CHARTA LEARNING · LESSON NOTES

Convertible bonds and credit risk basics

Fixed Income

1 A bond with a ticket to the shares

- Convertible = a bond plus the right to swap into shares
- Conversion ratio: shares received for each bond
- Two routes to value: the shares and the plain bond
- Premiums: what the buyer pays above each route

IN THE LESSON

- Imagine you lend a friend one thousand dollars, and the friend writes an IOU that says something extra.
- Instead of taking the cash back, you may swap this IOU for twenty shares of my company, whenever you like.
- That IOU is a convertible bond.
- It pays a coupon like any bond, and it repays the face value at the end, but the holder also owns a choice.
- Which route is worth more, the shares or the bond?
- The answer changes with the share price.
- In this first part we measure both routes, find the floor under the convertible, and then measure how much extra a buyer pays above each route.
- Those extras are called premiums.

2 Two routes and the floor under them

Conversion value = Ratio × Share price
Minimum value = higher of conversion value and straight value

Conversion ratio	Shares per bond
Share price	Today's price of one share
Conversion value	What converting is worth
Straight value	Same bond, no swap right
Minimum value	The higher of the two

IN THE LESSON

- Let's put numbers on the two routes.
- Take Harlow Instruments, a made-up company.
- Its convertible has a face value of one thousand dollars and a conversion ratio of twenty, so each bond swaps for twenty shares.
- Route one is the shares.
- The conversion value is the conversion ratio times the share price.
- Route two is the bond without the swap right, called the straight value.
- That is what an otherwise identical plain bond from the same issuer would be worth, found by discounting its payments at a yield suited to that issuer.
- The convertible should not trade below either route, so its minimum value is the higher of the two.

3 Example 1: the floor under the Harlow convertible

Harlow convertible: face 1,000, ratio 20, shares at 44

Conversion value	20 × 44	**880**
Straight value	plain bond, same issuer	**930**
Minimum value	higher of the two	**930**
Market price	what it trades at	**1,060**

IN THE LESSON

- Here is our first case.
- Priya Nandakumar, an analyst at Ashby Capital, is looking at the Harlow convertible.
- The shares trade at forty-four dollars.
- With a conversion ratio of twenty, the conversion value is twenty times forty-four, which is eight hundred and eighty dollars.
- A plain Harlow bond would be worth nine hundred and thirty dollars.
- The minimum value is the higher of the two, so it is nine hundred and thirty, and here the straight value is the floor.
- The conversion price, the face value divided by the ratio, is fifty dollars, and the shares sit below it, so converting now would be a poor move.
- Yet the market charges one thousand and sixty dollars.
- That gap is what buyers pay for the chance that the shares rise.

4 The floor and what sits on it

Value ($)

Conversion value	**880**
Straight value	**930**
Market price	**1,060**

Floor = the higher of the first two bars

IN THE LESSON

- Here are those three numbers as bars.
- The vertical axis is value in dollars.
- The first bar is the conversion value, eight hundred and eighty.
- The second is the straight value, nine hundred and thirty, a little taller, so it is the floor.
- The third is the market price, one thousand and sixty.
- The gap between the floor and the price is one hundred and thirty dollars.
- That is the price of a right to convert into shares that may yet rise.
- If the shares climbed above forty-six dollars and fifty cents, which is nine hundred and thirty divided by twenty, the first bar would overtake the second and become the floor instead.
- Until then this convertible trades much like a bond with some equity upside on top.

5 The two premiums, each on its own base

Market conversion price = Price ÷ Ratio
Premium ratio = (Mkt conv. price − Share price) ÷ Share price
Premium over straight = Price ÷ Straight value − 1

Market conversion price	Price paid per share
Share price	Base of the premium ratio
Premium ratio	How far shares must rise
Premium over straight value	How far to the bond floor

IN THE LESSON

- Now the premiums.
- The market conversion price is what you effectively pay per share if you buy the convertible and then convert it: the bond's price divided by the conversion ratio.
- One thousand and sixty divided by twenty is fifty-three dollars.
- Compare that with the forty-four dollars it costs to buy a share in the market.
- The difference, nine dollars, is the market conversion premium per share.
- Divide it by the share price and you have the premium ratio.
- There is a second premium too, the bond's price over its straight value, minus one.
- It measures the distance down to the bond floor.
- Each premium has its own base, and mixing the bases is the classic slip.

6 Example 2: the Harlow premiums

Harlow convertible priced at 1,060, shares at 44

Market conversion price	1,060 ÷ 20	**53.00**
Premium per share	53 − 44	**9.00**
Premium ratio	9 ÷ 44	**20.5%**
Premium over straight	1,060 ÷ 930 − 1	**14.0%**

Wrong base: 9 ÷ 50 = 18.0%

IN THE LESSON

- Let's run the Harlow numbers.
- The market conversion price is one thousand and sixty over twenty, fifty-three dollars.
- The premium per share is fifty-three minus forty-four, nine dollars.
- The premium ratio is nine over forty-four, twenty point five per cent.
- Read it like this: the shares must rise about twenty point five per cent, from forty-four to fifty-three, before converting is worth what you paid, and that ignores the coupons you collect on the way.
- Now the second premium.
- One thousand and sixty over nine hundred and thirty, minus one, is fourteen point zero per cent.
- That is how far the price sits above the bond floor, not how far it could fall.
- Watch the base trap.
- Divide the nine dollars by the conversion price of fifty and you get eighteen per cent, which understates the rise needed.

7 Example 3: the same bond after the shares rise

Shares at 60, convertible at 1,230, straight value still 930

Conversion value	20 × 60	**1,200**
Minimum value	higher of 1,200 and 930	**1,200**
Premium ratio	(61.5 − 60) ÷ 60	**2.5%**
Premium over straight	1,230 ÷ 930 − 1	**32.3%**

IN THE LESSON

- Now suppose the shares climb to sixty dollars and the convertible trades at one thousand two hundred and thirty.
- The conversion value is twenty times sixty, twelve hundred dollars, which is now well above the straight value of nine hundred and thirty.
- So the conversion value is the floor.
- The market conversion price is one thousand two hundred and thirty over twenty, sixty-one dollars and fifty cents.
- The premium per share is only a dollar fifty, and the premium ratio is two point five per cent.
- The bond now moves almost one for one with the shares.
- But look at the other premium.
- One thousand two hundred and thirty over nine hundred and thirty, minus one, is thirty-two point three per cent.
- The bond floor is a long way down, so the protection has become thin.

8 Reading the premiums: upside and downside

- Small premium ratio: behaves like the shares
- Small premium over straight value: floor is close
- Premium ratio: divide by the share price
- The floor moves with rates and the issuer's credit

IN THE LESSON

- Put the two premiums side by side and you can read the bond's character.
- A small premium ratio means you are paying little over the shares, so the convertible acts like the shares.
- A small premium over straight value means the floor is close, so downside is limited and the bond acts like a bond.
- Harlow at forty-four dollars had twenty point five per cent and fourteen per cent: a hybrid.
- At sixty dollars it had two point five per cent and thirty-two point three per cent: share-like.
- Two cautions.
- The premium ratio always uses the share price as its base.
- And the straight value is not fixed.
- It falls when rates rise or the issuer's credit weakens, so the floor is only as solid as the issuer.

9 A convertible is a package of parts

- Straight bond: the base
- Call on shares: the holder's conversion right
- Issuer's call: the issuer's right to redeem early
- Holder's put: the holder's right to sell back

IN THE LESSON

- Picture a travel package.
- The base is a flight, and that is the plain bond.
- On top, you get a free upgrade you may claim whenever you like, and that is the conversion right, an option on the shares.
- The airline, though, keeps the right to move you to a different flight, and that is the issuer's call on the bond.
- And you also hold a right to cancel for a refund at a set date, and that is the holder's put.
- Pricing the whole package as one lump is hard.
- Pricing each part separately is easier, and the arbitrage-free approach says the package must be worth the sum of its parts, otherwise someone could trade the difference for a risk-free profit.

10 Building the convertible from its parts

Whose option is it?

Straight bond 930 → + Call on shares 150 → − Issuer's call 40 → + Holder's put 20

Add it up

930 + 150 − 40 + 20 → Model value 1,060

IN THE LESSON

- Here is the build, in boxes.
- Start on the left with the straight bond, worth nine hundred and thirty dollars in our next example.
- Move right.
- The holder owns the conversion right, a call on the shares covered by the conversion ratio, worth one hundred and fifty, so it is added.
- Next, the issuer owns the right to call the bond, and what belongs to the other side hurts you, so forty comes off.
- Last, the holder owns a put, worth twenty, so it is added back.
- The second row adds it up: nine hundred and thirty plus one hundred and fifty minus forty plus twenty is one thousand and sixty.
- For each part, ask who owns it, and let the answer set the sign.

11 The arbitrage-free value of a convertible

Plain convertible = Straight value + Call on shares
Convertible = Straight value + Call on shares − Issuer's call + Holder's put

Straight value	Bond with no options
Call on shares	Holder's option: added
Issuer's call	Issuer's option: subtracted
Holder's put	Holder's option: added

IN THE LESSON

- Here is the build as a formula.
- A plain convertible, with no call and no put, is worth its straight value plus a call option on the shares covered by the conversion ratio.
- If the issuer can also call the bond, subtract the issuer's call.
- If the holder can put it back, add the holder's put.
- The volatilities matter, and they differ by part.
- Higher volatility in the share price raises the value of the call on the shares.
- Higher volatility in interest rates raises the value of both bond options, the issuer's call and the holder's put.
- An option is worth more when its underlying can swing more, whoever owns it.

12 Example 4: value the Brindle Marine convertible

Brindle Marine: straight 930, call on shares 150, issuer's call 40, put 20

Straight bond	no options	**930**
Add call on shares	930 + 150	**1,080**
Take off issuer's call	1,080 − 40	**1,040**
Add holder's put	1,040 + 20	**1,060**
Market price 1,045	1,060 − 1,045	**15 cheap**

IN THE LESSON

- Back to Priya, who now looks at a different bond, a Brindle Marine convertible that is callable and putable.
- Her option model gives the pieces.
- The straight bond is worth nine hundred and thirty dollars.
- The call on the shares is worth one hundred and fifty, so adding it gives one thousand and eighty.
- The issuer's call is worth forty, so subtracting it gives one thousand and forty.
- The holder's put is worth twenty, so adding it gives one thousand and sixty.
- That is the model value.
- The bond actually trades at one thousand and forty-five dollars, fifteen below the model, so it looks cheap.
- If her inputs are right, she could buy the convertible and hedge the share risk by selling shares short.

13 Example 5: which volatility moves which part

Starting from a model value of 1,060

Share volatility up	call on shares 150 to 185	**1,095**
Change in value	1,095 − 1,060	**+35**
Rate volatility up	issuer's call 55, put 26	**1,051**
Change in value	1,051 − 1,060	**−9**

IN THE LESSON

- Now Priya asks what happens if volatility rises.
- First, share price volatility.
- Suppose the call on the shares rises from one hundred and fifty to one hundred and eighty-five, and nothing else moves.
- The value becomes nine hundred and thirty plus one hundred and eighty-five, minus forty, plus twenty, which is one thousand and ninety-five, a gain of thirty-five dollars.
- Second, interest rate volatility.
- Suppose it lifts the issuer's call from forty to fifty-five and the holder's put from twenty to twenty-six.
- The value becomes nine hundred and thirty plus one hundred and fifty, minus fifty-five, plus twenty-six, which is one thousand and fifty-one, a loss of nine.
- Both bond options gained, but the issuer's call gained more, and it is the one you subtract.

14 Why practitioners use trees, not just sums

- Parts interact: the call depends on shares and rates
- The conversion option depends on rates too
- Models use trees or simulation for both rates and shares
- Sum of parts: good for signs and reasoning

IN THE LESSON

- Adding separately priced parts is a good way to understand a convertible, but the final price comes from modelling the whole bond.
- The parts are not independent.
- The issuer's call is often used to force holders to convert, so its value depends on the share price as well as on rates.
- The conversion option depends on rates too, because the bond you give up to get the shares is itself worth more or less as rates move.
- So practitioners use a tree or a simulation that moves interest rates and share prices together, and read the parts off the result.
- In an exam, expect to use the sum of parts for reasoning and for the signs, and to be told that the real value needs a model.

15 Mistakes so far

- Premium ratio base: the share price
- Minimum value: the higher of the two routes
- Issuer's call subtracted, holder's put added
- Share volatility: equity call. Rate volatility: bond options

IN THE LESSON

- Before we move on, here are the slips that cost marks.
- First, the base of the premium ratio.
- It is the share price, not the conversion price, and not the straight value.
- Second, the minimum value is the higher of the conversion value and the straight value, not the lower, and not their sum.
- Third, the signs.
- The issuer's call is subtracted and the holder's put is added, because the option belongs to the other side or to you.
- Fourth, matching volatility to option.
- Share price volatility drives the call on the shares.
- Interest rate volatility drives both bond options, so its net effect depends on which option gains more.

16 A hybrid car on a changing road

- Busted: shares far below conversion price, bond-like
- Equity-like: shares far above, moves with the shares
- Hybrid in between: part of the upside, a cushion below
- Versus bond: lower coupon. Versus shares: less upside

IN THE LESSON

- Think of a hybrid car.
- On a flat road it runs on petrol, and on a hill it leans on the battery.
- A convertible works the same way, and the road is the share price.
- When the shares are far below the conversion price, the swap right is nearly worthless and the bond runs on its coupons and credit.
- Traders call that a busted convertible.
- When the shares are far above, the swap right is deep in the money and the convertible follows the shares.
- In between, it takes part of the upside, with the bond cushioning part of the fall.
- This part asks how that mix compares with owning the bond alone or the shares alone.

17 Three regimes, one security

Chart: Value ($) vs Share price ($)

- Straight value 920
- Conversion value
- Shares 27
- Shares 45
- Convertible

IN THE LESSON

- Here is the picture.
- Along the bottom is the share price, from low on the left to high on the right.
- Up the side is value in dollars.
- The flat line is the straight value, nine hundred and twenty.
- The steep rising line is the conversion value, the ratio times the share price.
- The thick curve is the convertible.
- On the left, in the busted zone, it hugs the flat line, so it moves with interest rates and credit rather than with the shares.
- On the right, in the equity-like zone, it hugs the rising line, with a thin premium.
- In the middle it bends between them.
- The two red lines mark the share prices we will test next, twenty-seven and forty-five.

18 Example 6: the Linden Systems convertible

Ratio 25, price 1,050, shares 36, straight value 920

Conversion value today	25 × 36	**900**
Shares +25% to 45	1,190 ÷ 1,050 − 1	**+13.3%**
Shares −25% to 27	950 ÷ 1,050 − 1	**−9.5%**
Coupon given up	5% − 2% of 1,000	**30 a year**

IN THE LESSON

- Now a worked case.
- Priya studies a Linden Systems convertible with a conversion ratio of twenty-five.
- It trades at one thousand and fifty dollars with the shares at thirty-six, and its straight value is nine hundred and twenty.
- It pays a two per cent coupon, while Linden's plain bonds pay five per cent.
- Suppose the shares rise twenty-five per cent to forty-five.
- Her model says the convertible would trade at one thousand one hundred and ninety, a gain of thirteen point three per cent.
- If the shares fall twenty-five per cent to twenty-seven, it would trade at nine hundred and fifty, a loss of nine point five per cent.
- The straight value of nine hundred and twenty held it up.
- The shares themselves would gain or lose a full twenty-five per cent either way.

19 Most of the upside, less of the downside

Change in value (%)

Shares up	**+25.0**
Convertible up	**+13.3**
Shares down	**−25.0**
Convertible down	**−9.5**

Linden Systems: shares move 25% either way

IN THE LESSON

- Here are those results as bars, measured as the change in value in per cent.
- The first bar is the shares if they rise, twenty-five per cent.
- The second is the convertible in the same case, thirteen point three per cent, about half of the shares' gain.
- The third bar is the shares if they fall, a loss of twenty-five per cent.
- The fourth is the convertible in that case, a loss of only nine point five per cent, under two-fifths of the shares' loss.
- That asymmetry is what the convertible offers.
- Compared with the shares, it gives up some upside for downside protection.
- Compared with the straight bond, it pays a lower coupon, thirty dollars a year less here, for the equity option.

20 Example 7: why the convertible lags on the upside

Premium of price over conversion value (the premium ratio)

Today, shares at 36	(1,050 − 900) ÷ 900	**16.7%**
Shares at 45	(1,190 − 1,125) ÷ 1,125	**5.8%**
Shares at 27	(950 − 675) ÷ 675	**40.7%**
At 27, over straight	950 ÷ 920 − 1	**3.3%**

IN THE LESSON

- Why does the convertible gain only about half when the shares rise?
- Because its premium shrinks.
- Today the conversion value is twenty-five times thirty-six, nine hundred, so the premium over it is one hundred and fifty, sixteen point seven per cent.
- If the shares rise to forty-five, the conversion value is eleven hundred and twenty-five, and the price of eleven hundred and ninety leaves a premium of only sixty-five, five point eight per cent.
- The option is deeper in the money and worth less over and above conversion value.
- If the shares fall to twenty-seven, the conversion value is six hundred and seventy-five and the premium balloons to two hundred and seventy-five, forty point seven per cent, because the bond value is doing the work.
- There, the price is only three point three per cent above the straight value.

21 Traps in comparing the three

- Not all the upside, not all the protection
- The bond floor falls with rates and weak credit
- Name the regime: busted, hybrid or equity-like
- Price: lower coupon and a premium buy the option

IN THE LESSON

- There are three traps here.
- First, the claim that a convertible gives you all of the upside of the shares and protects you on the way down.
- It gives part of the upside and part of the protection, and you pay for both with a lower coupon and a premium.
- Second, the bond floor is not a guarantee.
- It is the value of a bond, so it falls when rates rise or the issuer's credit weakens, and a collapsing share price often arrives together with weak credit.
- Third, which behaviour you get depends on where the shares sit.
- Busted means bond-like, equity-like means share-like, and in between you hold a hybrid.
- Say which regime before you compare risk and return.

22 Credit risk: what could go wrong with the payments

- Expected exposure: what is at stake on a date
- Probability of default: how likely, date by date
- Loss given default: what is lost after recovery
- Together they give the credit valuation adjustment

IN THE LESSON

- Now to credit risk.
- Suppose you lend to a neighbour and ask three questions.
- How much is owed at the moment things go wrong?
- How likely is it that things go wrong at all?
- And if they do, how much would you actually lose, after getting some of it back?
- Those are the three building blocks of credit risk.
- Expected exposure is the amount at stake on a date, before any recovery.
- The probability of default is the chance the default happens on that date.
- Loss given default is what you lose after recovering a share of the exposure.
- Multiply them together date by date, discount, add up, and you have the credit valuation adjustment, the amount credit risk takes off a bond's value.

23 Four pieces, one adjustment

The pieces

Exposure → Loss given default → Expected loss → CVA

How each is built

Value of what is left → × (1 − recovery) → × chance of default → Discount, then add

IN THE LESSON

- Here is the chain, in boxes.
- Follow one date along.
- The top row names each piece.
- Exposure is the amount at stake if the issuer defaults on that date, which for a bond is the value of all the payments still to come.
- Loss given default takes that exposure and keeps only the part you do not recover.
- Expected loss multiplies it by the chance of default on that date.
- The credit valuation adjustment discounts the expected losses from every date and adds them up.
- The second row shows the operation behind each box: value what is left, multiply by one minus the recovery rate, multiply by the probability of default, then discount and add.
- Each box has its own input, so you can challenge one assumption without touching the others.

24 Loss given default and expected loss

Loss given default = Exposure × (1 − Recovery rate)
Expected loss = Loss given default × Probability of default

Expected exposure	Value of payments still owed
Recovery rate	Share recovered
Loss given default	What is lost if default
Probability of default	Chance default is this date
Expected loss	Loss weighted by chance

IN THE LESSON

- Two formulas first.
- Expected exposure at a date is the value, at that date, of the payments still owed, discounted at the risk-free rate.
- It includes the payment due that day, because we assume the default happens just before that payment is made.
- Loss given default equals the exposure times one minus the recovery rate.
- The recovery rate is the share of the exposure holders get back, so with forty per cent recovery, you lose sixty per cent of what was at stake.
- Expected loss for the date is then the loss given default times the probability of default on that date.
- This is the same as any expected value: a loss, weighted by the chance it happens.

25 Probability of default, and the adjustment

Probability of default = Hazard rate × Survival to start of year
CVA = sum of Expected loss ÷ (1 + risk-free rate) to the date
Fair value = Default-free value − CVA

Hazard rate	Chance of default in a year
Survival to start of year	Chance still alive at start
Credit valuation adjustment	Discounted expected losses
Default-free value	Value if no default
Fair value	Default-free value less CVA

IN THE LESSON

- Next, the probability of default for a year.
- The hazard rate is the chance of default in a year, given that the issuer has survived to the start of it.
- Suppose it is two per cent.
- The probability of default in a year equals the hazard rate times the probability of surviving to the start of that year.
- In year one, survival is certain.
- In year two, you must have survived year one first, which has probability ninety-eight per cent.
- So the probability of default falls over time, even with a constant hazard, because an issuer that has already defaulted cannot default again.
- Then the credit valuation adjustment is the sum of each date's expected loss, discounted at the risk-free rate.
- Fair value is the default-free value minus that adjustment.

26 Example 8: exposure at each date (Tarrant Freight)

Bond: 5% coupon, face 100; risk-free 3.5%; hazard 2%; recovery 40%

Exposure, year 1	5 + 5 ÷ 1.035 + 105 ÷ 1.035^2	**107.85**
Exposure, year 2	5 + 105 ÷ 1.035	**106.45**
Exposure, year 3	the last payment	**105.00**

IN THE LESSON

- Here is the worked case.
- Priya values a three-year bond from Tarrant Freight, a made-up company.
- It pays a five per cent annual coupon on a face value of one hundred.
- The risk-free rate is three point five per cent at every maturity, the hazard rate is two per cent, and the recovery rate is forty per cent.
- Defaults can happen only on payment dates, just before the payment.
- At the end of year one, the exposure is the five due that day, plus the five due a year later discounted, plus the final one hundred and five discounted two years.
- That comes to one hundred and seven point eight five.
- At year two it is five plus one hundred and five discounted one year, one hundred and six point four five.
- At year three it is the last payment, one hundred and five.
- Exposure falls as payments are made.

27 Example 8: loss given default

Loss given default = exposure × (1 − 40%) = exposure × 0.60

Loss given default, year 1	107.85 × 0.60	**64.71**
Loss given default, year 2	106.45 × 0.60	**63.87**
Loss given default, year 3	105 × 0.60	**63.00**

IN THE LESSON

- Next, the loss given default.
- With a recovery rate of forty per cent, the loss share is sixty per cent.
- Multiply each exposure by point six.
- Year one: one hundred and seven point eight five times point six is sixty-four point seven one.
- Year two: one hundred and six point four five times point six is sixty-three point eight seven.
- Year three: one hundred and five times point six is sixty-three.
- Read the first one this way.
- If Tarrant defaults at the end of year one, holders were owed about one hundred and eight, they get back about forty-three, and they lose about sixty-five.
- The loss given default is the amount lost if the default happens, not the amount we expect to lose, because we have not yet weighted it by a chance.

28 Probability of default falls with a constant hazard

Probability of default (%)

Year 1	**2.00**
Year 2	**1.96**
Year 3	**1.92**

Hazard rate 2% each year; survival 100%, 98%, 96.04%

IN THE LESSON

- Now the probabilities.
- The vertical axis is the probability of default in per cent.
- Year one: the hazard rate times survival of one hundred per cent, so two per cent.
- Year two: the hazard rate times ninety-eight per cent survival, so one point nine six.
- Year three: two per cent times ninety-six point zero four per cent survival, which is one point nine two.
- The bars are almost level, but they slope gently down.
- The hazard rate is the same two per cent every year.
- What shrinks is the chance of still being around to default.
- Across the three years, the total chance of default is a little under six per cent, which is one minus the survival to the end, about five point eight eight.

29 Example 8: expected loss and the adjustment

Expected loss = loss given default × probability of default

Year 1	64.71 × 2.00%, discounted	**1.25**
Year 2	63.87 × 1.96%, discounted	**1.17**
Year 3	63.00 × 1.9208%, discounted	**1.09**
CVA	1.25 + 1.17 + 1.09	**3.51**

IN THE LESSON

- Now multiply and discount.
- Expected loss in year one is sixty-four point seven one times two per cent, one point two nine.
- Discounted one year at three point five per cent, it is one point two five.
- Year two: sixty-three point eight seven times one point nine six per cent, one point two five, discounted two years, one point one seven.
- Year three: sixty-three times one point nine two per cent, one point two one, discounted three years, one point zero nine.
- Add the three present values and the credit valuation adjustment is three point five one.
- Notice that we discount at the risk-free rate, not at the bond's own yield.
- The risk of default is already in the expected losses, and a risky discount rate would count it twice.

30 Example 8: fair value, yield and spread

Default-free value less the credit valuation adjustment

Default-free value	5, 5 and 105 at 3.5%	**104.20**
Credit valuation adjustment	from the last slide	**3.51**
Fair value	104.20 − 3.51	**100.69**
Yield on fair value	solves the price	**4.75%**
Credit spread	4.75% − 3.5%	**1.25%**

IN THE LESSON

- Last step.
- The default-free value is the three payments discounted at three point five per cent: five over one point zero three five, five over that squared, and one hundred and five over that cubed.
- That is one hundred and four point two zero.
- Subtract the credit valuation adjustment of three point five one and the fair value is one hundred point six nine.
- As a check, find the yield that gives that price.
- It is about four point seven five per cent.
- Subtract the risk-free three point five and the credit spread is about one point two five per cent.
- A quick rule agrees: the hazard rate of two per cent times the sixty per cent loss is one point two.
- Credit risk shows up as a lower price and a higher yield.

31 Traps in the credit pieces

- Exposure: what is still owed, not the face value
- Loss given default uses 1 − recovery, not recovery
- Later-year default chance = hazard × survival
- Discount expected losses at the risk-free rate

IN THE LESSON

- Four traps.
- First, exposure is not the face value.
- It is the value of what is still owed at that date, so it changes as payments are made.
- Second, loss given default uses one minus the recovery rate.
- Using the recovery rate itself gives the amount you get back.
- Third, the probability of default for a later year is the hazard rate times the survival probability, not the hazard rate on its own.
- Fourth, the discounting is at the risk-free rate.
- And watch how the answer moves.
- Raise recovery from forty to sixty per cent and the credit valuation adjustment falls from three point five one to two point three four.
- Double the hazard rate to four per cent and it rises to about six point eight nine.

32 Recap: what to carry into the exam

- Minimum value = higher of conversion and straight value
- Convertible = straight + call on shares − issuer's call + put
- Part of the upside, part of the protection, lower coupon
- Exposure to LGD to expected loss to CVA

IN THE LESSON

- Let's pull it together.
- A convertible's conversion value is the ratio times the share price, and its minimum value is the higher of that and the straight value.
- The premium ratio divides by the share price, and the premium over straight value measures the distance to the bond floor.
- In an arbitrage-free framework a callable, putable convertible is the straight value plus the call on the shares, minus the issuer's call, plus the holder's put.
- Compared with shares it gives up upside for protection, and compared with a bond it pays a lower coupon for the option.
- In credit, exposure times one minus recovery gives loss given default, times probability of default gives expected loss, and discounting and adding gives the credit valuation adjustment.

33 Try three quick questions

1. Ratio 40, price 1,100, shares 24. Premium ratio?
   Answer: (27.50 − 24) ÷ 24 = 14.6%
2. Straight 900, call 120, issuer's call 30, put 10. Value?
   Answer: 900 + 120 − 30 + 10 = 1,000
3. Exposure 90, recovery 50%, default chance 3%. Expected loss?
   Answer: 90 × 0.5 × 3% = 1.35

IN THE LESSON

- Now try three questions yourself, pausing after each one.
- Question one.
- A convertible with a conversion ratio of forty is priced at one thousand one hundred dollars, and the shares are at twenty-four.
- What is the premium ratio?
- The market conversion price is eleven hundred over forty, twenty-seven fifty.
- Subtract twenty-four to get three fifty, then divide by twenty-four: fourteen point six per cent.
- Question two.
- Straight value nine hundred, call on shares one hundred and twenty, issuer's call thirty, holder's put ten.
- What is the value?
- Nine hundred plus one twenty, minus thirty, plus ten, is one thousand.
- Question three.
- Exposure ninety, recovery fifty per cent, probability of default three per cent.
- What is the expected loss?
- Loss given default is forty-five, and times three per cent is one point three five.