Option Greeks and hedging

Derivatives

1 A dashboard with five dials

- Option value depends on several inputs
- Each Greek: change one input, hold the rest
- Together they form the option dashboard
- Dealers manage whole books with them

IN THE LESSON

- Imagine a car dashboard with five dials.
- One shows speed, one shows fuel, one shows engine temperature, and so on.
- Each dial answers a single question: how is this one thing changing?
- Option dealers have a dashboard like that.
- An option's value depends on the share price, on how fast the share price swings, on time, and on interest rates.
- A dealer wants to know how much the value moves when just one of those inputs changes, while the others stand still.
- Each answer has a Greek name.
- Together they are called the Greeks, and they are how a book of options is watched and managed.

2 Five Greeks, five questions

Each Greek changes one input, holds the rest still

Greek	What it answers
Delta	Option value per 1 move in the share
Gamma	Change in delta per 1 move in the share
Vega	Value per 1 point of volatility
Theta and rho	Value per day; per 1 point of rate

IN THE LESSON

- Here are the five, each with the question it answers.
- Delta asks: if the share price rises by one, how much does the option's value change?
- Gamma asks: how much does delta itself change when the share moves by one?
- So gamma is the speed of delta.
- Vega asks: if volatility rises by one percentage point, what happens to value?
- Theta asks: what does one day of passing time do to the value?
- And rho asks: what does a one point rise in the interest rate do?
- Notice that gamma is a second order idea, a change in a change, while the other four are first order.

3 Delta of a call and of a put

Call delta = e^(−δT) × N(d1)
Put delta = Call delta − e^(−δT)

Call delta	Between 0 and 1
Dividend discount factor	Equals 1 with no dividend
N of d one	A normal probability, 0 to 1
Put delta	Between −1 and 0

IN THE LESSON

- Delta has a formula in the pricing model.
- For a European call, delta equals the dividend discount factor times a number called N of d one.
- N of d one is a probability read from the normal distribution, so it is always between zero and one.
- The dividend discount factor is a number just under one that adjusts for dividends, and it equals one if the share pays none.
- So a call's delta lies between zero and one.
- For a put, delta equals the call's delta minus that same discount factor.
- That makes it a number between minus one and zero.
- A put gains when the share falls, which is why its delta is negative.

4 Example 1: delta of an at-the-money option

Share 100, strike 100, volatility 20%, rate 5%, one year

Call and put values	from the pricing model	**10.45 and 5.57**
d1	(ln 1 + 0.07) ÷ 0.20	**0.35**
N(d1)	normal table	**0.6368**
Call delta	0.6368 × 1	**+0.64**
Put delta	0.6368 − 1	**−0.36**

IN THE LESSON

- Example one.
- An analyst values a one year call and a one year put on a share priced at one hundred dollars.
- The strike is one hundred, volatility is twenty per cent, the risk free rate is five per cent, and there are no dividends.
- Under the pricing model the call is worth ten dollars forty five cents and the put five dollars fifty seven.
- The model gives d one equal to zero point three five.
- N of that is zero point six three six eight.
- With no dividends, the call's delta is zero point six four.
- The put's delta is that minus one, which is minus zero point three six.
- Check: the call's delta minus the put's delta is exactly one.

5 Delta against the share price

Chart: Delta (zero line in the middle) vs Share price (strike 100 in the middle)

- Call: 0 to 1
- Put: −1 to 0
- Steepest at the strike

IN THE LESSON

- Here is delta drawn out.
- The share price runs along the bottom and delta goes up the side.
- The middle line is zero.
- The rising green curve is the call's delta.
- Far to the left, where the call is deep out of the money, it is close to zero, because a small move in the share hardly matters.
- Far to the right, deep in the money, it nears one, because the call then moves almost one for one with the share.
- The red curve is the put, and it is the same curve slid down by one.
- It runs from near minus one on the left to near zero on the right.
- Both curves are steepest in the middle, near the strike.
- Remember that steepness.

6 Gamma against the share price

Chart: Gamma (relative) vs Share price (strike 100 in the middle)

- Near expiry
- One year to run
- Strike

IN THE LESSON

- Now gamma, the speed at which delta changes.
- Same axes: share price along the bottom, gamma up the side.
- The tall, narrow hump is an option very close to expiry.
- Gamma is biggest right at the strike, because that is where delta swings from near zero to near one in a tiny stretch of price.
- Away from the strike, gamma is close to nothing.
- The lower, wider hump is an option with a year to run.
- Its delta changes more gently, so the peak is lower and the shape is spread out.
- So gamma is highest near the money and close to expiry.
- Gamma is the same number for a call and a put with the same terms.

7 Example 2: the other Greeks, same option

Same call and put; each Greek on its own

Gamma, call and put	per $1 move in the share	**0.0188**
Vega, call and put	per 1 point of volatility	**+0.38**
Theta, call	per day	**−0.018**
Theta, put	per day	**−0.005**
Rho, call and put	per 1 point of rates	**+0.53 / −0.42**

IN THE LESSON

- Example two uses the very same call and put.
- Gamma is zero point zero one eight eight, and it is identical for both.
- So if the share rises one dollar, each option's delta rises by about two hundredths.
- Vega is zero point three eight for the call and zero point three eight for the put.
- A rise of one point in volatility adds thirty eight cents to either one.
- Theta is minus one point eight cents a day for the call and minus half a cent a day for the put.
- Rho is plus fifty three cents for the call and minus forty two cents for the put, for a one point rise in rates.
- Now we have all five on one page.

8 Reading the signs, calls against puts

Long call / long put; the writer has the opposite sign

Greek	Long call / long put
Delta, rho	positive / negative
Gamma, vega	positive / positive
Theta	usually negative / usually negative

IN THE LESSON

- Look at the signs side by side.
- Delta and rho have opposite signs for calls and puts, because they depend on direction.
- A call gains when the share rises and when rates rise, since a higher rate cuts the present value of the strike you will pay.
- A put does the reverse.
- Gamma and vega are positive for both.
- They come from the shape of the option, a value that curves upward whichever way the option points, and from the fact that more volatility makes a big move more likely, which both buyers like.
- Theta is usually negative for a long option, because time running down shrinks the chance of a big move.
- The writer collects that decay.

9 The trap: put vega is not negative

- Call minus put = share minus PV of strike
- Volatility is in neither side of that rule
- So call vega equals put vega, both positive
- Opposite signs: delta, rho. Same: gamma, vega

IN THE LESSON

- Here is the trap that catches many candidates.
- A call and a put move in opposite directions when the share price moves, so it is tempting to think they must also move in opposite directions when volatility moves.
- They do not.
- Put call parity says the call minus the put equals the share minus the present value of the strike.
- Neither the share price nor the strike depends on volatility.
- So if volatility changes, the call and the put must change by the same amount.
- That is why both vegas equal thirty eight cents in our example.
- Higher volatility makes both options worth more.
- Remember: opposite on delta and rho, same on gamma and vega.

10 Why dealers hedge

- Selling calls creates exposure to the share
- Dealer wants the fee, not a bet on the share
- Hold other positions so total delta is zero
- Delta neutral: small moves leave value unchanged

IN THE LESSON

- Picture a baker who agrees to sell a thousand loaves next month at a fixed price, but does not yet own the flour.
- If flour jumps in price, the baker loses.
- So the baker buys flour today to cover the promise.
- Option dealers face the same thing.
- A dealer who sells call options has promised something whose value rises when the share rises.
- The dealer does not want a bet on the share.
- The dealer wants to earn a fee for making the market.
- So the dealer holds just enough of the share to cancel the exposure.
- That is delta hedging.
- The aim is a position whose deltas add up to zero, called delta neutral.

11 The hedge size formula

Units = −(portfolio delta) ÷ (delta of one hedge unit)

Portfolio delta	Delta of what you hold now
Delta of one hedge unit	Share = 1; put = its delta
Hedge units to trade	Positive = buy, negative = sell

IN THE LESSON

- Here is the recipe for how much to trade.
- First, work out the delta of the whole position you are hedging.
- Then pick the instrument you will hedge with, often the share itself, and find the delta of one unit of it.
- The number of units to trade equals minus the portfolio delta, divided by the delta of one hedge unit.
- The minus sign makes the new positions cancel the old.
- Notice that you divide.
- The share has a delta of exactly one, so dividing by it changes nothing.
- A put has a delta that is a fraction, so you divide by that fraction, and the answer is a larger number of puts than shares.

12 Example 3: a dealer hedges short calls

Short 10,000 calls, each with delta 0.55

Delta of the short calls	10,000 × (−0.55)	**−5,500**
Delta of one share	always	**1**
Units to trade	−(−5,500) ÷ 1	**+5,500**
Action	buy shares	**buy 5,500**
Total delta after	−5,500 + 5,500	**0**

IN THE LESSON

- Example three.
- A dealer has sold ten thousand call options on a share.
- Each call has a delta of zero point five five.
- The dealer is short the calls, so the position's delta is ten thousand times minus zero point five five, which is minus five thousand five hundred.
- The book behaves like being short five thousand five hundred shares.
- To cancel that, the dealer needs plus five thousand five hundred of delta.
- A share has a delta of one, so the units to trade are minus the minus five thousand five hundred, divided by one.
- The dealer buys five thousand five hundred shares.
- Total delta is now zero.

13 Example 4: a shareholder hedges with puts

Own 10,000 shares; each put has delta −0.40

Delta of the shares	10,000 × 1	**+10,000**
Units to trade	−10,000 ÷ (−0.40)	**+25,000**
Action	buy puts	**buy 25,000**
Trap: multiply instead	10,000 × 0.40	**4,000 puts**
Trap leaves unhedged	10,000 − 4,000 × 0.40	**8,400**

IN THE LESSON

- Example four.
- A client holds ten thousand shares and wants protection using put options.
- Each put has a delta of minus zero point four zero.
- The client's delta is plus ten thousand, one for each share.
- Apply the formula: minus ten thousand, divided by minus zero point four zero.
- A negative divided by a negative is positive, so the answer is twenty five thousand puts to buy.
- Now the trap.
- Many people multiply ten thousand by zero point four zero and buy four thousand puts.
- Four thousand puts only cancel one thousand six hundred of delta, leaving the client with eight thousand four hundred of delta, still mostly unhedged.

14 The hedge is a cycle, not a one-off

Set the hedge

Measure portfolio delta →
Units = −delta ÷ hedge delta →
Trade; delta is now zero

Keep it right

Share moves or time passes →
Delta drifts from zero →
Rebalance with new trade

IN THE LESSON

- A delta hedge is never finished, and here is why.
- It is built on the delta at a particular price and a particular moment.
- But delta changes when the share price moves, and it also changes as time passes.
- So the hedge only holds for small moves.
- The picture is a cycle.
- Measure the portfolio delta.
- Work out the units with the formula.
- Trade them.
- Then wait.
- The share moves, or a day goes by, and the delta has drifted away from zero.
- So the dealer measures again, and trades again.
- That repeated trading is called rebalancing, and it costs money every time the dealer does it.

15 Example 5: rebalancing after a rise

Short 10,000 calls; share rises; call delta 0.62

New delta of the calls	10,000 × (−0.62)	**−6,200**
Shares held	from example three	**5,500**
Shortfall	6,200 − 5,500	**700**
Action	buy more shares	**buy 700**
Share holding now	5,500 + 700	**6,200**

IN THE LESSON

- Example five picks up the dealer from example three.
- The share rises.
- At the new price each call's delta is zero point six two.
- The dealer is still short ten thousand calls, so the exposure is now ten thousand times zero point six two, which is six thousand two hundred shares' worth.
- But the dealer holds only five thousand five hundred shares.
- The hedge is seven hundred short.
- Apply the formula again: the dealer must buy another seven hundred shares.
- Notice the direction.
- A dealer short calls buys after the share has risen, at the higher price.
- If the share falls, delta falls, and the dealer sells.

16 Four mistakes so far

- 1. Put vega taken as negative; it is positive
- 2. Delta range for calls and puts mixed up
- 3. Multiplying by put delta instead of dividing
- 4. Selling when the hedge needs buying

IN THE LESSON

- Let me gather the slips from the first half.
- One, thinking put vega is negative because puts and calls move in opposite directions with the share.
- Both vegas are positive.
- Two, mixing up the unit of delta.
- Delta is value change per one dollar move, and a call's lies between zero and one while a put's lies between minus one and zero.
- Three, multiplying by the put delta when you should divide, which leaves you under hedged.
- Four, selling shares when the hedge requires buying.
- A dealer short calls holds shares, and buys more after a rise.
- Work out the sign of the exposure first, and the direction of the trade follows.

17 Delta neutral does not mean safe

- A delta hedge assumes delta stays fixed
- Big moves change delta, hedge goes stale
- The leftover risk is gamma risk
- Delta neutral does not mean no price risk

IN THE LESSON

- A delta hedge is like wearing a seat belt that only works at low speed.
- For gentle bumps it holds.
- For a crash it is not enough.
- Why?
- Because the hedge treats delta as a fixed number.
- We saw that delta moves when the share moves.
- A dealer who is delta neutral is protected against a tiny move, but a big move changes delta, and the hedge is suddenly the wrong size.
- The leftover risk has a name: gamma risk.
- Gamma, remember, is the speed at which delta changes.
- It is what a delta hedge leaves behind.
- So a delta neutral book can still lose money, and the exam loves that point.

18 Estimating the change in value

Change in value ≈ Delta × move + ½ × Gamma × move²

Change in option value	Estimated change in option value
Delta effect	Straight part; hedge cancels
Gamma effect	Curved part; gain if long
Move in the share	Dollar move in the share

IN THE LESSON

- To measure what a move does to an option, we use two terms.
- The first is delta times the move in the share.
- That is the straight line part, and it is what the delta hedge cancels.
- The second is one half times gamma times the move squared.
- That is the curved part.
- Because the move is squared, it is always positive for a positive gamma, whichever way the share goes.
- The delta also changes, by about gamma times the move.
- So for an option you own, a big move helps you, in either direction, by the gamma term.
- For an option you have written, the same term hurts you in either direction.

19 The gap a delta hedge leaves

Chart: Call value vs Share price (hedge set at 100)

- Call value
- Delta line
- Gap = gamma

IN THE LESSON

- Here is gamma as a picture.
- The share price runs along the bottom and the option's value goes up the side.
- The green curve is the call's value.
- It bends upward.
- The gold straight line touches it at one hundred, where the hedge was set, and it is what the delta hedge assumes: that value rises in a straight line with the share.
- Close to the touching point, the line and the curve are nearly the same.
- Move either way and the curve pulls away above the line.
- That gap is gamma at work.
- If you own the call, the gap is a gain whichever way the share goes.
- If you wrote the call, it is a loss whichever way.

20 Example 6: a delta neutral book loses either way

Short 1,000 calls (delta 0.50, gamma 0.04); 500 shares at $100

Call change, share +$5	0.50×5 + ½×0.04×25	**+$3.00**
Book, share +$5	500×5 − 1,000×3.00	**−$500**
Call change, share −$5	0.50×(−5) + ½×0.04×25	**−$2.00**
Book, share −$5	−500×5 + 1,000×2.00	**−$500**

Delta neutral, yet a loss in both directions

IN THE LESSON

- Example six.
- A dealer is short one thousand at the money calls.
- Each has a delta of zero point five and a gamma of zero point zero four.
- The dealer is delta hedged with five hundred shares at one hundred dollars.
- Now the share jumps five dollars up.
- Each call gains zero point five times five, plus a half times zero point zero four times twenty five, which is three dollars.
- The shares gain two thousand five hundred, but the calls cost three thousand.
- Net, minus five hundred.
- Now suppose it falls five dollars instead.
- Each call loses only two dollars, so the calls give back two thousand, but the shares lose two thousand five hundred.
- Net, minus five hundred again.

21 Rebalancing: buying high, selling low

Shares needed to stay hedged

Share at 95	**300**
Share at 100	**500**
Share at 105	**700**

Buy 200 at 105 after a rise; sell 200 at 95 after a fall

IN THE LESSON

- Now see what the dealer must do next, shown as bars.
- Each call's delta changes by about gamma times the move.
- After the share rises five dollars, delta goes from zero point five to zero point seven.
- A thousand calls now need seven hundred shares, not five hundred.
- So the dealer buys two hundred more at one hundred and five dollars.
- After a fall of five dollars, delta drops to zero point three, so only three hundred shares are needed, and the dealer sells two hundred at ninety five.
- The middle bar is where the hedge started.
- Buying after rises and selling after falls is how a short gamma book locks in its losses.

22 Long gamma and short gamma

- Long options: long gamma, gains from big moves
- Pays for it through time decay, theta
- Short options: short gamma, collects theta
- Only other options can change gamma, not shares

IN THE LESSON

- Every position falls on one side of this.
- Someone who owns options is long gamma.
- They gain from big moves in either direction, and they trade the right way round: they sell after rises and buy after falls, selling high and buying low.
- But long gamma is not free.
- They pay for it through time decay, theta, losing value every quiet day.
- Someone who writes options is short gamma.
- They earn the time decay, but they are hurt by big moves and rebalance the wrong way round.
- Last point: you cannot cancel gamma with the share, because the share's gamma is zero.
- Only another option, which has gamma, will do it.

23 Example 7: neutralising gamma with options

Short 1,000 calls (gamma 0.04); hedge option gamma 0.02, delta 0.30

Book gamma	−1,000 × 0.04	**−40**
Options to buy	40 ÷ 0.02	**2,000**
Delta they add	2,000 × 0.30	**+600**
Shares to sell	offset the +600	**sell 600**
Shares held now	500 − 600	**−100**

IN THE LESSON

- Example seven shows the fix.
- The dealer from example six is short a thousand calls with gamma of zero point zero four each, so the book's gamma is minus forty.
- Another option is available with a gamma of zero point zero two and a delta of zero point three.
- To get plus forty of gamma the dealer buys forty divided by zero point zero two, which is two thousand of them.
- That also adds two thousand times zero point three, so six hundred of delta.
- The book is no longer delta neutral, so the dealer sells six hundred shares.
- Gamma is now zero and delta is zero.
- The share never touched the gamma, only the options did.

24 Prices in volatility, not in money

- Money prices differ by strike, maturity, price
- Implied volatility: the volatility the price implies
- Found by working backwards from market price
- A common scale for comparing options

IN THE LESSON

- When you buy fruit, you compare a price per kilo, not the price of each bag.
- It makes different bags comparable.
- Option traders do the same.
- An option's price in money depends on the strike, the maturity and the share price, so it is hard to compare two options.
- Instead they quote volatility.
- Implied volatility is the volatility that, when you put it into the pricing model, gives back the market price of the option.
- The market price is the output of buying and selling.
- The volatility is what the market is implying.
- You work backwards from price to volatility.
- Then every option is on the same scale, a price per kilo.

25 Example 8: finding the implied volatility

Call at $7.18, share 100, strike 100, 6 months, rate 4%

Try volatility 20%	model gives $6.63	**too low: raise**
Try volatility 25%	model gives $8.01	**too high: lower**
Try volatility 22%	model gives $7.18	**match**
Implied volatility	the search ends here	**22%**

IN THE LESSON

- Example eight.
- A six month at the money call on a one hundred dollar share trades at seven dollars eighteen cents, and the risk free rate is four per cent.
- The model has no formula to run backwards, so we search by trial.
- Try twenty per cent volatility.
- The model gives six dollars sixty three.
- That is below the market price, so raise the volatility.
- Try twenty five per cent.
- The model gives eight dollars and one cent, above the market, so lower it.
- Try twenty two per cent.
- The model gives seven dollars eighteen, a match.
- Implied volatility is twenty two per cent.
- Because value rises steadily with volatility, only one answer exists.

26 Value rises with volatility

Chart: Call value vs Volatility, 15% to 30%

- Market price $7.18
- Implied: 22%
- Model value

IN THE LESSON

- Here is why the search always finds one answer.
- Volatility runs along the bottom, from fifteen to thirty per cent, and the call's model value goes up the side.
- The green curve climbs steadily, never flat and never turning back.
- The red horizontal line is the market price, seven dollars eighteen.
- Follow it across until it meets the curve, then drop straight down.
- You land on twenty two per cent.
- That is the implied volatility.
- If the curve ever doubled back, there could be two answers, but it never does.
- At eighteen per cent, the model gives six dollars eight cents, which is the value on a trader's own view.

27 Using implied volatility to find a rich option

The view

Implied 22% against expected 18% → Option looks rich by about $1.10

The trade

Sell the call → Delta hedge with shares → Profit if volatility is lower

IN THE LESSON

- How do traders use this?
- They compare implied volatility with the volatility they expect the share to actually deliver.
- In our example the call implies twenty two per cent, but the trader expects only eighteen per cent.
- At eighteen, the call would be worth six dollars eight cents, about a dollar ten less than the market asks.
- So the option looks rich, or dear.
- The trader sells it.
- But selling a call alone is a bet that the share falls, and the trader has no such view.
- So the trader delta hedges, which leaves a position that profits if the share turns out less volatile than the price implies.
- It is a trade on volatility, not on direction.

28 A smile across strikes (illustration)

Chart: Implied volatility vs Strike price (low to high)

- Flat line: one volatility
- Higher at low strikes

IN THE LESSON

- Now a refinement.
- The pricing model assumes one volatility for a share.
- If that were true, every option on the same share and expiry would imply the same number, and the picture would be a flat line.
- Here is a made up illustration of what markets actually show.
- Strike runs along the bottom, implied volatility up the side.
- The curve dips near the current share price and rises on both sides.
- That shape is a smile.
- For equity indices the left side is often higher, so it is more of a smirk.
- Draw it for many maturities and you get a surface.
- Traders judge an option as cheap or rich against its neighbours on the surface.

29 Parity, and the last trap

- Same strike and expiry: call and put share one IV
- Different values would allow a free profit
- IV is an output of market prices, not an input
- Compare it with your own forecast of volatility

IN THE LESSON

- Two last points on implied volatility.
- First, put call parity.
- A European call and put with the same strike and expiry are tied together by parity.
- If they implied different volatilities, you could combine one with the share and some borrowing or lending to beat the other, and make a risk free profit.
- So they must carry the same implied volatility.
- Second, the trap.
- Students often treat implied volatility as an input that the trader types into the model.
- It is the reverse.
- The market price comes first.
- Implied volatility is the output you work out from that price, then compare with your own forecast.

30 Recap: the whole lesson in four lines

- Greeks: delta, gamma, vega, theta, rho
- Hedge units = −portfolio delta ÷ hedge delta
- Gamma risk: delta neutral is not risk free
- Implied volatility is an output of the price

IN THE LESSON

- Let me pull it together.
- First, the Greeks.
- Delta is the value change per move in the share, zero to one for calls and minus one to zero for puts.
- Gamma is the change in delta, the same for calls and puts.
- Vega is positive for both, theta is usually negative for a long option, and rho is positive for calls and negative for puts.
- Second, a delta hedge uses units equal to minus the portfolio delta, divided by the hedge delta, and needs rebalancing.
- Third, gamma is what the hedge leaves behind, so a delta neutral book can still lose.
- Fourth, implied volatility is read from the market price, not typed in.

31 Try three quick questions

1. Own 5,000 shares; puts have delta −0.25. How many puts?
   Answer: −5,000 ÷ (−0.25) = buy 20,000 puts.
2. Short 2,000 calls, each delta 0.40. How many shares to buy?
   Answer: 2,000 × 0.40 = buy 800 shares.
3. Delta 0.5, gamma 0.04, share +$3. Option change?
   Answer: 0.5×3 + ½×0.04×9 = 1.50 + 0.18 = $1.68

IN THE LESSON

- Now three quick questions.
- Pause after each one.
- Question one.
- An investor owns five thousand shares and hedges with puts that each have a delta of minus zero point two five.
- How many puts does the investor buy?
- Minus five thousand divided by minus zero point two five is twenty thousand puts.
- Question two.
- A dealer is short two thousand calls, each with a delta of zero point four zero.
- How many shares does the dealer buy?
- Two thousand times zero point four zero is eight hundred shares.
- Question three.
- An option has a delta of zero point five and a gamma of zero point zero four.
- The share rises three dollars.
- By about how much does the option rise?
- One point five plus zero point one eight, so one dollar sixty eight.
- Well done.