| **Cost available** | 2,000 + 3,600 | **$5,600** |
| **Units available** | 200 + 300 | **500** |
| **Average cost per unit** | 5,600 ÷ 500 | **$11.20** |
| **Cost of goods sold** | 400 × 11.20 | **$4,480** |
| **Ending inventory** | 100 × 11.20 | **$1,120** |

*FIFO would give COGS $4,400; LIFO $4,600*

IN THE LESSON

- Careful, this next term is a look-alike.
- Weighted average cost, without the word capital, is an inventory method.
- A shop buys goods at different prices.
- When it sells some, which price counts?
- Weighted average cost gives every unit sold and every unit left the same average cost of the goods available.
- Suppose a shop opens with two hundred units at ten dollars and buys three hundred at twelve.
- The cost available is five thousand six hundred dollars for five hundred units, an average of eleven dollars twenty.
- It sells four hundred.
- Cost of goods sold is four thousand four hundred eighty, and the hundred left cost one thousand one hundred twenty.
- F I F O would give four thousand four hundred, and L I F O four thousand six hundred, so the average sits between.
- You will meet this in financial reporting.

## 27 Expected value: the probability-weighted average

*E(X) = Σ p × x: three outcomes for next year's return*

| | | |
|---|---|---|
| **Good year, 20% chance** | 0.20 × 30% | **6.0%** |
| **Middling year, 50% chance** | 0.50 × 10% | **5.0%** |
| **Poor year, 30% chance** | 0.30 × (−10%) | **−3.0%** |
| **Expected return** | 6.0 + 5.0 − 3.0 | **8.0%** |

*A sure 8% has the same expected value, with no downside*

IN THE LESSON

- Now we turn to returns, and first to uncertain ones.
- Expected value is the probability weighted average of all the possible outcomes.
- It is what you would average if you could repeat the situation many times.
- It need not be a value that can actually happen.
- Say a project by Merrin Works has three outcomes for next year's return.
- A good year, a twenty per cent chance of thirty per cent.
- A middling year, a fifty per cent chance of ten per cent.
- A poor year, a thirty per cent chance of minus ten per cent.
- Multiply each outcome by its chance and add: six plus five minus three is an expected eight per cent.
- A sure eight per cent shares that value but carries no downside.

## 28 Geometric mean: the average that compounds

*Returns: +30%, −20%, +10% in three years*

| | | |
|---|---|---|
| **Growth factors** | 1.30, 0.80, 1.10 | |
| **Multiply** | 1.30 × 0.80 × 1.10 | **1.1440** |
| **Cube root, three years** | 1.1440^(1/3) | **1.0459** |
| **Geometric mean** | 1.0459 − 1 | **4.59%** |
| **Arithmetic mean, for contrast** | (30 − 20 + 10) ÷ 3 | **6.67%** |

IN THE LESSON

- When returns follow each other year after year, the ordinary average misleads.
- The geometric mean is the average that compounds correctly.
- A fund returns thirty per cent, then minus twenty, then ten.
- Turn each into a growth factor: one point three zero, zero point eight zero and one point one zero.
- Multiply them to get one point one four four.
- A hundred dollars becomes one hundred fourteen forty.
- Take the cube root, one point oh four five nine, and subtract one: four point five nine per cent a year.
- The simple average, six point six seven, overstates it.

## 29 Gross return and net return

*Fund: start $100, end $112 before fees, fee 1.5% of $100*

| | | |
|---|---|---|
| **Start of year** | | **$100** |
| **End of year, before fees** | | **$112** |
| **Gross return** | (112 − 100) ÷ 100 | **12.0%** |
| **Fee** | 1.5% × 100 | **$1.50** |
| **Net return** | (112 − 1.50 − 100) ÷ 100 | **10.5%** |

IN THE LESSON

- Next, how much of a return reaches the investor.
- The gross return is the return before management fees, custody and other administrative costs are deducted, though after trading costs.
- It measures the manager's investing.
- The net return is what remains after those fees come off.
- It is closer to what the investor received, though still before tax and inflation.
- A fund starts the year at one hundred dollars and ends at one hundred twelve before fees, a gross return of twelve per cent.
- The fee is one and a half per cent of the starting assets, which is one dollar fifty.
- The net return is ten and a half per cent.

## 30 Real return: taking inflation out

**Real return = (1 + nominal) ÷ (1 + inflation) − 1**
**Example: 1.105 ÷ 1.03 − 1 = 7.28% (shortcut: 7.5%)**

| | |
|---|---|
| **Nominal return** | Return before inflation |
| **Inflation** | Rise in prices over the period |
| **Real return** | Change in purchasing power |

IN THE LESSON

- There is one more thing that eats a return: rising prices.
- A return of ten per cent means little if everything also costs ten per cent more.
- The real return is the return with inflation taken out, so it measures the change in purchasing power.
- The nominal return is the one you actually see, before inflation is removed.
- A rough shortcut subtracts inflation from the nominal return.
- The exact method divides one plus the nominal return by one plus inflation, then takes one away.
- Take our net return of ten and a half per cent, with inflation of three per cent.
- One point one zero five divided by one point oh three, minus one, is seven point two eight per cent.
- The shortcut says seven and a half.

## 31 Time-weighted and money-weighted return

**Time-weighted = $(1 + r_1) \times (1 + r_2) \times \ldots - 1$**
**Money-weighted = IRR of the investor's cash flows**

| | |
|---|---|
| **Return of each piece** | Return between cash flows |
| **Investor's cash flows** | Money in, money out, end value |
| **Rate making NPV zero** | The rate making their NPV zero |

IN THE LESSON

- Last, two ways to measure a portfolio when money moves in and out.
- The time-weighted return is the compound growth of one unit over the period, linking the returns between cash flows so that their timing and size have no effect.
- It is the standard measure for a manager who does not control the flows.
- The money-weighted return is the internal rate of return on the portfolio's actual cash flows, so it reflects when money was added or withdrawn.
- It is the fair measure when the manager, such as a private equity general partner, controls that timing.

## 32 Same portfolio, two different answers

*Buy at 100; buy again at 150; holding worth 240 at the end*

| | | |
|---|---|---|
| **Year 1 return** | 150 ÷ 100 − 1 | **+50%** |
| **Year 2 return** | 120 ÷ 150 − 1 | **−20%** |
| **Time-weighted, two years** | 1.50 × 0.80 − 1 | **20.0%** |
| **Time-weighted, per year** | 1.20^(1/2) − 1 | **9.54%** |
| **Money-weighted, per year** | IRR of −100, −150, +240 | **−2.88%** |

IN THE LESSON

- Here is one portfolio measured both ways.
- An investor buys one share at one hundred dollars.
- A year later it is worth one hundred fifty, and the investor buys a second share.
- In year two the price falls to one hundred twenty, so the two shares are worth two hundred forty.
- Year one returned fifty per cent.
- Year two returned minus twenty per cent.
- Linking them, one point five times zero point eight, gives a time-weighted return of twenty per cent over the two years, or nine point five four per cent a year.
- The investor's cash flows are minus one hundred, minus one hundred fifty, plus two hundred forty.
- Their internal rate of return is minus two point eight eight per cent a year.
- The manager did well; the investor lost.

## 33 Recap: the key relationships

**PV = FV ÷ (1 + r)ⁿ Annuity = PMT × AF**
**Perpetuity = PMT ÷ r**
**NPV = PV of inflows − outlay WACC = wd·rd·(1 − t) + we·re**
**Annual = (1 + r)^m − 1 Real = (1 + nom) ÷ (1 + inf) − 1**

| | |
|---|---|
| **Discounting** | Present value, discount factor |
| **Streams of payments** | Annuity factor, perpetuity |
| **Project tests** | NPV in money, IRR as a rate |
| **Cost of funds** | WACC, required return, hurdle |
| **Returns** | Gross, net, real, TWR, MWR |

IN THE LESSON

- Let us tie it together.
- Discounting turns later money into today's money: present value is the future amount divided by one plus the rate to the power of the periods, and the discount factor is that division done once for a single unit.
- Streams of payments use the annuity factor, which is the sum of the discount factors, or the payment over the rate for a perpetuity.
- The project tests are net present value, in money, and the internal rate of return, a rate to compare with the hurdle.
- The cost of funds is the weighted average cost of capital.
- And returns come gross, net and real, measured time-weighted or money-weighted, annualised by compounding.

## 34 Try three quick questions

1. Yield 5.10% to 5.50%: how many basis points?
   **Answer:** 0.40 percentage points × 100 = 40 basis points.
2. Which return judges the manager, which the investor?
   **Answer:** Time-weighted: manager. Money-weighted: investor.
3. Perpetuity of $50 a year at 5%: present value?
   **Answer:** 50 ÷ 0.05 = $1,000.

IN THE LESSON

- Now three quick questions.
- Pause after each one.
- Question one.
- A bond yield moves from five point one zero per cent to five point five zero per cent.
- How many basis points is that?
- Forty, because each hundredth of a percentage point is one basis point.
- Question two.
- Which return judges the manager, and which describes the investor?
- The time-weighted return judges the manager, because it ignores the timing of cash flows.
- The money-weighted return describes the investor, because it is the internal rate of return on the investor's own flows.
- Question three.
- A perpetuity pays fifty dollars a year and the discount rate is five per cent.
- What is its present value?
- Fifty divided by zero point oh five is one thousand dollars.
- Well done, and thank you for working through these.