CHARTA LEARNING · LESSON NOTES

Binomial option pricing

Derivatives

1 A story: the stall owner's promise

- An option is a promise that depends on a share
- Tomorrow the share can do only two things
- Hold some shares so the risk cancels out
- A riskless position has one fair price

IN THE LESSON

- Imagine Maya, who runs a market stall.
- Tomorrow is either wet or dry, and nobody knows which.
- A customer asks Maya for a promise: if it is wet, she must hand over a coat at a fixed price.
- What should Maya charge for that promise?
- Here is the clever idea.
- She can keep some umbrellas in stock, just enough that she ends up with the same money whether it is wet or dry.
- A position with no risk has a known price, and that pins down the price of the promise.
- An option on a share works in exactly the same way.

2 The words of the model

- S = share today; S×u or S×d after one period
- Call payoff = max(0, share price − X)
- r = risk-free rate for one period
- Example: S 40, u 1.25, d 0.80, X 42, r 4%

IN THE LESSON

- Let us name the pieces.
- Today the share costs S.
- After one period it moves to S times u, where u is the up factor, or to S times d, where d is the down factor.
- Each possible price is called a node.
- The option here is a call with strike price X, so at the end it pays the share price minus X if that is positive, and nothing otherwise.
- Money can be lent or borrowed at the risk-free rate r for one period.
- Our running example is a share at forty, with u of one point two five, d of zero point eight, a strike of forty-two and a rate of four per cent.

3 What the call pays at the end

Chart: Call payoff vs Share price at expiry

- Strike 42
- Down: 32, pays 0
- Up: 50, pays 8

IN THE LESSON

- Here is the call's payoff drawn.
- Along the bottom is the share price at expiry.
- Up the side is what the call pays.
- Follow the line from the left.
- While the share is below the strike of forty-two, the call is worth nothing, so the line lies flat along the bottom.
- Once the share passes forty-two, the call pays one for one with every extra unit of price, so the line climbs.
- Now mark our two possible outcomes.
- If the share falls to thirty-two, the call pays nothing.
- If the share rises to fifty, the call pays eight.
- Those two numbers, eight and nothing, are all the model needs.

4 One period, two paths

If the share goes up

Share 40 today → Share 50 (u = 1.25) → Call pays 8

If the share goes down

Share 40 today → Share 32 (d = 0.80) → Call pays 0

IN THE LESSON

- Now the picture of the tree.
- Read each row from left to right.
- On the top row the share starts at forty, goes up to fifty, and the call pays eight.
- On the bottom row the same share starts at forty, goes down to thirty-two, and the call pays nothing.
- We are not saying which row will happen.
- In fact the model never asks.
- All it needs is the two prices the share might reach, and the two payoffs the option would then have.
- With those four numbers we can build a hedge, and the hedge is what gives us the price of the call.

5 The hedge ratio

h = (c+ − c−) ÷ (S+ − S−)

Hedge ratio	Shares held per call written
Call if up	Call payoff after an up move
Call if down	Call payoff after a down move
Share if up	Share price after an up move
Share if down	Share price after a down move

IN THE LESSON

- The hedge comes first.
- Suppose you write, meaning sell, one call, and you buy h shares to protect yourself.
- How many shares do you need so that your position is worth the same in both outcomes?
- The answer is the gap between the call's two payoffs, divided by the gap between the share's two prices.
- Think of it as how much the call moves for each unit the share moves.
- If the call swings by eight while the share swings by eighteen, you need eight eighteenths of a share.
- That number is called the hedge ratio, and it is also the call's delta.

6 Example 1: price the call by hedging

S = 40, u = 1.25, d = 0.80, X = 42, r = 4%

Payoffs c+, c−	50 − 42, and 0	**8, 0**
Hedge ratio h	8 ÷ (50 − 32)	**0.4444**
Hedged payoff	0.4444 × 32	**14.22**
Present value	14.22 ÷ 1.04	**13.68**
Call value	0.4444 × 40 − 13.68	**4.10**

IN THE LESSON

- Let us price the call.
- The payoffs are eight and zero.
- The hedge ratio is eight divided by eighteen, which is zero point four four four four of a share per call.
- Check the hedge.
- In the down state, those shares are worth zero point four four four four times thirty-two, which is fourteen point two two, and the call costs nothing to settle.
- In the up state, the shares are worth twenty-two point two two, but you owe eight on the call, which leaves fourteen point two two again.
- The position is riskless, so it must grow at four per cent.
- Today's value of fourteen point two two is thirteen point six eight.
- The call is the cost of the shares, seventeen point seven eight, less thirteen point six eight, which is four point one zero.

7 The same answer, with probabilities

π = (1 + r − d) ÷ (u − d)
c = [π c+ + (1 − π) c−] ÷ (1 + r)
No arbitrage: d < 1 + r < u

Risk-neutral probability	A pricing weight, not a forecast
Risk-free rate	Per-period rate on a safe loan
Up factor	Share price factor if up
Down factor	Share price factor if down
Call value today	Option price now

IN THE LESSON

- There is a shortcut.
- Instead of building the hedge, give the up move a special weight called the risk-neutral probability, written pi.
- It equals one plus r, minus d, all divided by u minus d.
- The down move gets one minus pi.
- The call is then the weighted average of its two payoffs, discounted for one period at the risk-free rate.
- This works only if there is no arbitrage, which needs the risk-free growth factor to sit strictly between d and u.
- If it did not, you could borrow to buy the share, or sell the share short, and win either way.
- Hedge and probabilities are the same argument written in two ways.

8 Example 2: the shortcut, and a trap

Same call: price it with probabilities

Risk-neutral π	(1.04 − 0.80) ÷ (1.25 − 0.80)	**0.5333**
Expected payoff	0.5333 × 8 + 0.4667 × 0	**4.2667**
Discount one period	4.2667 ÷ 1.04	**4.10**
Analyst's 70% instead	0.70 × 8 ÷ 1.04	**5.38**

Pi is a pricing weight, not the real chance of rising

IN THE LESSON

- Now the shortcut on the same call.
- Pi is one point zero four minus zero point eight, divided by one point two five minus zero point eight.
- That is zero point two four over zero point four five, or zero point five three three three.
- Weight the up payoff of eight by that and you get four point two six six seven.
- The down payoff is zero.
- Divide by one point zero four and the call is four point one zero, matching the hedge.
- Now the trap.
- An analyst believes the share goes up with seventy per cent chance.
- Using that, the value would be zero point seven times eight, over one point zero four, which is five point three eight.
- That price is too high, and anyone could sell it and hedge for four point one zero.

9 Why the weights make sense

- d = 0.80
- 1 + r = 1.04
- u = 1.25

IN THE LESSON

- Here is a picture of why pi is a sensible weight.
- Draw a line from the share's worst factor on the left to its best factor on the right.
- The worst is d, zero point eight.
- The best is u, one point two five.
- Now place the growth factor of a safe loan, one point zero four.
- For there to be no arbitrage, it must land somewhere between the ends, and it does.
- The fraction of the way along from d to u is exactly pi.
- That is zero point five three three three.
- So pi is simply where the safe return sits between the share's two outcomes.
- It says nothing about how likely each move really is.

10 From one period to two

- A European option can be exercised only at expiry
- Two periods give three end prices
- Value = expected end payoff, discounted
- Weights are risk-neutral, rate is risk-free

IN THE LESSON

- Our second outcome asks you to see an option's price as an average.
- Think of a lottery ticket.
- Its fair price is the prize you expect, discounted to today.
- A European option is like that: it can be exercised only at expiry, so only the end matters.
- Now stretch the tree to two periods.
- After one move the share is at one of two prices, and after a second move it is at one of three, because up then down lands where down then up lands.
- The price today is the expected payoff at the end, using risk-neutral probabilities, discounted over both periods at the risk-free rate.

11 The three end nodes

S = 50, u = 1.20, d = 0.80, X = 45, r = 4% a period

End node and share price	Call pays
Two ups: 72	27
Up then down, or down then up: 48	3
Two downs: 32	0

IN THE LESSON

- Here is the new example.
- A share starts at fifty.
- Each period it moves up by a factor of one point two or down by zero point eight, and the safe rate is four per cent a period.
- We value a two-period European call with a strike of forty-five.
- Two ups take the share to seventy-two, so the call pays twenty-seven.
- One up and one down take it to forty-eight, so the call pays three.
- Two downs take it to thirty-two, so the call pays nothing.
- Notice that the middle price can be reached by two different routes, up then down, or down then up.
- That matters when we assign the weights.

12 The two-period value

Weights: π^2, $2\pi(1-\pi)$, $(1-\pi)^2$
c = (weights × payoffs, added up) ÷ $(1+r)^2$

Risk-neutral probability	Risk-neutral chance of an up move
Path weights	Three end-node probabilities
End-node payoffs	What the option pays at each end
Risk-free rate	Per-period risk-free rate
Call value today	Option price today

IN THE LESSON

- To weight the end nodes, multiply the probabilities along each path.
- Two ups have probability pi squared.
- Two downs have one minus pi, squared.
- The middle node has probability two times pi times one minus pi, because there are two paths to it, each with the same chance.
- Check that these three weights add up to one, as probabilities must.
- Then multiply each weight by its payoff, add them up, and discount.
- We discount over two periods, so we divide by one plus r, squared.
- That is the whole method for a European option, however many periods the tree has.

13 Example 3: the two-period European call

S = 50, u = 1.20, d = 0.80, X = 45, r = 4%

Risk-neutral π	(1.04 − 0.80) ÷ (1.20 − 0.80)	**0.60**
Weights	π², 2π(1 − π), (1 − π)²	**0.36, 0.48, 0.16**
Expected payoff	0.36 × 27 + 0.48 × 3 + 0.16 × 0	**11.16**
Discount factor	1.04 squared	**1.0816**
Call today	11.16 ÷ 1.0816	**10.32**

IN THE LESSON

- Let us compute.
- Pi is one point zero four minus zero point eight, over one point two minus zero point eight, which is zero point six.
- The weights are zero point six squared, zero point three six; two times zero point six times zero point four, zero point four eight; and zero point four squared, zero point one six.
- They add up to one.
- The expected payoff is zero point three six times twenty-seven, plus zero point four eight times three, plus zero for the last node.
- That is nine point seven two plus one point four four, which is eleven point one six.
- Now discount over both periods.
- One point zero four squared is one point zero eight one six, and eleven point one six divided by that is ten point three two.

14 Where the price comes from

Value (currency)

Two ups	**9.72**
Up-down	**1.44**
Two downs	**0**
Expected	**11.16**
Today	**10.32**

Each node: weight times payoff; then discount twice

IN THE LESSON

- Here are the pieces as bars.
- Read them from left to right, in the order of the calculation.
- The first bar is the contribution of the top node, zero point three six times twenty-seven, which is nine point seven two.
- The second is the middle node, zero point four eight times three, one point four four.
- The third is the bottom node, which contributes nothing.
- Together they make the expected payoff at expiry, eleven point one six, in the fourth bar.
- The last bar is today's price.
- It is shorter, because we have discounted two periods of interest.
- The top node does most of the work, since it pays most, even though it is the least likely of the paying nodes.

15 Example 4: the forecast trap

Same call, using the analyst's 70% chance of up

Her weights	0.7², 2 × 0.7 × 0.3, 0.3²	**0.49, 0.42, 0.09**
Expected payoff	0.49 × 27 + 0.42 × 3	**14.49**
Discounted at 4%	14.49 ÷ 1.0816	**13.40**
Arbitrage-free value	risk-neutral route	**10.32**

Investors' risk attitudes already sit in the share price

IN THE LESSON

- Now see what goes wrong with forecasts.
- An analyst thinks the share rises with seventy per cent chance each period.
- Her end weights are zero point seven squared, which is zero point four nine; two times zero point seven times zero point three, which is zero point four two; and zero point three squared, zero point zero nine.
- The expected payoff is zero point four nine times twenty-seven, plus zero point four two times three, which is fourteen point four nine.
- Discount at four per cent for two periods and the call is thirteen point four zero.
- That is wrong, not because her forecast is silly, but because a risky payoff is being discounted at the safe rate.
- The arbitrage-free price stays at ten point three two.

16 Spotting a mispriced option

- Model value = cost of the replicating portfolio
- Market above model: the option is dear, sell it
- Market below model: the option is cheap, buy it
- Trade the opposite way in the replica

IN THE LESSON

- Our third outcome is to spot an options arbitrage and set out the trades.
- Think of two shops selling the same sandwich, one at two dollars and one at three.
- Buy at the cheap shop, sell at the dear one, and pocket the difference with no risk.
- An option is a sandwich we can rebuild from shares and a loan, which we call the replicating portfolio.
- So we compare the market price of the option with its model value.
- If the market is above the model, the option is dear: sell it.
- If the market is below, it is cheap: buy it.
- In both cases the other side of the trade is the replica.

17 Example 5: a call that is too dear

Call quoted 4.60, model value 4.10: sell 900 calls

Sell 900 calls	900 × 4.60	**+4,140**
Buy 400 shares	400 × 40	**−16,000**
Borrow	12,800 ÷ 1.04	**+12,307.69**
Kept today	4,140 − 16,000 + 12,307.69	**+447.69**

IN THE LESSON

- Back to the one-period call, worth four point one zero in the model.
- Suppose the market quotes it at four point six zero, and we trade nine hundred calls.
- The call is dear, so we sell nine hundred and take in four thousand one hundred and forty.
- To hedge, we need the replica, which is four ninths of a share for each call, so we buy four hundred shares for sixteen thousand.
- We finance this by borrowing the present value of what the shares are worth in the worst state, twelve thousand eight hundred, which is twelve thousand three hundred and seven point six nine today.
- Add up the cash: four thousand one hundred and forty, minus sixteen thousand, plus twelve thousand three hundred and seven point six nine.
- We keep four hundred and forty-seven point six nine.

18 Check: nothing owed at expiry

Both outcomes end with zero: the gap is pure profit

State at expiry	Net cash
Up: 20,000 − 7,200 − 12,800	0
Down: 12,800 − 0 − 12,800	0
Kept up front, today	+447.69

IN THE LESSON

- Now check that this really is risk-free.
- If the share rises to fifty, our four hundred shares are worth twenty thousand.
- We owe eight on each of nine hundred calls, seven thousand two hundred, and we repay the loan, which has grown to twelve thousand eight hundred.
- Twenty thousand minus seven thousand two hundred minus twelve thousand eight hundred is zero.
- If the share falls to thirty-two, the shares are worth twelve thousand eight hundred, the calls expire worthless, and the loan takes all the shares are worth.
- Again zero.
- So the profit of four hundred and forty-seven point six nine is ours today, and nothing is owed later.

19 Which way do the trades go?

- Call too dear: sell it, buy h shares, borrow
- Call too cheap: buy it, short h shares, lend
- American option below exercise value: buy, exercise
- Parity broken: buy the cheap side, sell the dear side

IN THE LESSON

- A simple rule keeps the direction straight: buy what is cheap, sell what is dear, and make the other side the replica.
- If the call is cheap, say it is quoted at three point seven zero, reverse everything.
- Buy nine hundred calls for three thousand three hundred and thirty, short four hundred shares to take in sixteen thousand, and lend twelve thousand three hundred and seven point six nine.
- The profit today is three hundred and sixty-two point three one.
- Two more checks belong in the kit.
- An American call quoted below what it would pay if exercised now can be bought and exercised at once.
- And when put call parity fails, buy the cheap side and sell the dear side.

20 Four mistakes so far

- 1. Reading π as the real chance of going up
- 2. Middle node counted once, or one discount only
- 3. Arbitrage direction backwards
- 4. Gap measured against an undiscounted payoff

IN THE LESSON

- Before we go on, four common slips.
- One.
- Treating pi as a forecast of the real chance of an up move.
- It is only a pricing weight, and using a forecast gives a value that is not free of arbitrage.
- Two.
- Forgetting the middle node counts twice in a two-period tree, or discounting only once.
- Three.
- Getting the direction backwards, selling the cheap option or buying the dear one.
- Always ask first: is the market price above or below the model value?
- Four.
- Measuring the gap against the undiscounted expected payoff.
- The model value is already discounted, and that is the number to compare with the market.

21 American options: the right to act early

- European: exercise only at expiry
- American: exercise on any day
- Roll back with π, one period at a time
- At each node take the larger: held or exercised

IN THE LESSON

- Our fourth outcome compares American and European options in a two-period tree.
- A European option can be used only on the last day.
- An American option can be used on any day.
- Imagine a season ticket you may cash in whenever you like, against one you may cash in only at the end of the year.
- The flexible one can never be worth less.
- To value it we still find the end prices, work out the payoffs, and roll back one step at a time with pi, discounting one period at a time.
- The only change is a check at every earlier node: the value is the larger of the rolled-back value and the value of exercising right now.

22 The roll-back rule

Rolled back = [π V up + (1 − π) V down] ÷ (1 + r)
American value = max(rolled back, exercise now)

Risk-neutral probability	Risk-neutral up probability
Risk-free rate	Per-period risk-free rate
Rolled-back value	Held-on value at the node
Exercise value now	What exercising would pay
Value at the node	Value used at that node

IN THE LESSON

- Here is the rule written out.
- At any node, the rolled-back value is the up value times pi, plus the down value times one minus pi, divided by one plus r for one period.
- For a European option, that is the node value, full stop.
- For an American option, compare it with the exercise value, which is what the option would pay if used now.
- A call pays the share price minus the strike, a put pays the strike minus the share price, and neither can be below zero.
- Take the bigger of the two.
- Then carry that value back to the previous node and repeat until you reach today.

23 Example 6: European and American puts

S = 50, u = 1.20, d = 0.80, X = 52, r = 4%, π = 0.60

Up node (S = 60)	(0.6 × 0 + 0.4 × 4) ÷ 1.04	**1.54**
Down node, held	(0.6 × 4 + 0.4 × 20) ÷ 1.04	**10.00**
Down node, American	max(10.00, 52 − 40)	**12.00**
European put today	(0.6 × 1.54 + 0.4 × 10) ÷ 1.04	**4.73**
American put today	(0.6 × 1.54 + 0.4 × 12) ÷ 1.04	**5.50**

IN THE LESSON

- Same share, started at fifty, up by twenty per cent or down twenty, four per cent a period, pi of zero point six.
- Value puts with a strike of fifty-two.
- The end payoffs are zero at seventy-two, four at forty-eight and twenty at thirty-two.
- At the up node, the share is sixty.
- Rolling back gives zero point six times zero plus zero point four times four, over one point zero four, which is one point five four.
- At the down node the share is forty, and holding is worth zero point six times four plus zero point four times twenty, over one point zero four, which is ten.
- But exercising pays fifty-two minus forty, twelve.
- The American takes twelve.
- Rolling back to today gives four point seven three for the European and five point five zero for the American.

24 The price of the early right

Put value (currency)

Held, down	**10.00**
Exercise, down	**12.00**
European today	**4.73**
American today	**5.50**

Early exercise adds 0.77 to the put today

IN THE LESSON

- The bars show where the difference arises.
- The first bar is the down node if we simply hold the European put: ten.
- The second is the American put at the same node: twelve, because exercising beats waiting.
- The third is the European put today, four point seven three.
- The fourth is the American put today, five point five zero.
- The extra point seven seven is the value of being allowed to take the strike early.
- Why does early exercise pay for a put?
- Because the most a put can ever pay is the strike.
- When the share is far below it, the put already pays almost all it can, and receiving the cash now lets you earn interest on it.

25 The call that is never exercised early

- No dividends: never exercise an American call early
- So the American call equals the European call
- Early exercise of a put can pay: strike earns interest
- Roll back with the check, not from end nodes alone

IN THE LESSON

- Now the call.
- Take an American call on a share that pays no dividends.
- Should you ever exercise early?
- No.
- Exercising means paying the strike today, which loses the interest you could have earned on that cash.
- It also throws away the option's remaining value, the insurance that the share might fall.
- Selling the call, or simply holding it, is always worth at least as much.
- So on a share with no dividends, the American call equals the European call, and you can value it from the end nodes alone.
- The trap runs the other way for puts.
- Valuing an American put from terminal payoffs alone gives only the European value, which misses the early-exercise premium.

26 Options on interest rates

- Underlying is a rate, not a share price
- Call pays if the rate ends above the strike
- Put pays if the rate ends below the strike
- Tree of one-period rates, one half on each branch

IN THE LESSON

- Our last outcome is an option on a rate.
- Think of a mortgage rate.
- You worry it will climb, so you buy protection that pays if it rises above four per cent.
- That is a call on the rate, like a single piece of an interest rate cap, a caplet.
- The put pays if the rate falls below a strike, like a floorlet.
- There is one big change from share options.
- The thing that moves is the interest rate itself, so the discount rate is no longer fixed.
- We use a tree of one-period rates, built to be arbitrage free, with probability one half on each branch, and each node carries its own rate.

27 Payoffs of a rate call and put

Chart: Payoff at that rate vs Rate when set

- Strike 3.50%
- Call: pays above
- Put: pays below

IN THE LESSON

- Here are the two payoffs.
- Along the bottom is the rate when it is set.
- Up the side is the payoff.
- The strike is three point five per cent, where the lines meet the axis.
- The solid line is the call.
- It is flat at zero while the rate is below the strike, then climbs as the rate rises above it, paying the notional times the gap.
- The second line, in red, is the put, and it works the other way.
- It pays when the rate is below the strike, and falls to zero as the rate reaches it.
- In both cases the payment is scaled by the accrual period, so a half-year period would halve it.

28 The rate tree and the roll back

Rates on the tree

Today: 3.00% → Year one: 4.20% or 2.80% → Year two: 5.40%, 3.60%, 2.40%

Order of the work

Payoffs at year two → Average, discount at node rate → Today: discount at 3.00%

IN THE LESSON

- Here is the tree for our example.
- On the top row we read the rates.
- Today's one-year rate is three per cent.
- At year one it is either four point two per cent or two point eight per cent.
- At year two it is five point four, three point six or two point four per cent.
- Notice the middle value is reached from both nodes.
- The bottom row is the order of the work.
- First we find the payoffs at year two.
- Then we step back, averaging the two values that follow each node and discounting at the rate on that node.
- Finally we reach today and discount at three per cent.
- Each step uses its own rate.

29 Rate option payoffs and values

Call pays N × max(0, rate − X) × accrual
Put pays N × max(0, X − rate) × accrual
Node value = 0.5 × (V up + V down) ÷ (1 + node rate)

Notional	Amount the rate is applied to
Rate when set	Rate set at that date
Strike rate	Rate you are protected at
Accrual period	Fraction of a year paid for
Rate at that node	Discount at this node, not today's

IN THE LESSON

- The formulas follow the picture.
- A call pays the notional, times the larger of zero and the rate minus the strike, times the accrual period.
- A put pays the notional, times the larger of zero and the strike minus the rate, times the accrual period.
- Then roll back.
- The value at a node is one half of the up value plus one half of the down value, divided by one plus the rate at that node.
- Our rates are annual, so the accrual period is one year and the last factor is one.
- The trap is discounting everything at today's rate, which ignores that rates move.

30 Example 7: a two-year call on the rate

N = 10m, X = 3.50%, accrual 1 year, 0.5 per branch; paid when rate is set

Year-two payoffs	10m × (rate − 3.50%)	**190k, 10k, 0**
Year one, up (4.20%)	0.5 × (190,000 + 10,000) ÷ 1.042	**95,969**
Year one, down (2.80%)	0.5 × (10,000 + 0) ÷ 1.028	**4,864**
Today (3.00%)	0.5 × (95,969 + 4,864) ÷ 1.03	**48,948**
Flat 3.00%, wrong	52,500 ÷ 1.03^2	**49,486**

IN THE LESSON

- Let us value a call on the one-year rate.
- The notional is ten million, the strike is three point five per cent, and the accrual period is one year.
- At year two the payoffs are ten million times one point nine per cent, which is one hundred and ninety thousand; ten thousand at three point six; and nothing at two point four.
- At the up node of year one, the average is one hundred thousand, divided by one point zero four two, which is ninety-five thousand nine hundred and sixty-nine.
- At the down node, five thousand divided by one point zero two eight is four thousand eight hundred and sixty-four.
- Today, the average of those is fifty thousand four hundred and seventeen, divided by one point zero three, forty-eight thousand nine hundred and forty-eight.
- Discounting at a flat three per cent would give forty-nine thousand four hundred and eighty-six, which is too high.

31 Recap: the formulas to remember

h = (c+ − c−) ÷ (S+ − S−)
π = (1 + r − d) ÷ (u − d)
Value = weighted payoffs ÷ (1 + r)^n

Hedge ratio	Shares per option, the delta
Risk-neutral probability	Pricing weight on an up move
End-node payoffs	Weighted, then discounted
Exercise value now	American: take the larger
Rate at that node	Rate options: node-by-node

IN THE LESSON

- Let us pull it together.
- The hedge ratio is the change in the option's payoff over the change in the share price.
- Pi is one plus r minus d, over u minus d, and it is a pricing weight, not a forecast.
- The value of a European option is the weighted end payoffs, discounted at the risk-free rate for every period.
- An American option takes the larger of holding and exercising at every earlier node.
- A call on a non-dividend share is never exercised early.
- To trade a mispricing, sell what is dear and buy what is cheap, with the replica on the other side.
- For rate options, discount at each node's own rate.

32 Try three quick questions

1. Call pays 10 if up, 0 if down; share 60 or 40. Hedge ratio?
   Answer: 10 ÷ (60 − 40) = 0.5 share per call
2. u = 1.30, d = 0.90, r = 5%. What is π?
   Answer: (1.05 − 0.90) ÷ 0.40 = 0.375
3. Rate call: N = 5m, X = 4.00%, rate set at 4.50%, 1 year. Payoff?
   Answer: 5m × 0.50% × 1 = 25,000

IN THE LESSON

- Now try three quick questions.
- Pause after each one.
- Question one.
- A call pays ten after an up move and zero after a down move.
- The share ends at sixty or forty.
- What is the hedge ratio?
- Ten divided by twenty is zero point five.
- Question two.
- The up factor is one point three, the down factor is zero point nine, and the rate is five per cent.
- What is pi?
- One point zero five minus zero point nine is zero point one five, over zero point four, which is zero point three seven five.
- Question three.
- A rate call has notional five million, strike four per cent, and the rate is set at four point five per cent over one year.
- What does it pay?
- Five million times zero point five per cent is twenty-five thousand.