Growth phases and multistage dividend models

Equity Investments

1 A story: the orchard

- Young orchard: plant more trees, take little home
- Neighbours arrive: each new tree earns less
- Full orchard: slow, steady growth, most fruit taken home
- A company follows the same three stages

IN THE LESSON

- Picture a farmer who plants an orchard.
- In the first years the young trees give little fruit, and the farmer spends nearly everything on more trees, water and fences.
- Nothing is taken home, but the orchard grows fast.
- Later the neighbours plant orchards too, good land runs short, and each new tree earns less than the last.
- The farmer plants fewer trees and starts taking fruit home.
- Finally the orchard is full.
- Growth is slow and steady, and nearly all of the fruit is taken home.
- A company is much the same.
- Early on it keeps its profits to grow, then it keeps less as the best chances are used up, and in the end it pays most of its profits out.

2 Three phases, drawn

Chart: Growth and payout (%) vs Time

- Growth rate
- Payout ratio
- Growth
- Transition
- Maturity

IN THE LESSON

- Here is the picture.
- The horizontal axis is time, the vertical axis is a rate, and everything is an illustration.
- Follow the green line from the left.
- It is the growth rate of earnings.
- It starts high in the growth phase, falls through the transition phase, and levels off low in the mature phase.
- Now the gold line, the payout ratio, which is the share of earnings paid out as dividends.
- It does the opposite.
- It starts close to zero, because the company needs its profits for growth, then climbs as the best investments run out, and settles high in maturity.
- The two grey lines mark where one phase ends and the next begins.

3 What each phase looks like

Wren Devices, a made-up maker of medical sensors

Phase	What happens
Growth	Fast; payout low
Transition	Slowing; payout up
Maturity	Stable; payout high
Return on equity	Falls towards r

IN THE LESSON

- Now the same story in words.
- In the growth phase, a company has a real edge, perhaps a new product.
- Earnings grow fast, margins are high, and the company reinvests almost everything, so dividends are small or nil and free cash flow may even be negative.
- In the transition phase, competitors arrive.
- Growth slows towards the rate of the whole economy, margins narrow, investment needs ease, and the payout ratio rises.
- In the mature phase, the company earns a return on equity close to what investors require, growth is stable, and payout is high.
- Multistage dividend models copy these phases, with a separate growth assumption for each.

4 Growth comes from what is kept

g = b × ROE, where b = 1 − payout ratio

Retention b	Share of earnings kept
ROE	Return earned on kept equity
Growth g	Sustainable growth rate

IN THE LESSON

- A one line reminder from Level One.
- A company that keeps part of its earnings and earns a return on the equity it keeps can grow at a rate set by those two choices.
- The retention rate is one minus the payout ratio.
- Multiply it by the return on equity, said R O E, and you have the sustainable growth rate.
- This formula explains the phases.
- In the growth phase, the return on equity is high and the retention rate is high, so growth is fast.
- As competition cuts the return on equity and the company keeps less, growth falls.
- In maturity, both are lower, and growth settles at a modest rate.

5 Example 1: Wren Devices through its life

Wren's required return r is 11%

Growth phase	ROE 24%, payout 10%	**21.6%**
Transition phase	ROE 16%, payout 45%	**8.8%**
Mature phase	ROE 11%, payout 70%	**3.3%**

IN THE LESSON

- Example one.
- Wren Devices has a required return of eleven per cent.
- In the growth phase, Wren earns twenty-four per cent on equity and pays out ten per cent, so it keeps ninety per cent.
- Growth is zero point nine times twenty-four, which is twenty-one point six per cent.
- In the transition phase, return on equity has fallen to sixteen per cent and payout has risen to forty-five per cent.
- Growth is zero point five five times sixteen, eight point eight per cent.
- In maturity, return on equity has settled at eleven per cent, equal to the required return, and payout is seventy per cent.
- Growth is zero point three times eleven, three point three per cent.

6 The trap: freezing the growth phase

- Growth phase: 21.6% growth against 11% required return
- r − g = 11 − 21.6 = −10.6: a negative gap
- A one-stage model gives a nonsense price
- Model the fade in stages instead

IN THE LESSON

- The classic trap is to take a company in its growth phase and assume that its return on equity and growth will last for ever.
- Try it on Wren.
- A single stage model needs the required return to be larger than the growth rate.
- Wren's required return is eleven per cent, and its current growth is twenty-one point six per cent.
- The gap, required return minus growth, is minus ten point six, a negative number.
- The formula would give a negative price, which is nonsense.
- Growth above the required return cannot last for ever, because competitors would pile in.
- So you must model the fade, and that is why multistage models exist.

7 A story: the cliff and the slope

- Cliff: full speed, then a sudden drop to the floor
- Slope: a little slower with every step
- Same floor at the end, very different journey
- Match the model to the shape

IN THE LESSON

- Imagine two hikers walking along a high ridge.
- The first ridge ends at a cliff.
- The hiker walks at full speed until the edge, then drops straight to the valley floor.
- The second ridge slopes gently down, and the hiker slows a little with every step until reaching the valley floor.
- Both end on the same floor, but the cliff hiker spent far more time up high.
- Companies lose their advantage in these two ways.
- A patent that ends on a known date is a cliff.
- A brand that rivals wear away year by year is a slope.
- The model you pick should match the shape of the ridge.

8 Three growth paths, drawn

Chart: Dividend growth (%) vs Years from now

- Two-stage: cliff
- H-model: slope
- Three-stage
- Long-run rate

IN THE LESSON

- Now the shapes as a picture.
- The horizontal axis is years, the vertical axis is the dividend growth rate.
- Start with the cliff, the two stage model: growth stays at fourteen per cent for eight years, then drops straight to the long run rate of four per cent.
- Next the slope, the H model: growth falls in a straight line from fourteen to four over those same eight years.
- Last, the plateau and slope, the three stage model: growth stays high for two years, then falls in a straight line over eight years, then holds at four.
- Notice that the cliff line sits above the others for years.
- That extra growth is worth a lot of money.

9 Choosing the model

- Advantage ends on a known date: two-stage
- Advantage erodes gradually: H-model, an approximation
- High, then a fade, then steady: three-stage
- Irregular payout or rates: spreadsheet

IN THE LESSON

- Each model encodes a pattern of growth, so how do you choose?
- The two stage model is a cliff, the H model a straight slope, the three stage model a plateau and then a slope, and a spreadsheet can draw any path.
- Ask what the advantage is, and how it will end.
- If it ends on a known date, such as a patent expiry, the cliff of the two stage model is honest.
- If competition wears the advantage away gradually, the slope of the H model is better, but remember it is an approximation, and it works best when the fade is short or the gap between the two rates is small.
- If growth stays high for a while and then fades, use three stages.
- If payout, margins or the required return will change in an irregular way, build a spreadsheet.
- The trap is choosing an abrupt drop when growth will really fade gradually.
- That overstates the value, as we will see.

10 A story: selling the house

- Rent for three years, then sell the flat
- Value today = rent collected + sale price, both discounted
- The sale price at the horizon is the terminal value
- It stands in for every later dividend

IN THE LESSON

- Suppose you buy a flat to rent out and plan to sell it in three years.
- What is the flat worth to you today?
- It is the rent you collect for three years, plus the price you expect to sell it for, brought back to today.
- A multistage dividend model works the same way.
- You forecast dividends in detail for a few years, then assume you sell the share at the end of that horizon.
- The price at the horizon is called the terminal value.
- It stands in for every dividend after the last one you forecast.
- It is often most of the value, so we must learn the two ways to set it and the mistakes people make.

11 Terminal value by constant growth

TV(n) = D(n+1) ÷ (r − gL), then discount n years

Next dividend	Dividend one year past horizon
Required return	Required return
Long-run growth	Long-run growth, below r
Terminal value	Value at the horizon

IN THE LESSON

- The first way uses the constant growth model from Level One, the Gordon model, at the horizon.
- Say the horizon is year n.
- From then on, dividends grow at a steady long run rate that is below the required return.
- The value at year n is the dividend of year n plus one, divided by the required return minus the long run growth rate.
- Notice the dividend: it is the next one, the one one year past the horizon, because the price at any date is the value of the dividends that come after that date.
- Then you discount this value back to today by n years, not n plus one.

12 Example 2: Heron Foods, constant growth

Heron: D1 1.50, D2 1.65, D3 1.80; r 9%; gL 4% after year 3

PV of years 1 to 3	1.376 + 1.389 + 1.390	**4.155**
Year 4 dividend	1.80 × 1.04	**1.872**
Terminal value, year 3	1.872 ÷ 0.05	**37.44**
PV of terminal value	37.44 ÷ 1.09^3	**28.911**
Value today	4.155 + 28.911	**33.07**

IN THE LESSON

- Example two.
- Heron Foods is forecast to pay one point five zero, one point six five and one point eight zero in years one to three.
- The required return is nine per cent, and after year three dividends grow at four per cent for ever.
- The present value of the three dividends is one point three seven six, plus one point three eight nine, plus one point three nine zero, a total of four point one five five.
- The year four dividend is one point eight zero times one point zero four, one point eight seven two.
- The terminal value is that divided by zero point zero five, thirty-seven point four four.
- Discounted three years, it is twenty-eight point nine one.
- Heron is worth thirty-three point zero seven.

13 Where the value sits

Present value per share

Years 1 to 3	**4.15**
Terminal value	**28.91**

Terminal value is 87% of Heron's value

IN THE LESSON

- Look at where the value of Heron sits.
- The first bar is the present value of the three dividends you forecast, one at a time: about four dollars fifteen.
- The second bar is the present value of the terminal value: about twenty-eight dollars ninety-one.
- The terminal value is more than eighty-seven per cent of the total.
- The years you worked so hard to forecast are a small slice, and almost everything depends on the long run growth rate and the required return behind that one number.
- If you change long run growth by one point, the value moves a great deal.
- So treat the terminal value as the most important assumption in the model, and test it.

14 Example 3: terminal value by a multiple

Heron: year-3 EPS 3.10; peer P/E 14; r 9%

Terminal value by multiple	14 × 3.10	**43.40**
PV of terminal value	43.40 ÷ 1.09^3	**33.51**
Value by multiple	4.155 + 33.513	**37.67**
Value by Gordon, from before		**33.07**
P/E implied by the Gordon value	37.44 ÷ 3.10	**12.08**

IN THE LESSON

- The second way applies a multiple to the earnings forecast for the horizon.
- Say peers trade at fourteen times earnings, the P E ratio, and Heron's earnings per share in year three are forecast at three point one zero.
- The terminal value is fourteen times three point one zero, forty-three point four zero.
- Discounted three years, thirty-three point five one.
- Add the dividends, and the value is thirty-seven point six seven, higher than the constant growth answer.
- The multiple that the Gordon value implies is only twelve point zero eight.
- A multiple ties the terminal value to market evidence, but it also imports any mispricing in the peers.
- If peers are overpriced, so is your terminal value.

15 Example 4: two slips in terminal value

Heron again; correct value is 33.07

Correct value	4.155 + 28.911	**33.07**
Used D3, not D4	4.155 + 27.799	**31.95**
Discounted four years	4.155 + 26.523	**30.68**

Slips cost 3.4% and 7.2%

IN THE LESSON

- Now the two usual slips, on Heron's numbers.
- The correct value is thirty-three point zero seven.
- First slip: using the year three dividend instead of the year four dividend.
- Divide one point eight zero by zero point zero five and the terminal value is thirty-six, not thirty-seven point four four.
- Heron then comes out at thirty-one point nine five, about three per cent too low.
- Second slip: discounting the terminal value for four years instead of three.
- The terminal value sits at the end of year three, so it moves back three years only.
- Discount it four years and the value becomes thirty point six eight, over seven per cent too low.

16 Mistakes so far

- Single-stage model on a company still in its growth phase
- Abrupt drop when growth will fade gradually
- Using D(n) instead of D(n+1) in the terminal value
- Discounting the terminal value n+1 years
- Long-run growth at or above r; trusting a peer P/E blindly

IN THE LESSON

- Let us pause on the mistakes you are most likely to meet.
- One: freezing the growth phase, valuing a fast grower with a single stage model, which gives nonsense when growth beats the required return.
- Two: choosing an abrupt drop in growth when the advantage will really fade gradually, which overstates the value.
- Three: using the dividend of the horizon year in the terminal value formula, instead of the dividend one year later.
- Four: discounting the terminal value one period too many.
- Five: choosing a long run growth rate at or above the required return, which breaks the formula.
- And six: trusting a peer multiple without asking whether the peers are fairly priced.

17 The two-stage model

V0 = Σ D(t) ÷ (1+r)^t + TV(n) ÷ (1+r)^n

Fast-years dividends	Dividends in the fast years
Terminal value	D(n+1) ÷ (r − gL)
Discounting	Back n years, not n+1

IN THE LESSON

- Now we value shares with the models, starting with the two stage model.
- Stage one is a run of years of high growth, in which you forecast each dividend and discount it.
- Stage two begins when growth drops to the long run rate.
- You capture all of stage two in the terminal value at the end of stage one, using the constant growth formula, and discount that value back by the same number of years as stage one.
- The share value today is the sum of the present values of the stage one dividends plus the present value of the terminal value.
- It is just the Heron calculation with a growth pattern behind it.

18 Example 5: the fast-years dividends

Orla: D0 2.00; 12% for 5 years; r 9%

Dividend ÷ 1.09 to the year	PV
Years 1, 2: 2.24, 2.51	2.06, 2.11
Year 3: 2.81 ÷ 1.09^3	2.17
Year 4: 3.15 ÷ 1.09^4	2.23
Year 5: 3.52 ÷ 1.09^5	2.29

IN THE LESSON

- Example five.
- Orla Tools has just paid a dividend of two dollars.
- Its patent protects it for five more years, in which dividends grow at twelve per cent a year.
- After that growth falls to four per cent for ever.
- The required return is nine per cent.
- First the dividends: two point two four in year one, two point five one in year two, two point eight one, three point one five, and three point five two in year five.
- Each one is divided by one point zero nine, raised to its year.
- The present values are two point zero six, two point one one, two point one seven, two point two three and two point two nine.
- Together they come to ten point eight six.

19 Example 5 continued: the terminal value

Orla: gL 4% from year 6; r 9%

PV of dividends, years 1 to 5		**10.86**
Year 6 dividend	3.52 × 1.04	**3.67**
Terminal value, year 5	3.67 ÷ 0.05	**73.31**
PV of terminal value	73.31 ÷ 1.5386	**47.65**
Value today	10.86 + 47.65	**58.51**

IN THE LESSON

- Now stage two.
- The year five dividend is three point five two.
- The year six dividend is that times one point zero four, three point six seven.
- The terminal value at the end of year five is three point six seven divided by zero point zero five, seventy-three point three one.
- Discount it five years, dividing by one point five three eight six, and it is worth forty-seven point six five today.
- Add the ten point eight six from the fast years, and Orla Tools is worth fifty-eight point five one.
- The terminal value is over eighty-one per cent of that.
- Even with five years of detail, the part after the patent expires dominates.

20 The H-model

V0 = D0(1+gL) ÷ (r − gL) + D0 × H × (gS − gL) ÷ (r − gL)

Stable part	Value at the long-run rate
Fade premium	Extra value from the fade
H	Half the fade length, in years

IN THE LESSON

- Now the H model, for the slope.
- It values a share whose dividend growth starts at a high rate and falls in a straight line to the long run rate.
- It has a closed form, so no year by year work is needed.
- The first part is the value if growth were the long run rate from the start: the last dividend times one plus the long run growth, over the required return minus the long run growth.
- The second part is a premium for the extra growth during the fade.
- It is the last dividend, times H, times the gap between the high and long run rates, over the same denominator.
- H is half the length of the fade, in years.

21 Example 6: Quill Textiles, a gentle fade

Quill: D0 1.50; 14% fading to 4% over 8 years; r 10%

H	8 ÷ 2	**4**
Stable part	1.50 × 1.04 ÷ 0.06	**26.00**
Fade premium	1.50 × 4 × 0.10 ÷ 0.06	**10.00**
Value today	26.00 + 10.00	**36.00**
Year-by-year check	same fade, exact	**34.94**

IN THE LESSON

- Example six.
- Quill Textiles has just paid one dollar fifty.
- Its dividend growth is now fourteen per cent, and rivals will wear that down in a straight line to four per cent over eight years.
- The required return is ten per cent.
- The fade lasts eight years, so H is four, half of eight.
- The stable part is one point five zero times one point zero four, over zero point zero six: twenty-six dollars.
- The fade premium is one point five zero times four times zero point one zero, over zero point zero six: ten dollars.
- The value is thirty-six.
- A year by year calculation of the same fade gives thirty-four point nine four, so the H model is a close approximation, a little high.

22 The cost of choosing the cliff

Value per share

Stable only	**26.00**
Exact fade	**34.94**
H-model	**36.00**
Abrupt drop	**48.74**

Same start, same end: only the path differs

IN THE LESSON

- Now the lesson of the cliff against the slope, on Quill's numbers.
- The first bar is the share if growth were four per cent from the start: twenty-six dollars.
- The second is the year by year fade, thirty-four ninety-four.
- The third is the H model, thirty-six.
- The fourth bar is an analyst who ignores the fade and keeps growth at fourteen per cent for eight years, then drops to four: forty-eight seventy-four.
- That is thirty-five per cent above the H model value.
- All four use the same start and the same end growth.
- The only difference is the path, and the path is worth many dollars.

23 The three-stage model, in order

Fast growth: explicit dividends →
Fade: H-model value at end of stage one →
Stable growth: inside that value →
Discount back to today

IN THE LESSON

- Now the three stage model, which builds on both.
- Here is the order of work.
- Stage one is the fast growth years, a short run in which you forecast each dividend and discount it.
- Stage two is the fade.
- You do not forecast it year by year.
- At the end of stage one you apply the H model, which values the fade and the stable years together, as a single number at that date.
- Stage three, the stable growth, is already inside that H model value.
- Then you discount the H model value back to today.
- The most common slip is in that last step, so count the years carefully.

24 Example 7: Fenn Marine, three stages

Fenn: D0 1.00; 18% for 2 years; fade to 5% over 6; r 10%

PV of years 1 and 2	1.073 + 1.151	**2.22**
Stable part at year 2	1.3924 × 1.05 ÷ 0.05	**29.24**
Fade premium at year 2	1.3924 × 3 × 0.13 ÷ 0.05	**10.86**
Value at year 2, to today	40.10 ÷ 1.21	**33.14**
Value today	2.22 + 33.14	**35.36**

IN THE LESSON

- Example seven.
- Fenn Marine has just paid one dollar.
- Dividends grow at eighteen per cent for two years, then growth falls in a straight line over six years to five per cent.
- The required return is ten per cent.
- The dividends are one eighteen and one thirty-nine.
- Their present values are one point zero seven and one point one five, together two point two two.
- At the end of year two, H is three.
- The stable part is one point three nine two four times one point zero five, over zero point zero five: twenty-nine point two four.
- The fade premium is one point three nine two four times three times zero point one three, over zero point zero five: ten point eight six.
- The value at year two is forty point one zero.
- Back two years, thirty-three point one four.
- Fenn is worth thirty-five point three six.

25 Example 8: two slips in the three-stage model

Fenn again; correct value is 35.36

Correct value	H = 3, back 2 years	**35.36**
H = 6, the full fade	premium 21.72	**44.34**
Discounted three years	40.10 ÷ 1.331	**32.35**

H slip: +25%. Discount slip: −9%

IN THE LESSON

- Example eight: two classic slips on Fenn.
- The correct value is thirty-five point three six.
- First slip: using the whole six year fade for H instead of half of it.
- The premium doubles to twenty-one point seven two, and the share comes out at forty-four point three four, about twenty-five per cent too high.
- Second slip: the value at year two is correct, but it is discounted three years instead of two.
- The share comes out at thirty-two point three five, about nine per cent too low.
- Both slips are easy to make under exam pressure, so say it to yourself: H is half the fade, and the value at year two comes back two years.

26 A story: the household budget

- Budget each year on its own line
- Growth, margins and payout may differ every year
- Close the forecast with a terminal value
- Discount everything to today

IN THE LESSON

- Think of a household budget on a spreadsheet.
- You do not assume your income grows at one rate for ever.
- You list each year: a pay rise here, a child starting school there, a mortgage paid off in year ten.
- Every year has its own line, and the final result follows from the lines.
- A spreadsheet model for a share works the same way.
- You forecast each year's earnings and payout, or each dividend, one by one, so growth, margins, payout and even the required return can change from year to year.
- Then you add a terminal value at the end and discount everything to today.
- It can follow any life cycle, which no single formula can.

27 How the sheet is built

Every forecast year

Earnings → Payout → Dividends

At the horizon

Terminal value → Discount → Value today

IN THE LESSON

- Here is how the sheet is built.
- Each forecast year is a column.
- Start at the top left.
- Earnings are projected from growth rates you set for each year.
- A payout ratio, which can also change each year, turns earnings into dividends.
- Those three steps are repeated for every year in the forecast.
- Then the bottom row.
- At the end of the forecast you set a terminal value, by the constant growth formula or by a multiple.
- Next you discount each dividend and the terminal value at the required return.
- And the sum is the value of the share today.
- The method is the same as before, with more detail in the first steps.

28 Example 9: Maple Rail's forecast

Maple Rail: earnings per share × payout ratio

Year: EPS × payout	Dividend
Year 1: 3.00 × 20%	0.60
Year 2: 3.40 × 35%	1.19
Year 3: 3.80 × 50%	1.90
Year 4: 3.80 × 1.04 × 60%	2.37

IN THE LESSON

- Example nine.
- Maple Rail, a made up freight company, is in transition.
- The analyst forecasts earnings per share of three dollars, three forty and three eighty in the next three years.
- The payout ratio rises from twenty per cent, to thirty-five, to fifty per cent.
- So dividends are sixty cents, one nineteen and one ninety.
- From year four, growth is four per cent and the payout is sixty per cent, which gives a year four dividend of two dollars thirty-seven.
- The required return is nine per cent.
- Notice that the payout is not a straight line, and the earnings growth is not constant.
- No one formula model would capture this path, but the sheet handles it easily.

29 Example 9 continued: the value

Maple Rail: gL 4% from year 4; r 9%

PV of dividends, years 1 to 3		**3.02**
Year 4 dividend	3.80 × 1.04 × 0.60	**2.37**
Terminal value, year 3	2.371 ÷ 0.05	**47.42**
PV of terminal value	47.42 ÷ 1.295	**36.62**
Value today	3.02 + 36.62	**39.64**

Terminal value is 92% of the total

IN THE LESSON

- Now the value.
- The present value of the three dividends is three point zero two.
- The year four dividend is two point three seven.
- The terminal value at the end of year three is that divided by nine per cent minus four per cent, forty-seven point four two.
- Discounted three years, thirty-six point six two.
- Maple Rail is worth thirty-nine point six four.
- And again, the terminal value is the big piece: over ninety-two per cent.
- The careful three year forecast has barely mattered, compared with what we assumed about growth after year three.
- A detailed model can look very precise while resting on a few guesses.

30 Example 10: testing one assumption

Maple Rail: only the long-run growth changes

Long-run growth 3%	terminal value 39.14	**33.24**
Long-run growth 4%, base	terminal value 47.42	**39.64**
Long-run growth 5%	terminal value 59.85	**49.23**

Moving growth by one point shifts the value by −16% to +24%

IN THE LESSON

- A strength of the spreadsheet is that it makes testing easy.
- Change one input and watch the answer.
- Keep Maple Rail's forecast as it is and change only the long run growth rate.
- At three per cent, the value is thirty-three point two four.
- At four per cent, the base case, thirty-nine point six four.
- At five per cent, forty-nine point two three.
- One point of growth moves the value by about six or ten dollars, which is a swing of up to twenty-four per cent.
- All of that comes through the terminal value.
- A sensitivity table like this tells the reader how much to trust the answer, which is more honest than a single number.

31 Strengths, costs and the trap

- Gain: any path, easy sensitivity tests
- Cost: false precision; formula errors hide in big sheets
- Still needs a terminal value, often most of the value
- Detailed does not mean accurate

IN THE LESSON

- Let us weigh the spreadsheet fairly.
- The gains: flexibility, because the model can follow whatever path the life cycle suggests, and easy sensitivity testing of every assumption.
- The costs: with many inputs, a model can look precise while resting on guesses, and a formula error hides easily in a large sheet.
- And it still needs a terminal value, which, as we just saw, often dominates the result.
- So here is the trap.
- A detailed model is not the same as an accurate model.
- Forty lines of inputs do not make the forecast better than the thinking behind them.
- Choose the simplest model that captures the shape of the growth you actually expect.

32 Recap: the formulas to remember

g = b × ROE TV(n) = D(n+1) ÷ (r − gL)
H-model: D0(1+gL)÷(r−gL) +
D0×H×(gS−gL)÷(r−gL) H = fade ÷ 2

Growth g	Growth from what is kept
Terminal value	Next dividend over r − gL
Two-stage	Cliff: terminal value at year n
H-model	Slope: stable part plus premium
Three-stage	Fast years, then H-model

IN THE LESSON

- Let us pull the lesson together.
- Companies move from growth to transition to maturity, and the models copy that.
- Sustainable growth is retention times return on equity.
- The terminal value at the horizon is the next dividend, over the required return minus long run growth, discounted back exactly n years.
- The two stage model suits a cliff, with the terminal value at the end of stage one.
- The H model suits a slope, with a stable part plus a premium, and H is half the fade.
- The three stage model puts the H model at the end of the fast years.
- And the spreadsheet follows any path, but still needs a terminal value.
- Choose the model to fit the shape of the fade.

33 Try three quick questions

1. D0 2.00; growth 10% fading to 4% over 4 yrs; r 9%. H-model value?
   Answer: H = 2; 41.60 + 4.80 = 46.40.
2. D3 = 2.00, gL 3%, r 8%. Terminal value at year 3?
   Answer: 2.00 × 1.03 ÷ 0.05 = 41.20.
3. Patent expires on a known date. Which model?
   Answer: Two-stage: growth drops abruptly.

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- A share has just paid two dollars.
- Growth will fall in a straight line from ten per cent to four per cent over four years, and the required return is nine per cent.
- What is its value by the H model?
- H is two.
- The stable part is two times one point zero four over zero point zero five, forty-one point six.
- The premium is two times two times zero point zero six over zero point zero five, four point eight.
- The value is forty-six point four.
- Question two.
- The year three dividend is two dollars, long run growth is three per cent, and the required return is eight per cent.
- What is the terminal value at the end of year three?
- Two times one point zero three over zero point zero five, forty-one point two.
- Question three.
- A firm's key patent expires on a known date.
- Which model fits?
- The two stage model, because growth drops abruptly.