Pricing forwards and futures

Derivatives

1 A story: booking a delivery of flour

- A forward fixes today a price paid later
- Futures: the same promise, on an exchange
- Alternative: buy now and hold until delivery
- Fair forward price = the cost of that route

IN THE LESSON

- Picture a baker who knows she will need a sack of flour in nine months.
- She can wait and pay whatever the price is then.
- Or she can agree today, with a miller, to pay a fixed price at that future date.
- That agreement is a forward contract.
- A futures contract is the same promise, but traded on an exchange with standard terms.
- Now, what is the fair price for that promise?
- Here is the thought that unlocks everything.
- The baker could instead buy the flour today and store it.
- The fair forward price must equal the cost of doing exactly that.

2 Two jobs: price it, then value it

- Pricing: set the forward price on day one
- Valuing: what is the contract worth later?
- Price is fixed; value moves with the market
- Case: Alder Foods at 80, dividend 1.60 in 9 months

IN THE LESSON

- Level Two questions test two separate jobs, and mixing them up is the commonest slip.
- Pricing means choosing the forward price when the contract is signed, so that it is worth nothing to either side on day one.
- Valuing means working out what the contract is worth later, once the market has moved.
- The price is fixed for life.
- The value changes every day.
- Our running case is a fictional analyst, Mara Quinn, at Fenwick Capital.
- She follows a share called Alder Foods, which trades at eighty.
- Alder will pay a dividend of one dollar sixty in nine months, and the risk free rate is four per cent a year.

3 Two routes to the same share on delivery day

Route one: the forward

Agree price F zero today →
Cash earns interest →
Pay F zero, receive share

Route two: do it yourself

Borrow and buy the share →
Collect the dividend →
Repay loan with interest

IN THE LESSON

- Here is the no arbitrage argument as a picture.
- Take the top row.
- Route one is the forward: you agree a price today and pay it at delivery, so your cash stays invested meanwhile and you end up holding the share.
- Now the bottom row.
- Route two builds the same result by hand.
- You borrow money, buy the share today, collect any dividend it pays, and at delivery repay the loan with interest.
- Both routes leave you owning the share on the same day, so they must cost the same.
- If they did not, traders would buy the cheap route and sell the dear one for a risk free profit, until prices came back together.

4 Equity forward price: discrete dividends

F0 = (S0 − PV of dividends) × (1 + r)^T

F zero: forward price	Price agreed today
S zero: spot price	Today's share price
PV of dividends	Dividends, valued today
r: risk-free rate	Annual rate, compounded yearly
T: time to delivery	Years until delivery

IN THE LESSON

- Here is the first formula.
- The forward price equals the spot price, less the present value of the dividends paid before delivery, all grown at the risk free rate for the time to delivery.
- Take the terms one at a time.
- S zero is today's spot price.
- The present value of dividends is what the dividends are worth today, found by discounting each one back at the risk free rate.
- Subtracting it is sensible, because the forward buyer does not receive those dividends, so the price should be lower.
- The rate r and the time T, in years, then carry the remainder forward.
- Here, r is the annual rate with annual compounding, as in the curriculum.

5 Example 1: price the Alder forward

S0 = 80.00, dividend 1.60 in 9 months, r = 4%, T = 1 year

PV of the dividend	1.60 ÷ 1.04^0.75	**1.554**
Net spot price	80.00 − 1.554	**78.446**
Carry one year	78.446 × 1.04	**81.58**
Forward price F0	zero value at start	**81.58**

IN THE LESSON

- Example one.
- Mara prices a one year forward on Alder Foods.
- The dividend of one dollar sixty arrives in nine months, which is zero point seven five years.
- Step one: its present value is one point six zero divided by one point zero four to the power zero point seven five, which is one point five five four.
- Step two: take that off the spot price.
- Eighty minus one point five five four leaves seventy eight point four four six.
- Step three: carry it forward one year at four per cent.
- Seventy eight point four four six times one point zero four gives eighty one point five eight.
- That is the fair forward price, and the contract starts with zero value.

6 Equity forward price: continuous yield

F0 = S0 × e^((rc − δ) × T)

F zero: forward price	Forward index level
S zero: spot price	Spot index level
Continuous rate	Continuously compounded
Dividend yield	Delta: continuous yield
T: time to delivery	Years until delivery

IN THE LESSON

- Share indexes pay many small dividends from hundreds of companies, so we treat the dividends as a steady stream.
- That stream is measured by a dividend yield, written with the Greek letter delta, and it is continuously compounded.
- The index forward price is then the spot index times e to the power of the continuously compounded risk free rate, minus the dividend yield, times the time.
- The letter e is the constant two point seven one eight, and the e to the power button on your calculator does the work.
- The idea is unchanged.
- The yield is the income you give up by waiting, so you subtract it from the interest rate, which is the cost of waiting.

7 Example 2: a stock index forward

Index 4,000, yield 2%, rate 3.5%, both continuous, T = 0.5

Net carry rate	3.5% − 2.0%	**1.5%**
Exponent	0.015 × 0.5	**0.0075**
Forward index	4,000 × e^0.0075	**4030.11**
If yield ignored	4,000 × e^0.0175	**4070.62**

IN THE LESSON

- Example two.
- Mara prices a six month forward on the Harbor Four Hundred index, a made up index standing at four thousand.
- Its dividend yield is two per cent and the risk free rate is three point five per cent, both continuously compounded.
- The net carry rate is three point five minus two, which is one point five per cent.
- Over half a year that is zero point zero zero seven five.
- Raise e to that power, about one point zero zero seven eight, and multiply by four thousand.
- The forward is four thousand thirty point one one.
- Notice it sits only a little above spot.
- If you forget the yield you would get four thousand seventy point six two, which is too high.

8 Valuing the long forward later

Vt = St − PV of dividends left − F0 ÷ (1 + r)^(T − t)

V t: value of the long	Value to the long
S t: spot now	Share price now
PV of dividends	Dividends still to come
PV of F zero	Forward price, valued now
T: time to delivery	T minus t: time left

IN THE LESSON

- Now the second job: value.
- Some time t after the start, the contract is no longer worth zero.
- The long forward is worth the current spot price, less the present value of dividends still to come, less the present value of the original forward price.
- Think of it like this.
- Owning the share would give you the share and the future dividends.
- The long gets the share but not the dividends, so we take those off.
- The long also has to pay the forward price at delivery, but that payment is in the future, so we discount it.
- Crucially, we discount over the time that remains, T minus t, not the original term.

9 Example 3: value after six months

S now = 84.00, six months in, dividend 3 months away, r = 4%

PV of dividend left	1.60 ÷ 1.04^0.25	**1.58**
PV of forward price	81.58 ÷ 1.04^0.5	**80.00**
Value to the long	84.00 − 1.58 − 80.00	**2.42**

IN THE LESSON

- Example three.
- Six months have passed.
- Alder Foods now trades at eighty four, the dividend of one dollar sixty has not yet been paid, and the rate is still four per cent.
- Mara values the long position.
- The dividend is now three months away, so its present value is one point six zero over one point zero four to the power a quarter, which is one point five eight.
- The forward price of eighty one point five eight is due in six months, so we discount it over half a year: eighty one point five eight over one point zero four to the power a half is eighty point zero zero.
- The value is eighty four minus one point five eight minus eighty, which is two point four two.

10 Value of the long against the share price

Chart: Value of the long forward vs Share price at the valuation date

- Break even 81.58
- Long loses
- At 84.00: +2.42

IN THE LESSON

- Let us draw it.
- The horizontal axis is the share price at the valuation date.
- The vertical axis is the value of the long forward.
- Now the line.
- It slopes upward, because a rising share price helps the buyer, and it rises one for one with the share.
- The grey horizontal line marks zero value.
- Where the value line crosses it, we have the break even share price.
- For Alder that is eighty one point five eight, which is the dividend still to come plus the discounted forward price.
- Below it the long is losing money.
- At eighty four, our example, the long is ahead by two point four two.

11 Forward against futures: when value is paid

Same pricing formula; different settlement

Feature	Forward | Futures
Traded	Private | Exchange
Gain or loss paid	At end | Every day
Value after a day	Builds | Reset to 0
Price formula	Same | Same

IN THE LESSON

- One more contrast before we move on.
- A forward is a private deal, and nothing changes hands until delivery.
- So the gain or loss we just worked out stays inside the contract, as a value, until the end.
- A futures contract works differently.
- At the close of every day, the exchange marks the position to market.
- The gain or loss for the day is paid through the margin account, and the contract's value resets to zero.
- When interest rates are constant and known, futures prices and forward prices are equal.
- So the pricing formulas you have just seen apply to both.
- Only the valuation differs, because futures are settled every day.

12 Carry arbitrage: the idea

- Fair forward = cost of buy now, hold, deliver
- Financing cost = interest on money borrowed
- Benefits lower the price; costs raise it
- Quote off fair: trade it for a locked-in profit

IN THE LESSON

- Now the second outcome: the carry arbitrage model.
- An arbitrage is a risk free profit that needs no money of your own.
- The carry model says a forward must equal the cost of buying the asset now and financing it until delivery.
- The cost of financing is the interest on the money borrowed.
- If the asset pays benefits along the way, such as dividends, those reduce your cost.
- If it has costs, such as storage, those add to it.
- When no income or cost exists, the fair forward price is just the spot price grown at the risk free rate.
- Any quote away from fair can be exploited by trading the forward against the asset.

13 The carry model with cash flows

F0 = (S0 − PV of benefits + PV of costs) × (1 + r)^T

F zero: forward price	No-arbitrage forward
S zero: spot price	Spot price
PV of benefits	Income, valued today
PV of costs	Storage and the like
T: time to delivery	Years until delivery

IN THE LESSON

- Here is the general carry formula.
- The forward price equals the spot price, minus the present value of benefits, plus the present value of costs, all grown at the risk free rate to delivery.
- Benefits are cash the holder receives, such as dividends or coupons or a convenience of holding.
- Costs are cash the holder pays, such as storage or insurance.
- When the asset produces neither, both present values are zero and the formula collapses to the spot price times one plus r to the power T.
- Treat this as a master formula.
- The equity forward you priced earlier is just the case where the only benefit is a dividend.

14 Example 4: no cash flows, a quote too high

S0 = 100, no income or cost, r = 5%, T = 1, quote 106.50

Fair forward price	100 × 1.05	**105.00**
Quote versus fair	106.50 − 105.00	**1.50**
Today	borrow 100, buy, sell forward	**0 cash**
At delivery	106.50 − 105.00 repay	**1.50**

IN THE LESSON

- Example four, the simplest case.
- An asset trades at one hundred and pays nothing and costs nothing to hold.
- The risk free rate is five per cent, and a one year forward is quoted at one hundred six point fifty.
- The fair price is one hundred times one point zero five, which is one hundred five.
- The quote is one point fifty too high, so we sell the forward.
- Today we borrow one hundred and buy the asset.
- At the end of the year we deliver it and receive one hundred six fifty, repay the loan of one hundred five, and keep one point fifty.
- Nothing was invested and no risk was taken, and that is the arbitrage profit.

15 Which way the arbitrage runs

Quote above fair: carry arbitrage

Sell the forward → Borrow and buy asset → Deliver; repay loan

Quote below fair: reverse carry

Buy the forward → Short asset; lend cash → Take delivery; close short

IN THE LESSON

- There are two directions, and the picture shows both.
- Look at the top row first.
- When the quoted forward is above the fair price, the forward is dear, so you sell it.
- This is called carry arbitrage.
- You borrow, buy the asset, and hold it, and at delivery you hand it over at the high price and repay the loan.
- Now the bottom row.
- When the quote is below fair, the forward is cheap, so you buy it.
- This is reverse carry.
- You sell the asset short, lend the cash, and at delivery you take the asset and close the short.
- In both rows, the profit at expiry is the gap between the quoted and fair prices.

16 Example 5: the same case with income

S0 = 150, income 4.50 in 6 months, r = 6%, T = 1, quote 157.00

PV of income	4.50 ÷ 1.06^0.5	**4.37**
Net spot	150.00 − 4.37	**145.63**
Fair forward	145.63 × 1.06	**154.37**
Quote above fair by	157.00 − 154.37	**2.63**

IN THE LESSON

- Example five adds income.
- Mara looks at an asset priced at one hundred fifty.
- It pays income of four dollars fifty in six months.
- The risk free rate is six per cent, and the one year forward is quoted at one hundred fifty seven.
- First, the present value of the income: four point five over one point zero six to the power a half is four point three seven.
- Take it off the price, which leaves one hundred forty five point six three.
- Carry that for a year at six per cent, and the fair forward is one hundred fifty four point three seven.
- The quote of one hundred fifty seven is above fair, so the forward is dear.

17 Example 5, continued: the trade and the trap

Sell forward, borrow 150, buy the asset, hold to delivery

Delivery receipt	sell asset at the quote	**157.00**
Income grown	4.50 × 1.06^0.5	**4.63**
Loan repaid	150 × 1.06	**159.00**
Profit at expiry	157.00 + 4.63 − 159.00	**2.63**
Trap: price if income ignored	150 × 1.06	**159.00**

IN THE LESSON

- Now the trade.
- Because the quote is too high, Mara sells the forward, borrows one fifty, and buys the asset.
- At six months the income of four fifty arrives, and she invests it, so by the end of the year it has grown to four point six three.
- At delivery she hands over the asset for one hundred fifty seven, collects the grown income, and repays a loan that has grown to one hundred fifty nine.
- Net profit: two point six three, which is exactly the gap between quote and fair price.
- Now the trap.
- Ignore the income, and fair looks like one hundred fifty nine, so the quote looks cheap.
- You would buy the forward, which is the wrong direction.

18 Interest rate forwards: the forward rate agreement

- FRA: fixes today the rate for a future loan
- 3 × 9: starts in 3 months, ends in 9 months
- Long pays fixed, receives the market rate
- No loan is made; only the difference is paid

IN THE LESSON

- Now the third outcome, interest rate forwards.
- Imagine a treasurer, Dev Anand, who will borrow money in three months, for six months after that.
- Dev is worried that rates will rise before the borrowing starts, and can fix the rate today with a forward rate agreement, an agreement between two parties on an interest rate for a future period.
- No loan is made.
- At the start of the period the two sides just settle the difference between the agreed rate and the market rate on that day.
- We name it by its dates: a three by nine agreement starts in three months and ends in nine months.
- The buyer, called the long, pays the fixed rate and receives the market rate.

19 The forward rate agreement rate

FRA rate = [(1 + long rate × t_long) ÷ (1 + short rate × t_short) − 1] ÷ t_gap

FRA rate	Annualised fixed rate
Long-dated rate	Spot rate to loan end
Short-dated rate	Spot rate to loan start
Accrual fraction	Days ÷ 360 for each
Notional	Notional only scales

IN THE LESSON

- How do we find the fixed rate?
- We use two spot market reference rates, one for the short period up to the start and one for the long period up to the end.
- Growing money for the long period must give the same result as growing it for the short period and then for the gap between them at the forward rate.
- So divide one plus the long rate times its accrual fraction by one plus the short rate times its accrual fraction, and take away one.
- That gives the rate for the gap period.
- Then divide it by the accrual fraction of the gap period, to annualise it.
- Accrual fraction means days divided by the year basis, such as three hundred sixty.

20 Example 6: the three by nine rate

90-day 2.8%, 270-day 3.4%, actual/360, notional 25m

Growth to day 270	1 + 0.034 × 270/360	**1.0255**
Growth to day 90	1 + 0.028 × 90/360	**1.0070**
Rate for the 180 days	1.0255 ÷ 1.0070 − 1	**1.837%**
Annualised FRA rate	1.837% ÷ 0.5	**3.674%**

IN THE LESSON

- Example six.
- The treasurer, Dev Anand, works for a fictional firm called Marlow Packaging, and wants to borrow twenty five million for one hundred eighty days, starting ninety days from now.
- Rates use the actual over three hundred sixty convention.
- The ninety day rate is two point eight per cent and the two hundred seventy day rate is three point four per cent.
- Growth to day two seventy is one plus zero point zero three four times three quarters, which is one point zero two five five.
- Growth to day ninety is one point zero zero seven.
- Divide, subtract one, and you get one point eight three seven per cent for the half year.
- Divide by a half, and the rate is three point six seven four per cent.

21 The forward rate sits above both

Annual rate (%)

90-day rate	**2.80%**
270-day rate	**3.40%**
3 × 9 FRA rate	**3.67%**

Illustrative reference rates

IN THE LESSON

- Look at the bars.
- The first is the ninety day rate, two point eight per cent.
- The second is the two hundred seventy day rate, three point four per cent.
- The third bar is the forward rate agreement rate we just found, three point six seven per cent.
- Why is it the tallest?
- Borrowing for two hundred seventy days at three point four must cost the same as ninety days at two point eight followed by one hundred eighty days at the forward rate.
- The first stretch is cheap, so the second stretch must be dearer, to bring the whole journey up to three point four.
- A simple average would be wrong here.

22 Valuing the agreement after rates move

Value of long = (new rate − old rate) × t_loan × notional ÷ (1 + spot × t_end)

Change in rate	New FRA rate minus old
Accrual fraction	Length of loan, in years
Notional	Amount the rate applies to
Discount to loan end	One plus spot rate × days/360

IN THE LESSON

- Now the value.
- Some days later, rates have changed.
- The long, who pays fixed, gains if the market rate for the same loan period has risen above the agreed rate.
- The gain is the new forward rate minus the original, times the accrual fraction of the loan, times the notional.
- That money is earned at the end of the loan, so we discount it back to today, using the spot rate for the full time until the loan ends, and its own accrual fraction.
- Two traps appear here.
- Leaving out the accrual fraction doubles or worse the answer.
- Discounting only to the start date of the loan, rather than its end, gives a wrong number too.

23 Example 7: the value thirty days later

New FRA rate 4.00%, 240-day spot 3.70%, long pays 3.674%

Rate gap	4.000% − 3.674%	**0.326%**
Gain at loan end	0.00326 × 0.5 × 25m	**40,750**
Discount factor	1 + 0.037 × 240/360	**1.02467**
Value of the long	40,750 ÷ 1.02467	**39,769**

IN THE LESSON

- Example seven.
- Thirty days later the market rate for the same loan period, now sixty days ahead and running one hundred eighty days, is four per cent.
- The two hundred forty day spot rate is three point seven per cent.
- The long locked in three point six seven four, so the rate gap is zero point three two six per cent.
- Multiply by a half and by twenty five million, and the gain at the end of the loan is forty thousand seven hundred fifty.
- It is earned two hundred forty days from now, so divide by one plus zero point zero three seven times two hundred forty over three sixty.
- That gives thirty nine thousand seven hundred sixty nine.
- Dev's agreement is in the money.

24 Example 8: a rate future is quoted as 100 minus the rate

Rate future, 1m notional, 90-day period, price falls 96.80 to 96.45

Rate before	100 − 96.80	**3.20%**
Rate after	100 − 96.45	**3.55%**
Move in the rate	3.55% − 3.20%	**0.35%**
Gain to the short, per contract	0.0035 × 0.25 × 1m	**875**

IN THE LESSON

- Interest rate futures are quoted differently.
- The quoted price is one hundred minus the rate, in per cent.
- A price of ninety six point eight zero means a rate of three point two zero per cent.
- Suppose it falls to ninety six point four five.
- The rate has risen to three point five five.
- Each contract is for one million over ninety days, so each basis point is worth twenty five dollars.
- The rate rose thirty five basis points, which is eight hundred seventy five per contract.
- Who gains?
- The short, because the quoted price fell.
- The long loses.
- This is the mirror image of the agreement, where rising rates helped the long.
- After this daily settlement, the position is worth zero again.

25 Mid-lesson check: five mistakes to avoid

- Discount F0 over the time left, not the original term
- Ignoring income overstates fair price, flips the trade
- Pricing is day one; valuing is any later day
- FRA: keep accrual fraction; discount to loan end
- Rate futures quote 100 − rate: rising rates help the short

IN THE LESSON

- Before the bond contracts, let us collect the traps so far.
- One: forgetting to discount the forward price when valuing, or discounting it over the original term instead of the time that remains.
- Two: leaving out the dividend or income, which overstates the fair price and can even send you the wrong way in the trade.
- Three: mixing up pricing and valuing.
- The price is fixed at the start, the value moves.
- Four: with the forward rate agreement, forgetting the accrual fraction, or discounting only to the loan's start.
- Five: forgetting that rate futures are quoted as one hundred minus the rate, so rising rates help the short.

26 Bond forwards and futures: three extra details

- Quoted price is clean; you finance the full price
- Coupons before delivery are income
- At delivery: clean again, take off accrued
- Futures: divide by the conversion factor

IN THE LESSON

- Now the fourth outcome: forwards and futures on bonds.
- The idea is still carry arbitrage, but three details make the arithmetic harder.
- First, bonds are quoted with a clean price, which leaves out the interest earned since the last coupon.
- The buyer still pays that accrued interest, so the amount you finance is the full price, clean plus accrued.
- Second, coupons may arrive before delivery, and they are income that reduces the cost.
- Third, futures let the seller choose which bond to deliver, so each candidate has a conversion factor to put them on a common footing.
- Quoted forward and futures prices are clean again at delivery.

27 From quoted price to futures price

Clean price plus accrued →
Grow at r to delivery →
Less coupons and accrued →
Divide by conversion factor

IN THE LESSON

- Follow the boxes from left to right.
- Start with the clean price you see quoted.
- Add the accrued interest to reach the full price, because that is what the buyer really pays and finances.
- Next, grow the full price at the risk free rate until delivery.
- Now take off two things.
- The first is the value of any coupon paid before delivery, grown to the delivery date, since you collect it.
- The second is the accrued interest at delivery, because the forward price is quoted clean.
- What remains is the forward price for that bond.
- For a futures contract, divide by the conversion factor of the bond that will be delivered.

28 The bond forward and futures price

F0 = (B0 + AI0) × (1 + r)^T − AI_T − FV of coupons
Futures price = F0 ÷ CF

Full price	Clean price plus accrued
Carry to delivery	One plus r, to the power T
FV of coupons	Coupons grown to delivery
Accrued at delivery	Interest accrued at delivery
Conversion factor	Exchange's published factor

IN THE LESSON

- Here is the formula.
- The forward price is the full price today, B zero plus the accrued interest at the start, grown to delivery at one plus r to the power T, less the future value of coupons paid before delivery, less the accrued interest at delivery.
- The quoted futures price divides that by the conversion factor.
- One caution about the symbols.
- The future value of coupons means each coupon grown from the day it is paid to the delivery date.
- Coupons paid after delivery are not included.
- And the conversion factor is a fixed number per bond, published by the exchange, usually below one when the bond's coupon is under the contract's notional coupon.

29 Example 9: a bond forward price

Coupon 5% semi-annual, clean 102.40, last coupon 4 months ago, r = 3%

Full price	102.40 + 1.67	**104.07**
Carried 5 months	104.07 × 1.03^(5/12)	**105.36**
Coupon grown 3 months	2.50 × 1.03^0.25	**2.52**
Less accrued at delivery	3 months of 2.50	**1.25**
Clean forward price	105.36 − 2.52 − 1.25	**101.59**

IN THE LESSON

- Example nine.
- Mara prices a five month forward on a bond with a five per cent coupon, paid half yearly, so each coupon is two dollars fifty.
- The bond is quoted at one hundred two point forty.
- Its last coupon was four months ago, so accrued interest is four sixths of two fifty, which is one point six seven.
- The full price is one hundred four point zero seven.
- The next coupon falls in two months, three months before delivery.
- Carry the full price five months at three per cent, giving one hundred five point three six.
- Subtract the coupon grown three months, two point five two, and the accrued at delivery, one point two five.
- The forward is one hundred one point five nine.

30 Example 10: the futures price

CTD conversion factor 0.92; forward from Example 9

Clean forward price	from Example 9	**101.59**
Futures price	101.59 ÷ 0.92	**110.42**
Trap: no conversion factor	stops at the forward	**101.59**
Trap: accrued left in	(105.36 − 2.52) ÷ 0.92	**111.78**

IN THE LESSON

- Example ten.
- Fenwick wants the futures price for a contract where this bond is the one that will be delivered.
- Its conversion factor is zero point nine two.
- The futures price is the forward price divided by that factor.
- One hundred one point five nine divided by zero point nine two gives one hundred ten point four two.
- Why is it larger than the forward?
- Because the factor is below one.
- The bond counts as less than a full contract bond, so each futures contract has to be quoted higher to compensate.
- Now see the traps.
- Leave out the factor and you quote one hundred one fifty nine.
- Forget to subtract the accrued interest and you quote one hundred eleven seventy eight.

31 Example 11: which bond is cheapest to deliver

Futures 118.00; clean prices; accrued interest ignored

Bond A: 105.00, CF 0.90	105.00 − 106.20	**−1.20**
Bond B: 98.50, CF 0.83	98.50 − 97.94	**+0.56**
Bond C: 112.00, CF 0.95	112.00 − 112.10	**−0.10**

IN THE LESSON

- The short in a futures contract chooses which eligible bond to deliver, and will choose the cheapest.
- Here is how to spot it.
- For each bond, compare what it costs in the market with what the short is paid on delivery, which is the futures price times that bond's conversion factor.
- The bond with the lowest cost minus receipt is cheapest to deliver.
- Take a futures price of one hundred eighteen, and ignore accrued interest to keep the numbers clean.
- Bond A costs one hundred five, and delivering it pays one hundred six point two, a gain of one point two.
- Bond B loses fifty six cents.
- Bond C is nearly level, so bond A wins.
- That bond's carry drives the futures price.

32 Example 12: value of a bond forward later

3 months in: clean 104.00, r = 3%, 2 months left, no coupon left

New forward price	104.42 × 1.03^(2/12) − 1.25	**103.68**
Change from original	103.68 − 101.59	**2.09**
Value of long forward	2.09 ÷ 1.03^(2/12)	**2.08**
Value of futures after settling	marked to market	**0**

IN THE LESSON

- Finally, value.
- Three months after the start, the bond is quoted at one hundred four, and the coupon has been paid.
- Accrued interest is a sixth of two fifty, or zero point four two.
- Mara recomputes the forward price for the two months that remain, with no coupon left.
- The full price is one hundred four point four two.
- Carried two months at three per cent, then less the one point two five accrued at delivery, the new forward price is one hundred three point six eight.
- The long agreed one hundred one point five nine.
- The gap is two point zero nine, due in two months, so we discount it, and the long's position is worth two point zero eight.
- A futures position would be worth zero after settlement.

33 Recap: what to remember

- Forward = (spot − PV income + PV costs) × (1 + r)^T
- Quote above fair: sell forward; below: buy it
- FRA value = rate gap × accrual × notional, discounted
- Bond futures = [full price carried − AI − FV coupons] ÷ CF

IN THE LESSON

- Let us pull the lesson together.
- One: price a forward by carrying the cost of the asset to delivery, net of the income it pays.
- Value it later by taking the share price, less the present value of dividends still to come, less the present value of the original forward price over the time that remains.
- Two: if a quote is above fair, sell the forward and carry the asset; if below, buy it and run reverse carry.
- Three: a forward rate agreement rate comes from two spot rates, and its value from the rate change, the accrual fraction and a discount.
- Four: for bonds, finance the full price, subtract coupons and accrued interest, and divide by the conversion factor.

34 Try three quick questions

1. Share 50, dividend 1.00 in 6 months, r 4%, one-year forward: fair price?
   Answer: (50 − 0.98) × 1.04 = 50.98
2. Asset 200, no income, r 5%, forward quoted 212: what trade?
   Answer: Fair 210. Quote is dear: sell forward, borrow, buy, earn 2.
3. Rate future falls from 97.00 to 96.50. Who gains?
   Answer: The short: the rate rose by 0.50 points.

IN THE LESSON

- Now try three quick questions.
- Pause after each one.
- Question one.
- A share is fifty, it pays a dividend of one in six months, the rate is four per cent, and the forward is for one year.
- What is the fair forward price?
- The present value of the dividend is about zero point nine eight.
- Fifty minus zero point nine eight is forty nine point zero two, times one point zero four is fifty point nine eight.
- Question two.
- An asset at two hundred pays nothing, the rate is five per cent, and the one year forward is quoted at two hundred twelve.
- Fair is two hundred ten, so the quote is dear: sell the forward, borrow, buy, and make two.
- Question three.
- A rate future falls from ninety seven to ninety six point five.
- Who gains?
- The short, because rates rose.