Embedded options: durations, convexity, floaters and convertibles

Fixed Income

1 A story: the seesaw with a stopper

- A seesaw with a stopper under one end
- One average swing is wrong on both sides
- Callable bond: price hits the stopper when rates fall
- One-sided durations report each side

IN THE LESSON

- Picture a seesaw in a playground with a rubber stopper bolted under one end.
- Push the other end down and it swings a long way.
- Push the stopper end down and it halts almost at once.
- If you tried to describe this seesaw with one number, the average swing, you would be wrong in both directions.
- A callable bond is that seesaw.
- When rates fall, the issuer can call the bond, and the price hits the stopper.
- When rates rise, nothing stops the price falling.
- Effective duration quietly averages the two swings into one number.
- One-sided durations simply refuse to average them.
- They report the two sides separately.

2 One-sided durations

Down = $(PV_{-} - PV_0) \div (\Delta \times PV_0)$ Up = $(PV_0 - PV_{+}) \div (\Delta \times PV_0)$

Value if the curve falls	Revalued on a recalibrated tree
Value today	Today's price
Value if the curve rises	Revalued on a recalibrated tree
Size of the shift	As a decimal: 50 bp = 0.005

IN THE LESSON

- Level One gave you effective duration, which shifts the whole curve down and up by the same small amount and averages the result.
- One-sided durations use the same two shifted values but keep them apart.
- Take the value if the curve falls, minus today's value, and divide by the size of the shift times today's value.
- That is the down-duration, how much the bond gains per unit of fall.
- Now take today's value minus the value if the curve rises, over the same divisor.
- That is the up-duration, how much the bond loses per unit of rise.
- Both come out positive for an ordinary bond.
- Effective duration is exactly the average of the two.

3 Example 1: Brindle Rail's callable bond

Callable at 100: price 99.00; curve ±50 bp (0.005)

Values: down / today / up	99.60 / 99.00 / 97.00	**gain 0.60**
Down-duration	0.60 ÷ (0.005 × 99.00)	**1.21**
Up-duration	2.00 ÷ (0.005 × 99.00)	**4.04**
Effective duration	average of the two	**2.63**
Loss on a rise	97.00 vs 99.00	**loss 2.00**

IN THE LESSON

- Meet Imogen Rask, an analyst at a fund.
- She studies a five-year bond of Brindle Rail, callable at one hundred, with the price close to the call price.
- The tree gives a value of ninety-nine point zero zero today.
- With the curve down fifty basis points the value is ninety-nine point six zero, and with the curve up fifty basis points it is ninety-seven point zero zero.
- The gain is only zero point six zero, because the call caps it, but the loss is two point zero zero.
- Divide each by the shift, zero point zero zero five, times ninety-nine point zero zero.
- The down-duration is one point two one, and the up-duration is four point zero four.
- Their average, the effective duration, is two point six three, which describes neither side.

4 Example 2: the same test on three bonds

Same issuer, same curve shifts of ±50 bp

Straight: down / up	4.60 and 4.40	**nearly equal**
Straight values	102.30 / 100.00 / 97.80	
Putable values	103.10 / 101.00 / 99.20	
Putable down-duration	2.10 ÷ (0.005 × 101.00)	**4.16**
Putable up-duration	1.80 ÷ (0.005 × 101.00)	**3.56**

IN THE LESSON

- Rask now runs the same test on a straight bond and a putable bond from the same issuer, both priced close to one hundred, the putable one with a put at one hundred.
- The straight bond is worth one hundred point zero zero, and moves to one hundred two point three zero and ninety-seven point eight zero.
- Its down-duration is four point six zero and its up-duration four point four zero, nearly the same.
- The small gap is ordinary positive convexity.
- The putable bond is worth one hundred one point zero zero, moving to one hundred three point one zero and ninety-nine point two zero.
- Now the pattern flips.
- Its down-duration is four point one six, but its up-duration is only three point five six.
- The put sets a floor under the price, which limits the loss when rates rise.

5 Down and up, side by side

Duration (years)

Straight down	**4.60**
Straight up	**4.40**
Callable down	**1.21**
Callable up	**4.04**
Putable down	**4.16**
Putable up	**3.56**

Option near the money: the capped side is the short bar

IN THE LESSON

- Here are those six numbers as bars.
- Start on the left with the straight bond.
- Its down bar is four point six zero and its up bar four point four zero, almost twins.
- Move to the callable bond.
- The down bar is short, only one point two one, because the call caps the gain when rates fall.
- The up bar is tall, four point zero four, because nothing caps the loss when rates rise.
- Now the putable bond.
- The shorter bar is on the up side, three point five six against four point one six, because the put limits the loss.
- The rule to remember is simple.
- Near the call price, the down-duration is the small one.
- Near the put price, the up-duration is the small one.

6 Key rate duration

Key rate duration = $(PV_{-} - PV_{+}) \div (2 \times \Delta key \times PV_{0})$

Value if the curve falls	One key rate down, others fixed
Value if the curve rises	One key rate up, others fixed
Shift in one key rate	Size of that one shift
Value today	Today's price
Key rate duration	Exposure to that curve point

IN THE LESSON

- One-sided durations ask which direction rates move.
- Key rate durations ask where on the curve they move.
- The idea is to shift just one point of the benchmark curve, say the three-year par rate, and leave every other point alone.
- Take the value with that one rate down, minus the value with it up, and divide by two times the size of the shift times today's value.
- That gives the key rate duration at that maturity.
- Do it for each key point on the curve.
- For a parallel move, the key rate durations add up to roughly the effective duration, so they split the total risk into pieces and show where it sits.

7 Example 3: where the callable bond is exposed

10-year bond, callable at par in year 3, deep in the money; shift ±25 bp

3-year rate down 25 bp		**104.65**
3-year rate up 25 bp		**103.35**
3-year key rate duration	1.30 ÷ (2 × 0.0025 × 104.00)	**2.50**
2y / 5y / 10y	0.10 / 0.30 / 0.20	**small**
Sum of key rate durations	close to effective duration	**3.10**

IN THE LESSON

- Now a ten-year bond, callable at par in year three, with a coupon far above today's yields, so the call is deep in the money.
- Everyone expects the issuer to call at year three, so the bond behaves like a three-year bond.
- Rask shifts the three-year par rate by twenty-five basis points.
- The value is one hundred four point zero zero today, one hundred four point six five with the rate down, and one hundred three point three five with it up.
- The key rate duration is the difference, one point three zero, divided by two times zero point zero zero two five times one hundred four point zero zero.
- That is two point five zero.
- The other points are tiny: zero point one zero at two years, zero point three zero at five and zero point two zero at ten.
- The sum is three point one zero.

8 The exposure sits at the call date

Key rate duration (years)

2-year	**0.10**
3-year	**2.50**
5-year	**0.30**
10-year	**0.20**

Callable at year 3, deep in the money; a plain 10-year bond has 5.30 at ten years

IN THE LESSON

- Here are the callable bond's key rate durations as bars, one for each point on the curve.
- The two-year bar is tiny, zero point one zero.
- The three-year bar towers over the rest at two point five zero, because that is the call date and the bond is priced to it.
- The five-year bar is zero point three zero and the ten-year bar only zero point two zero, though the bond runs ten years.
- Compare an ordinary ten-year bond with the same coupon and no option.
- Its ten-year key rate duration would be five point three zero, with a total of seven point two zero.
- The takeaway is that the option moves the risk.
- A deep in the money callable is a bet on the three-year rate, not the ten-year rate.

9 A story: the hammock and the roof

- Ordinary bond: price curve sags like a hammock
- Fall in yield adds more than a rise takes away
- Callable near its call price: bends like a roof
- Negative convexity is the bill for the call

IN THE LESSON

- Think of two shapes.
- A hammock hangs between two trees and sags in the middle.
- A roof peaks in the middle and slopes away on both sides.
- Now draw a bond's price against its yield.
- For an ordinary bond the curve sags like a hammock.
- It bends upward, so a fall in yield adds more to the price than an equal rise takes away.
- That bend is positive convexity, and holders love it.
- A callable bond near its call price flattens as yields fall and bends the other way, like the roof.
- That is negative convexity.
- The holder has sold the issuer a cheap right, and the bill arrives as a curve that bends the wrong way.

10 Effective convexity

Effective convexity = $(PV_{-} + PV_{+} - 2 \times PV_{0}) \div (\Delta^2 \times PV_{0})$

Value if the curve falls	Value, curve down
Value if the curve rises	Value, curve up
Value today	Today's price
Shift squared	Shift squared: 0.005^2 = 0.000025
Effective convexity	Positive: hammock. Negative: roof

IN THE LESSON

- Here is the formula, built from the same three values as duration.
- Add the value with the curve down and the value with the curve up, subtract two times today's value, and divide by the size of the shift squared times today's value.
- Look at the top.
- If the two shifted values average above today's value, the price curve sags upward and convexity is positive.
- If they average below today's value, convexity is negative.
- The shift is squared in the denominator, because convexity is a second-order measure, a bend rather than a slope.
- Convexity is a bigger number than duration, often in the hundreds, because the squared shift is tiny.

11 Price against yield: hammock and roof

Chart: Price vs Yield

- Straight bond
- Call price 112
- Callable: negative convexity

IN THE LESSON

- Here is the picture.
- Yield runs along the bottom, from low on the left to high on the right, and price runs up the side.
- The green curve is a straight twenty-year bond.
- It falls as yields rise, and it sags, bending upward everywhere.
- That is positive convexity.
- The brown curve is the same bond with a call.
- On the right, where yields are high, the call is far out of the money and the two curves lie on top of each other.
- Move left.
- As yields fall, the brown curve flattens against the call price, near one hundred and twelve, while the green one keeps climbing.
- In that zone the callable's curve bends downward.
- That is negative convexity.

12 Example 4: convexity of the three bonds

Convexity = $(PV_- + PV_+ - 2 \times PV_0) \div (0.000025 \times PV_0)$

Straight: top 0.10	0.10 ÷ (0.000025 × 100.00)	**40**
Callable: top −1.40	−1.40 ÷ (0.000025 × 99.00)	**−566**
Putable: top 0.30	0.30 ÷ (0.000025 × 101.00)	**119**
Order, highest to lowest	putable > straight > callable	**119 / 40 / −566**

IN THE LESSON

- Back to Rask's three bonds.
- For the straight bond, add one hundred two point three zero and ninety-seven point eight zero and take away two hundred.
- The top is zero point one zero.
- The bottom is zero point zero zero zero zero two five times one hundred.
- Convexity is forty, positive.
- For the callable bond the top is minus one point four zero, negative, so convexity is minus five hundred sixty-six.
- The two shifted values average below today's price, which is the call at work.
- The putable bond's top is zero point three zero, positive, giving one hundred nineteen.
- That is more than the straight bond, because the put limits the downside.

13 Contrasting the three bonds

Effective convexity near the option's strike

Bond	Effective convexity
Straight	Positive
Callable, near call price	Negative
Callable, far from call	Like straight
Putable, near put price	More than straight

IN THE LESSON

- Let us put the contrast in one place, using the language the exam likes.
- A straight bond, with no option, has positive convexity.
- Its price rises more when yields fall than it falls when yields rise.
- A callable bond that is far from its call price behaves like the straight bond, because the call is out of the money.
- But a callable bond close to its call price has negative convexity, because the call caps the upside.
- A putable bond keeps positive convexity, and near its put price it has more than the straight bond, because the put limits the downside.
- Notice the pattern: the option always helps the side that has bought it, and hurts the side that has sold it.

14 Example 5: duration alone misleads

Callable: effective duration 2.63, convexity −566, shift ±50 bp

Duration effect	2.63 × 0.50%	**±1.31%**
Convexity effect	½ × (−566) × 0.005^2	**−0.71%**
Rates fall 50 bp	+1.31% − 0.71%	**+0.61%**
Rates rise 50 bp	−1.31% − 0.71%	**−2.02%**

IN THE LESSON

- Why does this matter in practice?
- Rask's manager expects rates to move fifty basis points, either way, and uses the callable bond's effective duration of two point six three.
- That implies a gain or loss of one point three one per cent.
- But convexity takes zero point seven one per cent off in both directions, half the convexity times the shift squared.
- If rates fall, the estimate is zero point six one per cent, which matches the true change in value of zero point six one.
- If rates rise, it is minus two point zero two per cent, against a true minus two point zero two.
- Duration alone promised a gain of one point three one per cent.
- The callable delivers less than half of that.

15 Mistakes to avoid so far

- Square the shift in the convexity formula
- Keep the sign: callable tops are negative
- Effective duration divides by 2 × shift
- Callable: duration differs on the two sides
- Callable: down-duration small. Putable: up small

IN THE LESSON

- Before we move on, here are the slips that cost marks.
- First, in the convexity formula, forgetting to square the shift.
- Fifty basis points is zero point zero zero five, and squared it is zero point zero zero zero zero two five.
- Second, getting the sign of the top the wrong way round, which turns a callable bond's negative convexity positive.
- Third, dividing by the shift instead of by two times the shift when you compute effective duration.
- Fourth, calling a callable bond's effective duration the same on both sides.
- The whole point of one-sided durations is that it is not.
- And fifth, forgetting that the down-duration is the small one for a callable, but the up-duration is the small one for a putable.

16 A story: the rent that follows the market

- Floater: coupon resets to the market each period
- So the plain floater is worth par
- Cap: coupon never above a limit. Holder loses value
- Floor: coupon never below a limit. Holder gains value

IN THE LESSON

- Imagine a flat where the rent is reset every year to whatever rents nearby are.
- Your landlord can never be short-changed.
- Whatever the market does, the rent follows.
- A floating-rate bond works the same way.
- Its coupon is reset each period to the reference rate, set at the start of the period and paid at the end.
- Because the bond always pays what the market pays, its value on a reset date is its par value.
- Now add a cap into the lease: your rent can never rise above a limit.
- You are happy, the landlord is not, and the lease is worth less to the landlord by exactly the value of that cap.
- A floor does the opposite.
- That is the whole idea of this section.

17 Who owns the option?

- Capped floater = plain floater − cap (issuer owns cap)
- Floored floater = plain floater + floor (investor owns floor)
- Cap value = 100 − capped floater value
- Floor value = floored floater value − 100

IN THE LESSON

- Be clear about who holds what.
- A capped floater pays the reference rate, but never more than the cap.
- The investor who buys it has effectively sold the issuer a cap, so the issuer holds the cap, and the capped floater is worth less than par.
- The cap is the right to pay no more than the limit when rates are high.
- A floored floater pays the reference rate, but never less than the floor.
- The investor holds the floor, and the floored floater is worth more than par.
- Think of a cap as an option owned by the issuer and a floor as an option owned by the investor.
- That gives us two simple rules for value.

18 Backward through the tree

Node value = [½ × (V up + V down) + coupon] ÷ (1 + r)

Node value	Value at this node
Average of next values	The two values one step ahead
Coupon paid next	Cap: lower. Floor: higher
Node rate	This node's one-year rate

IN THE LESSON

- We value a floater on a binomial interest rate tree, working backward from maturity.
- At each node, take the two values one step ahead and average them, since up and down are equally likely.
- Add the coupon that will be paid at the end of this period.
- That coupon is set by the rate at this node: for a capped floater it is the lower of the node rate and the cap, and for a floored floater it is the higher of the node rate and the floor.
- Then discount the total by one plus the node rate.
- Apply the cap or floor at every node, including today's rate.
- With no cap or floor, every node is worth par.

19 The order of the work

Capped at 3.5%: work backward

Year two: three values → Year one: two values → Today: one value → Cap = 100 − value

IN THE LESSON

- Here is the order in which the numbers appear.
- Start on the left with the last year.
- At the three year-two nodes, the bond pays its final coupon and principal, so each value is one hundred plus the coupon, divided by one plus the node rate.
- Next, step back to the two year-one nodes.
- Each averages the two values it leads to, adds its own coupon and discounts.
- Then step back again to today, the single starting node, and do the same.
- The result is the value of the capped floater.
- The last box is the one the exam likes: subtract that value from one hundred and you have the value of the cap.
- Notice that every box obeys the same one-line rule.

20 Example 6: the capped floater, nodes

Capped at 3.5%; coupon at a node = lower of rate and cap

Year 2, rate 5.37%: cap binds	(100 + 3.5) ÷ 1.0537	**98.23**
Year 2, rate 3.98%: cap binds	(100 + 3.5) ÷ 1.0398	**99.54**
Year 2, rate 2.95%: no cap	(100 + 2.95) ÷ 1.0295	**100.00**
Year 1, rate 3.47%	[98.882 + 3.47] ÷ 1.0347	**98.92**
Year 1, rate 2.57%	[99.769 + 2.57] ÷ 1.0257	**99.77**

IN THE LESSON

- Teal Housing issues a three-year floater, capped at three point five per cent.
- The tree has two per cent today, three point four seven and two point five seven in year one, and five point three seven, three point nine eight and two point nine five in year two.
- Start at year two.
- At five point three seven the cap binds, so the coupon is three point five, and the value is one hundred three point five over one point zero five three seven, ninety-eight point two three.
- At three point nine eight it binds again, giving ninety-nine point five four.
- At two point nine five nothing binds, so the value is one hundred point zero zero.
- In year one, three point four seven gives ninety-eight point nine two and two point five seven gives ninety-nine point seven seven.

21 Example 6 finished: the value of the cap

Capped floater at the start of the tree

Average of year-1 values	½ × (98.92 + 99.77)	**99.347**
Add coupon, discount	(99.347 + 2.00) ÷ 1.02	**99.36**
Plain floater, same tree	coupon = rate at every node	**100.00**
Value of the cap	100.00 − 99.36	**0.64**

IN THE LESSON

- Now step back to today.
- The rate is two per cent, below the cap, so the coupon is the full two per cent.
- Average the two year-one values, ninety-eight point nine two and ninety-nine point seven seven, which gives ninety-nine point three four seven.
- Add two and divide by one point zero two.
- The capped floater is worth ninety-nine point three six.
- A plain floater on the same tree is worth exactly one hundred, which you can check at any node, because the coupon always equals the rate it is discounted at.
- So the cap is worth one hundred minus ninety-nine point three six, or zero point six four.
- The issuer owns that value.
- It is small, because the cap only bites in the two high-rate corners of the tree.

22 Example 7: the floored floater

Floored at 3.0%; coupon at a node = higher of rate and floor

Year 2, rate 2.95%: floor binds	(100 + 3.0) ÷ 1.0295	**100.05**
Year 1, rate 3.47%: no floor	[100.00 + 3.47] ÷ 1.0347	**100.00**
Year 1, rate 2.57%: floor binds	[100.024 + 3.0] ÷ 1.0257	**100.44**
Today, rate 2.00%: floor binds	[100.221 + 3.0] ÷ 1.02	**101.20**
Value of the floor	101.20 − 100.00	**1.20**

IN THE LESSON

- Now Teal Housing offers a floored version, with a floor of three per cent and no cap.
- This time the floor binds where rates are low.
- At the year-two node of two point nine five, the coupon is three, not two point nine five, and the value is one hundred three over one point zero two nine five, one hundred point zero five.
- The other year-two nodes sit above the floor and are worth one hundred.
- In year one, three point four seven gives one hundred point zero zero.
- Two point five seven is below the floor, so the coupon is three, and the value is one hundred point four four.
- Even today's two per cent is below the floor, so the coupon is three.
- The floored floater is worth one hundred one point two zero, and the floor is worth one point two zero, owned by the investor.

23 Traps with caps and floors

- Apply the cap or floor at every node, today's too
- Coupon is capped; discounting uses the full node rate
- Issuer owns the cap, investor owns the floor
- More volatility raises both option values

IN THE LESSON

- Here are the traps on this topic.
- Do not forget today's rate.
- If today's rate is below the floor, the coupon you receive this year is the floor, and that is easy to miss.
- Do not apply the cap only at the last year.
- A rate above the cap in year one also caps the coupon, and its effect flows back through the tree.
- Do not confuse the coupon with the discount rate.
- The cap changes the coupon, but you still discount at the uncapped node rate.
- Remember who owns what: the issuer owns the cap, so the cap lowers the bond's value, and the investor owns the floor, so the floor raises it.
- Finally, higher interest rate volatility raises the value of both a cap and a floor.

24 A story: a bond with a ticket to the shares

- Convertible = a bond plus a right to swap for shares
- Coupons and principal like any bond
- Swap terms fixed in advance
- Lower coupon in return for the right

IN THE LESSON

- Imagine buying a season ticket to a theme park that also comes with a coupon: if the park ever opens a new ride, you can swap your ticket for a share of the park at a price fixed today.
- You hold a safe claim, and you also hold a chance to share in success.
- A convertible bond is exactly that.
- It is a bond issued by a company, paying coupons and principal.
- But it also gives the holder the right to exchange it for a fixed number of the issuer's shares.
- In return for that right, the coupon is usually lower than a plain bond's.
- The terms of the swap are what we study now.

25 The key terms

Fennick Labs convertible (made-up example)

Term	Fennick Labs
Face value	$1,000
Conversion ratio	25 shares
Conversion price	$1,000 ÷ 25 = $40
Callable at (after protection)	$1,020

IN THE LESSON

- Let us name the terms.
- Face value is the amount repaid at maturity.
- The conversion ratio is the number of the issuer's shares you receive for each bond.
- The conversion price is the effective price of each share when bought through the bond: face value divided by the conversion ratio.
- The conversion period is the window of time during which the holder may convert.
- Conversion value is what the shares received are worth right now: the ratio times the market price of one share.
- We follow Fennick Labs, a made up company, which issues a convertible with a face value of one thousand dollars and a conversion ratio of twenty-five shares.

26 Conversion price and conversion value

Conversion price = Face ÷ Ratio Conversion value = Ratio × Share price

Face value	Repaid at maturity
Conversion ratio	Shares received per bond
Conversion price	Effective cost of each share
Share price	Market price of one share
Conversion value	Worth of the shares if swapped now

IN THE LESSON

- Two small formulas do most of the work.
- The conversion price equals the face value divided by the conversion ratio.
- It tells you what you effectively pay for each share when you convert.
- The conversion value equals the conversion ratio times the current share price.
- It tells you what the shares are worth if you swap right now.
- Compare the two prices.
- If the share price is above the conversion price, conversion is worth more than the face value.
- If it is below, converting would give you less than the bond's face value.
- Next, the call price enters.
- Many convertibles are callable by the issuer, and the decision on a call turns on comparing conversion value with the call price.

27 A convertible's life

From issue to maturity

Issue: ratio fixed →
Call protection: holder may convert →
Callable: issuer may force conversion →
Maturity: face value, or shares

IN THE LESSON

- Here is the life of a typical callable convertible, and the order in which each party can act.
- At issue, the terms are fixed: the conversion ratio and with it the conversion price.
- During call protection, a period at the start in which the issuer is not allowed to call, only the holder can act.
- The holder may convert at any time in the conversion period, but there is usually no reason to, since a bond that has not been called keeps paying coupons.
- Once the bond becomes callable, the issuer can act too.
- If the shares have risen well above the conversion price, the issuer calls.
- At maturity, any holder who has not converted receives the face value.

28 Example 8: the call that forces conversion

Callable at $1,020; ratio 25; shares at $48, or $36

Conversion value at $48	25 × $48	**$1,200**
Called at $1,020: take the higher	$1,200 vs $1,020	**convert**
Conversion value at $36	25 × $36	**$900**
Called at $1,020: take the higher	$1,020 vs $900	**take cash**
Break-even share price	$1,020 ÷ 25	**$40.80**

IN THE LESSON

- Fennick's shares have risen to forty-eight dollars, and the call protection has ended.
- The conversion value is twenty-five times forty-eight, which is one thousand two hundred dollars.
- The call price is only one thousand twenty.
- If the issuer calls, the holder must choose between taking one thousand twenty in cash and converting into shares worth one thousand two hundred, so the holder converts.
- The call has become a forced conversion: the issuer swaps its debt for shares rather than paying cash.
- Now suppose the shares were at thirty-six dollars instead.
- The conversion value is only nine hundred, so a call would be taken in cash.
- The rule is simple.
- When called, compare the call price with conversion value and take the higher.

29 Conversion value against the call price

Chart: Value of the bond to the holder ($) vs Share price ($)

- Conversion value
- Call price 1,020
- Break-even $40.80

IN THE LESSON

- Here is the same decision as a picture.
- The share price runs along the bottom, and the value of the bond to the holder runs up the side.
- The rising green line is the conversion value, twenty-five times the share price.
- The flat red line is the call price, one thousand and twenty.
- They cross at a share price of forty point eight zero dollars.
- To the left of that point, the line is below the call price, so a called holder takes the cash.
- To the right, the line is above, so the called holder converts.
- At forty-eight dollars, the green line stands at twelve hundred, well above the red line.
- The further right the share price drifts, the surer it is that a call will force conversion.

30 Example 9: splits and a takeover bid

Split 3-for-2; change-of-control put at $1,010

New ratio, new price	25 × 3 ÷ 2; $1,000 ÷ 37.5	**37.5; $26.67**
Conversion value after split	37.5 × $32	**$1,200**
Bid $44 in cash	25 × $44 vs put $1,010	**$1,100: convert**
Bid $38 in cash	25 × $38 vs put $1,010	**$950: put**

IN THE LESSON

- Two protections matter.
- First, the terms adjust the ratio for splits and special dividends, so that corporate actions cannot dilute the holder.
- Suppose Fennick does a three-for-two split.
- The ratio rises from twenty-five to thirty-seven point five, the conversion price falls to twenty-six point six seven, and the share price falls to thirty-two, so the conversion value stays at one thousand two hundred.
- Second, a change-of-control put lets holders sell the bond back, here at one thousand ten, if control of the issuer changes.
- A bidder offers forty-four dollars a share in cash: the shares give one thousand one hundred, more than the put, so convert.
- At thirty-eight dollars the shares give only nine hundred fifty, so use the put.

31 Other kinds of convertible

- Exchangeable: converts into another company's shares
- Mandatory: converts at maturity, no choice
- Contingent: converts when a trigger is hit
- Callable after protection: forces conversion

IN THE LESSON

- Convertibles come in a few flavours, and the exam likes to test the vocabulary.
- An exchangeable bond is exchanged for shares of a company other than the issuer, often a firm whose shares the issuer already owns.
- A mandatory convertible must convert into shares at maturity, so the holder has no choice.
- A contingent convertible converts automatically when a trigger is hit, for example when a bank's capital ratio falls below a set level, and it is a way of absorbing losses.
- Many ordinary convertibles are also callable, usually after a period of call protection, and some are putable.
- Issuers typically call once conversion value is well above the call price, to force holders to convert.

32 Recap: the formulas to remember

Down: $(PV_{-}-PV_0)\div(\Delta\times PV_0)$ Convexity: $(PV_{-}+PV_{+}-2PV_0)\div(\Delta^2\times PV_0)$ Conversion price: Face ÷ Ratio

Duration or convexity	Down- and up-duration, key rates
Effective convexity	Callable: negative near call price
Node value	Floaters: cap or floor, every node
Conversion price	Face ÷ ratio
Conversion value	Take the higher when called

IN THE LESSON

- Let us pull the lesson together.
- One-sided durations split effective duration into a down side and an up side.
- A callable bond has a small down-duration near its call price, and a putable bond a small up-duration near its put price.
- Key rate durations shift one point on the curve at a time, and a deep in the money callable is most sensitive at its call date.
- Effective convexity is the bend of the price curve.
- It is positive for straight and putable bonds and negative for a callable bond near its call price.
- A capped or floored floater is valued backward on a tree, applying the cap or floor at every node.
- And a convertible swaps into a fixed number of shares: when called, take the higher of call price and conversion value.

33 Try three quick questions

1. Values 101.50 / 100.00 / 97.50 for ±50 bp. Durations?
   Answer: Down 3.0, up 5.0: smaller down side, so looks callable.
2. Values 104.0 / 100.0 / 96.2, shift 1%. Effective convexity?
   Answer: (104.0 + 96.2 − 200) ÷ (0.0001 × 100) = 20.
3. Face 1,000, ratio 20, shares $62, call at 1,150. Action?
   Answer: Conversion price $50; value $1,240 beats $1,150: convert.

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- A bond is worth one hundred point zero zero today, one hundred one point five zero if the curve falls fifty basis points, and ninety-seven point five zero if it rises fifty.
- What are its down-duration and up-duration, and what might the bond be?
- Down is three point zero, up is five point zero.
- The down-duration is smaller, so it looks callable.
- Question two.
- Values are one hundred point zero today, one hundred four point zero down and ninety-six point two up, for a shift of one per cent.
- What is the effective convexity?
- The top is zero point two, the bottom is zero point zero one, so convexity is twenty.
- Question three.
- A bond of face value one thousand converts into twenty shares, priced at sixty-two dollars, and is called at eleven hundred and fifty.
- Convert, since twelve hundred and forty beats the call price.