Credit analysis models

Fixed Income

1 A story: the bank manager and the agency

- Score: a number from a model, quick and cheap
- Rating: a letter grade from analysis and judgement
- Both rank credit risk, safer to riskier
- They differ in who is judged and how

IN THE LESSON

- Picture two people judging whether a borrower will repay.
- The first is a bank clerk with a screen.
- She types in an applicant's income and debts, and out comes a number.
- Quick, cheap, and the same every time.
- That is a credit score.
- The second is an analyst at a rating agency, who reads a company's accounts, meets its managers, thinks about its industry, and then writes a letter grade.
- That is a credit rating.
- Both sort borrowers from safer to riskier, but they are built differently.

2 Scores and ratings side by side

Same job, different design

Feature	Score vs rating
Who it covers	Persons vs issuers
How it is made	Model vs analysis
Higher means	Lower risk (score)
Cycle	Point in time vs through

IN THE LESSON

- Let's set them side by side.
- Credit scores are used mainly for individuals and small businesses, where there are thousands of similar borrowers and a lender needs something mechanical.
- A higher score means lower risk.
- Ratings from agencies apply to companies, governments and structured securities such as asset backed bonds, one credit at a time, and they blend financial analysis with judgement.
- Now the point most people miss.
- A score is ordinal.
- It ranks borrowers, but a score of seven hundred is not a seven per cent chance of anything.
- It is not a probability.
- And a score does not adjust for the economic cycle: in a recession many scores fall together.

3 Through the cycle: stable, and therefore slow

- Agencies aim to look through the cycle
- Stable ratings suit mandates and collateral rules
- The cost: they react slowly
- Market spreads usually move first

IN THE LESSON

- Agencies design ratings to be stable through the cycle.
- Think of a ship's captain who does not change course for every wave, only for a change in the tide.
- The agency tries to look past the temporary ups and downs of the economy and rate the borrower's underlying strength over a long horizon.
- That stability is useful.
- Mandates and collateral rules can be written around ratings, because a rating does not flicker.
- But there is a price.
- Ratings react slowly.
- When conditions change fast, the market's credit spread usually moves first, and the rating follows later, if it follows at all.
- So ratings often lag the market, and that is by design, not by accident.

4 Spread first, rating later

Chart: Risk as seen vs Time

- Market spread
- Rating
- News

IN THE LESSON

- Here is the lag drawn.
- The horizontal axis is time, and the vertical axis is how risky the market or the agency thinks the issuer is.
- Start with the green line, the market spread.
- It sits low and calm, then, when bad news about the issuer arrives, it jumps up quickly and begins to ease back.
- Now the gold line, the rating.
- It does not move at all while the spread is racing upward.
- Only later does it step down, and it steps once, to a new level.
- The red line marks the moment the news arrived.
- The gap between the two lines is the lag.
- An investor who waits for the downgrade has already seen most of the move in price.

5 Example 1: one issuer, three ratings

Issuer rated BBB−. Senior spread 150bp → 260bp, ratings unchanged

Issuer rating	overall company	**BBB−**
Secured notes	better recovery	**BBB**
Subordinated notes	paid last	**BB+**
Senior spread moves	ratings unchanged	**+110bp**
Is it a sell call?	rating is no trade call	**No**

IN THE LESSON

- Now a vignette.
- An analyst covers a company whose issuer rating is triple B minus.
- That grade speaks for the company as a whole.
- But the company has three kinds of debt.
- Its secured notes are backed by collateral, so if it fails, lenders are likely to get more back.
- The agency notches them one step up, to triple B.
- Its subordinated notes are paid after everyone else, so they are notched one step down, to double B plus.
- Then the senior spread widens from one hundred fifty to two hundred sixty basis points while all three ratings stay put.
- A client asks whether this is a sell signal.
- The honest answer is no.
- Neither signal is a recommendation.

6 What a rating does not tell you

- Not a default probability: only a ranking
- Not a buy or sell recommendation
- Issue rating can differ from issuer rating (notching)
- Stable rating does not mean unchanged credit

IN THE LESSON

- Let's collect the traps for this outcome.
- First, a rating is not a probability of default.
- It is an ordinal ranking, with higher grades meaning less credit risk, and the agency does not promise a number.
- Second, a rating is not a recommendation to buy or sell.
- A bond can be rated well and still be too expensive, or rated poorly and still be cheap.
- Third, an issue rating is not the same as the issuer rating.
- Notching moves the issue above or below the issuer grade to reflect seniority and expected recovery.
- And fourth, do not read a stable rating as proof that the credit has not changed.
- It may only mean the rating is slow.
- Keep the spread in view alongside it.

7 A story: the yield is not what you will earn

- Yield assumes nothing happens to the credit
- In a year the rating may move up, down or stay
- A new rating means a new spread, so a new price
- Average the price effects by their probabilities

IN THE LESSON

- Imagine you buy a bond that yields four per cent, and you plan to hold it for a year.
- The screen says you will earn four per cent.
- But in that year the company may be upgraded, it may be downgraded, or it may stay exactly where it is.
- Each of those changes the credit spread the market demands, and a change in spread changes the bond's price.
- This movement from one rating to another is called rating migration.
- So your true expected return is the yield, plus the price effect of migration, averaged over everything that might happen, each outcome weighted by how likely it is.
- Let's build that, step by step.

8 The method in one picture

Gather

Year end ratings → Their probabilities → Spread of each

Combine

Change in spread → Price change → Weight and add

IN THE LESSON

- Here is the method as a chain.
- In the top row, we begin with the bond's rating today and its transition probabilities, which tell us how likely each year end rating is.
- For each possible year end rating, we look up the average spread that rating carries.
- Then, in the bottom row, we take the change between the new spread and the current spread.
- We multiply that change by minus the modified duration to get a percentage price change.
- Last, we weight each price change by its probability, add them all up, and add the total to the yield.
- The two things to guard are the change in the spread, not its level, and the full list of outcomes, upgrades included.

9 Price change from a spread change

Price change ≈ −ModDur × ΔSpread + ½ × Convexity × ΔSpread²
Expected return = Yield + Σ (probability × price change)

Modified duration	Price sensitivity to yield
Change in spread	New spread minus today's
Convexity term	Optional, softens big losses
Probability of the move	Chance of that year end rating
Yield	What the bond yields today

IN THE LESSON

- Now the formulas.
- The percentage price change for a move to a given rating is approximately minus the modified duration times the change in spread.
- Modified duration is how sensitive the price is to a one unit move in yield.
- The change in spread is the new rating's spread minus today's spread.
- A rise in spread gives a negative price change, which is why the minus sign is there.
- If you want more accuracy, add a convexity term, one half times convexity times the change in spread squared.
- The expected return is then the yield, plus the sum, across scenarios, of each probability times its price change.
- The result is the expected return over the horizon.

10 The data: a bond rated single A

Year-end rating: probability, spread (now 1.00%)

AA (upgrade)	**2%, 0.70%**
A (unchanged)	90%, 1.00%
BBB	6%, 1.70%
BB	1.5%, 3.50%
B	0.5%, 5.50%

IN THE LESSON

- Here is the data for our vignette.
- An analyst holds a bond rated single A.
- It yields four per cent, its current spread is one per cent, and at the end of the one year horizon its modified duration will be seven.
- The table shows each possible year end rating, with its transition probability, and the spread that rating carries.
- Read it down.
- Two per cent chance of an upgrade to double A, where the spread is zero point seven per cent.
- A ninety per cent chance of staying single A.
- Six per cent for triple B at one point seven, one and a half per cent for double B at three point five, and half a per cent for single B at five point five.
- The probabilities sum to one hundred per cent.

11 Example 2, part 1: upgrade and first downgrade

Modified duration 7.0; spread change = new spread − 1.00%

Stay at A	Δ = 0	**0**
Up to AA, 2%	−7 × (−0.30%) = +2.10%	**+0.042%**
Down to BBB, 6%	−7 × (+0.70%) = −4.90%	**−0.294%**

IN THE LESSON

- Go row by row.
- If the bond stays single A, the spread does not change, so the price change is zero.
- Contribution, nothing.
- An upgrade to double A takes the spread from one per cent to zero point seven, a change of minus zero point three.
- Minus seven times minus zero point three per cent is a gain of two point one per cent.
- Weighted by the two per cent probability, that adds zero point zero four two.
- Now the move to triple B.
- The spread rises from one to one point seven, a change of plus zero point seven.
- Minus seven times zero point seven is a loss of four point nine per cent.
- Weighted by six per cent, that costs zero point two nine four.

12 Example 2, part 2: the deep downgrades and the total

Same bond: the last two downgrades, then add up

Down to BB, 1.5%	−7 × (+2.50%) = −17.50%	**−0.263%**
Down to B, 0.5%	−7 × (+4.50%) = −31.50%	**−0.158%**
Migration effect	0.042 − 0.294 − 0.263 − 0.158	**−0.67%**
Expected return	4.00% − 0.67%	**3.33%**

IN THE LESSON

- Now the two deeper downgrades.
- A move to double B takes the spread from one to three point five, a rise of two and a half points.
- Minus seven times two point five is a loss of seventeen and a half per cent.
- Weighted by one and a half per cent, that costs zero point two six three.
- A move to single B takes the spread up four and a half points, so the loss is thirty one and a half per cent, and weighted by half a per cent it costs zero point one five eight.
- Add all the weighted pieces: the small gain, less the three losses, gives a migration effect of minus zero point six seven.
- So the expected return is four per cent minus zero point six seven, which is three point three three per cent.

13 Where the drag comes from

Effect on return, % points

To AA	**+0.04**
To BBB	**−0.29**
To BB	**−0.26**
To B	**−0.16**

Migration effect in total: about −0.67 points of return

IN THE LESSON

- Here are the weighted contributions as bars.
- The vertical axis is the effect on return, in percentage points, and each bar's height is the size of the effect.
- The first bar, in green, is the upgrade to double A.
- It is the only gain, and it is tiny, just zero point zero four.
- The next three bars, in red, are the downgrades.
- The move to triple B costs about zero point three.
- Double B costs about zero point twenty six.
- Single B, which is the rarest, still costs about zero point sixteen.
- Notice that the two deepest downgrades together cost more than the likeliest one, even though they happen only two per cent of the time.
- Losses can be large because spreads have much more room to widen than to narrow.

14 The trap in numbers: change, not level

Duration multiplies the CHANGE in spread, never the level

Wrong: use the level	−7 × 1.70%	**−11.9%**
Right: use the change	−7 × 0.70%	**−4.9%**
Omit the upgrade	−0.294 − 0.263 − 0.158	**−0.714%**
Include everything	adds back +0.042	**−0.672%**

IN THE LESSON

- The most common slip is to multiply duration by the new spread level instead of the change.
- Take the move to triple B again.
- The new level is one point seven per cent.
- Multiply minus seven by one point seven and you get minus eleven point nine per cent, which would suggest a one notch downgrade nearly wipes out three years of yield.
- The right number uses the change, zero point seven, and gives minus four point nine per cent.
- The second trap is leaving out the upgrade.
- Without it, the migration effect would be minus zero point seven one rather than minus zero point six seven.
- A convexity term would soften the big losses a little, but not change the sign.

15 Why migration usually costs return

- Downgrades tend to be likelier than upgrades
- Spreads widen far more than they narrow
- So migration often reduces expected return
- Size depends on rating, horizon and spreads

IN THE LESSON

- Why does migration usually reduce expected return?
- Two reasons work together.
- First, for an investment grade bond the transition matrix generally shows downgrades being more likely than upgrades.
- Second, the price effects are lopsided.
- A spread can widen a great deal on a downgrade, but it can only narrow a little on an upgrade, because spreads cannot fall below zero and a strongly rated bond has little room to tighten.
- So the loss side is larger and likelier.
- But be careful with the word usually.
- The size of the effect depends on the starting rating, the horizon, and the spreads, and a bond near the top of the scale, or one with a favourable outlook, can show a different picture.

16 A story: two ways to ask why a firm fails

- Structural: look inside, assets against debt
- Reduced form: watch the market, default arrives
- One explains why, the other fits prices
- Each is strong where the other is weak

IN THE LESSON

- Two doctors are asked whether a patient will fall ill.
- The first examines the patient closely and builds a theory of how illness happens.
- The second looks at thousands of patients and notes how often people like this one fall ill each year.
- She has no theory of why, only a reliable rate.
- Credit models split the same way.
- A structural model looks inside the company, at its assets and its debt, and says default happens when the assets are not enough to pay the lenders.
- A reduced form model looks at the pattern in market prices and treats default as a random event that arrives at some rate.

17 The structural model in one line

Equity = Call on assets (strike = debt)
Risky debt = Risk-free debt − Put on assets

Assets at maturity	Falls below debt: default
Face value of debt	The strike of the option
Equity: call on assets	Shareholders' claim
Put on the assets	Lenders' implied loss
Risk-free debt	Debt valued at the risk-free rate

IN THE LESSON

- Here is the structural idea.
- Shareholders own the company's assets, but lenders must be repaid first.
- Say the company has a single zero coupon debt issue with a face value of one hundred, due in a year.
- At maturity, if the assets are worth more than one hundred, the lenders are paid in full and the shareholders keep the rest.
- If the assets are worth less, the shareholders walk away and the lenders take what there is.
- So the shareholders hold a call option on the assets, with a strike equal to the face value of the debt.
- Default means the assets fall below the debt at maturity.
- And risky debt equals the same promise with no default risk, less a put option on the assets.

18 What each side receives at maturity

Chart: Payoff at maturity vs Asset value at maturity

- Lenders: capped
- Shareholders: a call
- Debt face value

IN THE LESSON

- Now the picture.
- The horizontal axis is the value of the company's assets at the maturity date.
- The vertical axis is what each side receives.
- The red vertical line marks the face value of the debt.
- First, the green line, the lenders.
- When the assets are low, the lenders take everything, so the line rises with the assets.
- Once the assets pass the face value, the lenders are paid in full and the line goes flat.
- Now the gold line, the shareholders.
- They get nothing until the assets pass the face value, and then they get every extra dollar.
- That is the shape of a call option.

19 Example 3: pricing the debt with the structural model

Assets 120, debt 100 due in 1 year, volatility 30%, r = 5%

Equity (call, K = 100)	option model	**28.88**
Risky debt	120 − 28.88	**91.12**
Risk-free debt	100 × e^(−0.05)	**95.12**
Put on the assets	95.12 − 91.12	**4.00**
Credit spread	ln(100 ÷ 91.12) − 5%	**4.30%**

IN THE LESSON

- Let's run it on a made up company.
- Its assets are worth one hundred twenty, and it has one zero coupon debt issue with a face value of one hundred, due in a year.
- Asset volatility is thirty per cent, and the continuously compounded risk free rate is five per cent.
- An option model prices the equity, a call with a strike of one hundred, at twenty eight point eight eight.
- Risky debt is what is left of the assets, one hundred twenty less twenty eight point eight eight, which is ninety one point one two.
- The same promise with no default risk would be worth one hundred times e to the minus zero point zero five, ninety five point one two.
- The gap, four point zero zero, is the put.
- The credit spread is the natural log of one hundred over ninety one point one two, minus five per cent: four point three per cent.

20 The reduced form view: default arrives as a surprise

- Default = a random event with a hazard rate
- Hazard can depend on company and macro variables
- Uses observable data, fits market prices
- Silent on why the company defaults

IN THE LESSON

- Now the other family.
- A reduced form model gives up on explaining why a company fails.
- It says only that default is a random event, and it arrives with a certain intensity, called the hazard rate.
- The hazard rate is the probability of default in the next period, given that the company has survived so far.
- It can be a constant, or it can move with company variables such as borrowing, and with macroeconomic variables such as growth.
- Since the model only needs a hazard rate and a recovery rate, it can use observable data and be fitted so that it reproduces the market prices of the company's bonds.
- The price paid for that convenience is that it cannot tell you what would make the company default.

21 Strong and weak points compared

Which model is strong where?

Point	Model
Explains why default occurs	Structural
Needs unobservable inputs	Structural
Fits market prices easily	Reduced form
No cause for default	Reduced form

IN THE LESSON

- Let's compare, row by row, with the assumptions in mind.
- The structural model assumes the company has a simple liability structure, often a single zero coupon debt issue, and that default occurs only when the assets fall below the debt at maturity.
- Its strength is that it explains why default happens and links credit risk directly to leverage and to asset volatility.
- Its weakness is that the market value of the assets and their volatility cannot be observed.
- The reduced form model assumes default is a surprise governed by a hazard rate.
- Its strength is that it uses observable inputs and fits market prices.
- Its weakness is that it gives no economic reason for default.

22 Example 4: a constant hazard rate over three years

Hazard rate 2% a year, constant

Survive 1 year	1 − 0.02	**98.00%**
Survive 2 years	0.98 × 0.98	**96.04%**
Survive 3 years	0.98^3	**94.12%**
Default within 3 years	1 − 0.9412	**5.88%**

IN THE LESSON

- A short example with a hazard rate.
- Suppose an analyst estimates a constant annual hazard rate of two per cent.
- The chance of surviving the first year is ninety eight per cent.
- Survival over two years is ninety eight per cent of ninety eight per cent, which is ninety six point zero four.
- Over three years it is ninety eight cubed, which is ninety four point one two per cent.
- So the probability of default sometime in the three years is one minus that, five point eight eight per cent.
- Notice it is a little less than three times two per cent.
- The reason is that the hazard rate is applied only to firms that survived to the start of each year, which is exactly the idea we use in the pricing table next.

23 Mistakes to avoid so far

- 1. Rating or score read as a probability or advice
- 2. Stable rating taken as unchanged credit
- 3. Duration × spread level; upgrades left out
- 4. Structural model: needs hidden inputs
- 5. Reduced form: fits prices, gives no cause

IN THE LESSON

- A pause to collect the slips from the first three outcomes.
- One.
- Treating a rating or a score as a default probability, or as a trade recommendation.
- Two.
- Believing a stable rating means the credit has not changed, when it may just be slow.
- Three.
- In expected return, multiplying duration by the new spread level rather than the change, and leaving out the upgrade scenarios.
- Four.
- Saying that the structural model needs no estimates.
- It needs the asset value and the asset volatility, which nobody can observe.
- And five.
- Saying the reduced form model explains why a company defaults.
- It does not.
- It fits prices, and the hazard rate is an input, not a reason.

24 A story: the bond with a small hole in it

- Start from the default-free value of the bond
- Estimate the expected loss from default, date by date
- Discount those losses: the credit valuation adjustment
- Fair value = default-free value − adjustment

IN THE LESSON

- Think of a bond with no default risk as a bucket that holds exactly the water it promises.
- A risky bond is the same bucket with a small hole.
- You only need to estimate how much water you expect to lose, and take it off the full bucket.
- In credit terms the leak is the expected loss from default, and its present value is called the credit valuation adjustment.
- So, to value a risky bond, start with its default free value, from discounting the promised payments at risk free rates.
- Then subtract the adjustment.
- What is left is the fair value.

25 The building blocks of the adjustment

LGD = Exposure × (1 − Recovery) PD = Hazard × Survival to start
CVA = Σ (LGD × PD × discount factor) V = default-free value − CVA

Exposure	Value of all payments still due
Loss given default	Exposure × (1 − recovery)
Probability of default	Hazard × prior survival
CVA	Sum of discounted expected losses
Fair value	Default-free value − CVA

IN THE LESSON

- Now the formulas.
- For each payment date, the exposure is the value on that date of every payment still due, including the one that is due.
- The loss given default is the exposure times one minus the recovery rate.
- The probability of default for a given year, in this discrete approximation, is the hazard rate times the probability of surviving to the start of that year.
- Multiply the loss by the probability to get the expected loss, discount it at the risk free rate, and add up across the dates.
- That sum is the credit valuation adjustment.
- The fair value is the default free value less that sum.
- And as a quick check, the credit spread is close to the hazard rate times one minus the recovery rate.

26 Example 5, part 1: exposure and loss given default

3-year bond, 5% coupon, risk-free 3%, hazard 1.5%, recovery 40%

Exposure, year 1	all payments due, at 3%	**108.83**
Exposure, year 2		**106.94**
Exposure, year 3		**105.00**
Loss given default	exposure × 0.60	**65.30/64.17/63.00**

IN THE LESSON

- Here is the full example.
- A three year bond pays a five per cent annual coupon on a face value of one hundred.
- The risk free rate is three per cent at every maturity, the hazard rate is one and a half per cent a year, and the recovery rate is forty per cent.
- Defaults can occur only on coupon dates, just before payment.
- First the exposure.
- In year one it is the coupon due plus all later payments, valued at three per cent: one hundred eight point eight three.
- In year two, one hundred six point nine four.
- In year three, just the final payment, one hundred five.
- Loss given default is the exposure times zero point six: sixty five point three, sixty four point one seven, and sixty three.

27 Example 5, part 2: probabilities and the adjustment

Probability × loss, then discount at 3%

Probability of default	1.5% × survival	**1.50/1.48/1.46%**
Expected loss	PD × LGD	**0.98/0.95/0.92**
Discounted	÷ 1.03, 1.03^2, 1.03^3	**0.95/0.89/0.84**
CVA	sum	**2.68**

IN THE LESSON

- Next, the probability of default each year.
- In year one it is the hazard rate, one point five per cent, because the bond is certain to be alive at the start.
- In year two it is one point five per cent times the chance of surviving year one, ninety eight and a half per cent, which is one point four seven seven five.
- In year three it multiplies by survival through two years, giving one point four five five.
- Multiply each probability by its loss given default to get the expected losses: zero point nine eight, zero point nine five and zero point nine two.
- Discount them at three per cent and you get zero point nine five, zero point eight nine and zero point eight four.
- Their sum, two point six eight, is the credit valuation adjustment.

28 Each year's share of the adjustment

Present value of expected loss

Year 1	**0.95**
Year 2	**0.89**
Year 3	**0.84**

The three bars add up to the adjustment: 2.68

IN THE LESSON

- Here is each year's present value of expected loss as a bar.
- The vertical axis is the present value, per hundred of face value.
- Year one is the tallest, at zero point nine five.
- Year two is lower, at zero point eight nine.
- Year three is lower again, at zero point eight four.
- Three things push the bars down.
- Exposure falls, because every coupon paid is one less payment that a default can take away.
- The probability falls slightly, because to default in year three the issuer first has to survive years one and two.
- And later losses are discounted for longer.
- None of this means the issuer becomes safer.
- The hazard rate is the same every year.

29 Example 5, part 3: fair value and the credit spread

Default-free value, less CVA, then solve for the yield

Default-free value	payments at 3%	**105.66**
Fair value	105.657 − 2.684	**102.97**
Yield to maturity	at a price of 102.97	**3.93%**
Credit spread	3.93% − 3.00%	**0.93%**
Quick check	1.5% × (1 − 0.40)	**0.90%**

IN THE LESSON

- Now the value.
- Without default risk, the bond's payments discounted at three per cent are worth one hundred five point six six.
- Take off the adjustment of two point six eight, and the fair value is one hundred two point nine seven.
- Now the step people skip.
- Find the yield to maturity at that price.
- Five per cent coupons on a price of one hundred two point nine seven give a yield of about three point nine three per cent.
- The credit spread is that yield less the risk free yield of three per cent, so zero point nine three per cent.
- The quick check is the hazard rate times one minus recovery, one and a half per cent times sixty per cent, which is zero point nine.
- The two agree.

30 Example 6: what happens when recovery falls

Recovery cut from 40% to 20%; hazard unchanged at 1.5%

Loss given default	exposure × 0.80	**was ×0.60**
CVA	was 2.68	**3.58**
Fair value	105.657 − 3.578	**102.08**
Credit spread	yield 4.25% − 3.00%	**1.25%**
Quick check	1.5% × (1 − 0.20)	**1.20%**

IN THE LESSON

- One more case.
- The same company now issues secured debt that ranks ahead of our bond, so the analyst cuts the recovery rate from forty per cent to twenty per cent.
- The hazard rate is unchanged.
- Loss given default rises from sixty per cent of exposure to eighty per cent.
- Every expected loss grows by a third, so the adjustment rises from two point six eight to three point five eight.
- The fair value falls to one hundred two point zero eight, the yield rises to about four point two five per cent, and the spread is one point two five.
- The quick check is one and a half per cent times eighty per cent, one point two.
- Same default risk, bigger spread.

31 Traps in the pricing table

- Spread = YTM − risk-free yield; solve for the yield
- Loss uses (1 − recovery), exposure includes the payment due
- PD = hazard × survival to the start of the year
- Discount the losses at the risk-free rate
- Wider spread is not always higher default risk

IN THE LESSON

- Check these before you hand in a pricing table.
- First, the spread is the yield on the risky bond less the risk free yield, not the adjustment divided by anything.
- You must solve for the yield.
- Second, exposure includes the payment due on that date, and loss uses one minus recovery, not the recovery rate itself.
- Third, the probability of default is the hazard rate times survival to the start of the year, not the hazard rate alone, except in year one.
- Fourth, discount the expected losses at the risk free rate, not at the bond's own yield.
- And fifth, a wider spread does not always mean a higher default probability.
- Recovery, liquidity and risk appetite also move it.

32 Recap: the ideas to remember

Return = Y + Σ p × (−ModDur × ΔSpread) Risky debt = Risk-free debt − Put
V = default-free value − CVA Spread ≈ Hazard × (1 − Recovery)

Modified duration	Duration times change in spread
Probability of the move	Weights, upgrades included
Put on the assets	Structural: debt less a put
CVA	Reduced form: sum of losses
Credit spread	Yield less risk-free yield

IN THE LESSON

- Let's pull it together.
- Scores and ratings both rank credit risk.
- They are ordinal, they are not probabilities, and ratings are slow by design.
- Expected return is the yield plus the sum of probability times minus duration times the change in spread.
- Structural models say equity is a call on the assets, so risky debt is risk free debt less a put, and they explain default but need hidden inputs.
- Reduced form models use a hazard rate, fit prices, and give no cause.
- Finally, fair value is the default free value less the credit valuation adjustment, and the spread is the yield on that value less the risk free yield, close to hazard times one minus recovery.

33 Try three quick questions

1. Duration 6, spread +0.5% with probability 10%: contribution?
   Answer: −6 × 0.5% × 10% = −0.30%
2. Assets at maturity 90, debt face value 100. Equity value?
   Answer: Call, strike 100: 0 (assets are below debt)
3. Hazard 3%, recovery 40%. Approximate spread?
   Answer: 3% × (1 − 0.40) = 1.80%

IN THE LESSON

- Now try three questions yourself.
- Pause after each one.
- Question one.
- A bond has a modified duration of six, and may be downgraded, which would widen its spread by half a per cent, with a probability of ten per cent.
- What does that scenario contribute to expected return?
- Minus six times half a per cent is minus three per cent, times ten per cent is minus zero point three per cent.
- Question two.
- A company has assets of ninety and zero coupon debt with a face value of one hundred at maturity.
- Its equity is worth what at maturity, in that scenario?
- A call with a strike of one hundred on assets of ninety is out of the money, so zero.
- Question three.
- A hazard rate of three per cent and a recovery rate of forty per cent give an approximate spread of what?
- Three per cent times sixty per cent, one point eight per cent.