Curve risk, yield volatility and no arbitrage

Fixed Income

1 Duration pulls one lever; the curve has many

- Portfolio: $50m, effective duration 8.0
- Duration assumes every yield moves together
- Real curves twist, tilt and bend
- We need exposure point by point

IN THE LESSON

- Dana runs a fifty million dollar bond portfolio for Marlowe Pensions.
- The portfolio has an effective duration of eight.
- Level One taught you what that means: if every yield rose by one percentage point together, the portfolio would lose about eight per cent.
- But think of the yield curve as a rope pegged down at several points.
- Duration assumes you lift the whole rope evenly.
- In real markets the short end may jump while the long end sleeps, or the middle may sag while both ends rise.
- A single duration number cannot tell you what happens then.
- Dana needs a way to see exposure at each point along the curve, and that is what we build now.

2 Three ways a curve can move

Chart: Yield vs Maturity (short to long)

- Today
- Level shift
- Steeper

IN THE LESSON

- Here is the picture.
- Maturity runs along the bottom, from short to long, and the yield sits up the side.
- The grey line is today's curve, rising gently.
- The dashed green line is a level change, or parallel shift: every maturity moves up by the same amount, so the shape is unchanged.
- The gold line is a steepness change.
- Short yields drop and long yields climb, so the gap between them widens.
- The curve tilts like a see saw.
- The third movement, curvature, is a change in the middle of the curve relative to its two ends, a bend rather than a tilt.
- Researchers find that these three movements explain most of what yield curves do.

3 Level, steepness and curvature

- Level: a parallel shift, all yields together
- Steepness: the gap between short and long
- Curvature: the middle against the two ends
- Exposure depends on where cash flows sit

IN THE LESSON

- Let's pin down the three factors.
- Level is a parallel shift.
- Every yield rises or falls together.
- This is the movement that duration alone captures, and it is the biggest of the three.
- Steepness is a change in the gap between short and long yields.
- If the gap widens, the curve steepens.
- If it narrows, the curve flattens.
- Curvature is a change in the middle of the curve relative to the short and long ends.
- If the middle yields rise while the ends hold still, the curve becomes more humped.
- A portfolio is exposed to each factor to a different degree, depending on where its cash flows sit along the maturity line.

4 Key rate duration: one point at a time

- Move one yield, hold the others fixed
- The value change gives that point's duration
- Sum of key rate durations = effective duration
- A profile of exposure along the curve

IN THE LESSON

- The tool is key rate duration.
- Pick one point on the curve, say the five year yield.
- Move only that yield by a small amount and hold every other key yield fixed.
- Then see how much the portfolio's value changes.
- That sensitivity is the key rate duration for that point.
- Do the same at two years, ten years, thirty years, and you have a profile of exposure along the curve.
- Here is a neat check.
- If you add the key rate durations together, you get the portfolio's effective duration.
- The profile simply breaks that one number into pieces.
- A big key rate duration at ten years means the portfolio is mostly a bet on what happens to ten year yields.

5 Dana's portfolio: where the duration sits

Key rate duration

2 years	**0.8**
5 years	**2.1**
10 years	**3.4**
30 years	**1.7**

Together they add up to 8.0

IN THE LESSON

- Here is Dana's profile.
- The bars are the key rate durations at four points on the curve.
- At two years the key rate duration is zero point eight.
- At five years it is two point one.
- At ten years it is three point four, the tallest bar.
- And at thirty years it is one point seven.
- Add them up: zero point eight plus two point one plus three point four plus one point seven makes eight point zero, exactly the effective duration we started with.
- So the profile is not a different measure, it is the same risk sliced up.
- Notice that most of the exposure sits in the middle and long end.
- Keep that in mind, because it decides how the portfolio reacts to a tilt in the curve.

6 Estimating the change in value

Change in value ≈ −Σ (key rate duration × yield change)

Change in value (%)	The answer, as a % of value
Key rate duration	Sensitivity at one point
Yield change at that point	Move in that yield, in % points

IN THE LESSON

- Here is the formula.
- The estimated percentage change in value equals minus the sum, over every key point, of the key rate duration multiplied by the yield change at that point.
- Take each term in turn.
- The key rate duration is the sensitivity at one point, like zero point eight at two years.
- The yield change at that point is how far that particular yield moves, in percentage points.
- Multiply them, then add the results across all the points.
- The minus sign out in front is the one that catches people.
- Duration is positive, but when yields rise, value falls.
- So rising yields give a negative number, and falling yields give a gain.
- Always check the sign last.

7 Example 1: a steepening curve

Yield changes: 2y −10 bp, 5y 0, 10y +15 bp, 30y +25 bp

Two-year	−0.8 × (−0.10%)	**+0.080%**
Five-year	−2.1 × 0	**0**
Ten-year	−3.4 × 0.15%	**−0.510%**
Thirty-year	−1.7 × 0.25%	**−0.425%**

Mind the sign: a falling yield is a gain

IN THE LESSON

- Example one.
- The curve steepens.
- The two year yield falls by ten basis points.
- The five year is unchanged.
- The ten year rises by fifteen basis points.
- The thirty year rises by twenty five.
- A basis point is one hundredth of a percentage point.
- Work through each point.
- At two years: minus zero point eight, times a fall of zero point one zero per cent, gives a gain of zero point zero eight zero per cent.
- At five years the yield did not move, so the contribution is zero.
- At ten years: minus three point four times zero point one five per cent is a loss of zero point five one zero.
- At thirty years: minus one point seven times zero point two five is a loss of zero point four two five.

8 Example 1: the total, and why the average misleads

Add the points, then compare with a parallel move

Total of the four points	0.080 + 0 − 0.510 − 0.425	**−0.855%**
On $50m	−0.855% × $50m	**−$427,500**
Parallel +10 bp, for contrast	−8.0 × 0.10%	**−0.80%**
Duration on the 7.5 bp average	−8.0 × 0.075%	**−0.60%**

Exposure sits where yields rose most, so the loss is bigger

IN THE LESSON

- Now add the four contributions.
- Plus zero point zero eight, plus nothing, minus zero point five one, minus zero point four two five.
- The total is minus zero point eight five five per cent.
- On fifty million dollars, that is a loss of four hundred and twenty seven thousand, five hundred dollars.
- Compare a plain parallel rise of ten basis points.
- Duration of eight times zero point one zero per cent gives minus zero point eight per cent, four hundred thousand dollars.
- Here is the trap.
- The average yield change across the four points was a rise of seven and a half basis points.
- Duration on that average would suggest a loss of only zero point six per cent.
- The real loss is bigger, because the exposure sits where yields rose most.

9 Example 2: managing curve risk

New key rate durations 1.4, 2.7, 2.9, 1.0 (still total 8.0)

Two-year	−1.4 × (−0.10%)	**+0.140%**
Ten-year	−2.9 × 0.15%	**−0.435%**
Thirty-year	−1.0 × 0.25%	**−0.250%**
Total	0.140 + 0 − 0.435 − 0.250	**−0.545%**
On $50m	−0.545% × $50m	**−$272,500**

IN THE LESSON

- Now management.
- Suppose Dana expects exactly that steepening.
- She can reshape exposure without changing the total duration.
- She sells some ten and thirty year bonds, and buys more two and five year bonds, so that the new key rate durations are one point four, two point seven, two point nine and one point zero.
- They still add up to eight point zero, so the level exposure is unchanged.
- Rerun the same scenario.
- Minus one point four times minus zero point one gives plus zero point one four.
- Five years adds nothing.
- Minus two point nine times zero point one five is minus zero point four three five, and minus one point zero times zero point two five is minus zero point two five.
- The total is minus zero point five four five per cent, a loss of two hundred and seventy two thousand, five hundred dollars, down from four hundred and twenty seven thousand.

10 Short yields jump about; long yields are anchored

- Yield volatility: how much a yield moves in a year
- Short yields track policy and fresh news
- Long yields reflect average rates over many years
- So the short end is typically most volatile

IN THE LESSON

- Next question: how much do yields themselves wobble?
- Yield volatility is how much a yield tends to move over a year, often measured in basis points.
- Here's an analogy.
- A kite on a short string whips around in every gust.
- A kite on a very long string swings slowly, because the weight of all that line damps it down.
- Short term yields are the short string.
- They respond directly to central bank policy and to every piece of economic news.
- Long term yields reflect expectations of average short rates over many years, so a surprise today barely shifts that average.
- The result is a term structure of yield volatility, and typically the short end is the most volatile.

11 Marlowe's yield volatility by maturity

Yield volatility (bp a year)

2-year	**95**
10-year	**70**
30-year	**60**

Illustrative numbers: the short end moves most

IN THE LESSON

- Here are the numbers Dana has from her risk system.
- These are made-up round numbers to illustrate the idea.
- The two year yield has a volatility of ninety five basis points a year, the tallest bar.
- The ten year yield has seventy.
- The thirty year yield has sixty.
- So the volatility falls as maturity grows, just as the kite story suggests.
- If you stopped here, you might guess the two year bond is the riskiest to hold.
- Hold that thought, because that guess is exactly the trap in this topic, and the next step shows why it fails.

12 From yield volatility to price volatility

Price volatility ≈ modified duration × yield volatility

Price volatility	Typical price swing, % a year
Modified duration	Sensitivity of price to yield
Yield volatility	Typical yield swing, % a year

IN THE LESSON

- Here is the link.
- Price volatility is roughly the modified duration multiplied by the yield volatility.
- Take the terms.
- Yield volatility is how much the yield moves, here ninety five basis points for the two year.
- Modified duration is the percentage price change for a one per cent move in yield, and it grows with maturity.
- Multiplying them turns a yield wobble into a price wobble.
- This is the same arithmetic you used for key rate durations, only now we use typical size of move rather than a specific scenario.
- Two things drive price risk, then, not one: how far the yield moves, and how sensitive the price is to each unit of movement.
- Both matter.

13 Example 3: three bonds, $10 million each

$10m in each bond; price volatility = duration × yield volatility

Two-year	1.9 × 0.95% = 1.805%	**$180,500**
Ten-year	8.5 × 0.70% = 5.95%	**$595,000**
Thirty-year	17.0 × 0.60% = 10.2%	**$1,020,000**

Calmest yield, biggest price risk

IN THE LESSON

- Example three.
- Dana holds ten million dollars in each of three government bonds.
- The two year has modified duration one point nine and yield volatility ninety five basis points, which is zero point nine five per cent.
- One point nine times zero point nine five is one point eight zero five per cent, about one hundred and eighty thousand dollars on ten million.
- The ten year has duration eight point five and volatility seventy basis points.
- Eight point five times zero point seven is five point nine five per cent, five hundred and ninety five thousand dollars.
- The thirty year has duration seventeen and volatility sixty basis points.
- Seventeen times zero point six is ten point two per cent, over one million dollars.
- The ranking of price risk is the exact opposite of the ranking of yield volatility.

14 Yield volatility versus price volatility

Price volatility (% a year)

2-year	**1.8**
10-year	**5.95**
30-year	**10.2**

The ranking flips: long bonds carry the price risk

IN THE LESSON

- Let's draw the answer.
- Same three bonds, but now the bars show price volatility in per cent a year.
- The two year is the small bar at one point eight.
- The ten year is five point nine five.
- The thirty year is ten point two, more than five times the two year.
- Look back at the yield volatility picture, where the order ran the other way.
- That is the lesson in one glance.
- Short yields are the jumpiest, yet they are attached to bonds with little duration.
- Long yields are calmer, yet they are attached to bonds with enormous duration.
- When a risk manager asks which position could hurt most in a year, the answer comes from this second picture, not the first.

15 Example 4: when volatility itself changes

Thirty-year bond, $10m, duration 17.0

Yield volatility 60 bp	17.0 × 0.60%	**10.2%**
Yield volatility 75 bp	17.0 × 0.75%	**12.75%**
Loss on $10m rises to	12.75% × $10m	**$1,275,000**
Higher volatility, callable bond	issuer's option worth more	**value falls**
Higher volatility, putable bond	investor's option worth more	**value rises**

Rule: price risk = duration × yield volatility

IN THE LESSON

- Two more points about volatility.
- First, for a given duration, price risk moves in proportion to yield volatility.
- If thirty year yield volatility rises from sixty to seventy five basis points, the price volatility rises from ten point two to twelve point seven five per cent, a quarter higher, and the loss on ten million dollars goes from one million twenty thousand to one million, two hundred and seventy five thousand.
- Second, the shape of the volatility curve matters when a bond has an embedded option.
- The option's value depends on assumed volatility.
- Higher volatility makes a call option held by the issuer worth more, so a callable bond is worth less.
- The same higher volatility makes a put option held by the investor worth more, so a putable bond is worth more.

16 From the economy to the curve

- Short end: anchored by the policy rate
- Policy depends on inflation and growth
- Long end: expected average short rates + term premium
- Premium rises with inflation doubt and borrowing

IN THE LESSON

- Now to the question every bond manager faces: what do I think rates will do, and where on the curve?
- Start with a simple split.
- The short end is anchored by the policy rate, the rate the central bank sets.
- So expected central bank action, which depends on inflation and growth, drives short yields.
- The long end is different.
- A long yield is roughly the expected average of short rates over many years, plus a term premium.
- That premium is extra compensation for the risk of holding a long bond.
- It rises when inflation is uncertain and when governments borrow heavily.
- Two different engines, then, one for each end of the curve.

17 Two engines behind the curve

Short end

Inflation and growth news → Expected policy rate changes → Short yields move

Long end

Inflation doubt, heavy borrowing → Term premium moves → Long yields move

IN THE LESSON

- Let me draw the chain of cause and effect.
- The top row is the short end.
- Inflation and growth news changes what the market expects the central bank to do.
- Expected policy changes move short yields, usually quickly.
- The bottom row is the long end.
- Doubt about future inflation, or a surge in government borrowing, raises the term premium.
- A higher premium, together with the path of expected short rates, moves long yields.
- Read each row left to right.
- When you form a view, always ask which row your news belongs to.
- A policy surprise is a top row story.
- A fear about inflation five years from now, or a big bond issue, is a bottom row story.

18 Scenario to curve move

Match the scenario to where the curve moves most

Scenario	Curve move
Credible tightening	Bear flatten
Easing into a slowdown	Bull steepen
Flight to quality	Spreads widen
Inflation fear, doubt on central bank	Bear steepen

IN THE LESSON

- Here is a map of common scenarios.
- First, a credible central bank tightening.
- Short yields jump, long yields rise by less because markets believe inflation will be tamed.
- Both rise, but the gap narrows.
- That is a bear flattening, bear because yields rise and prices fall.
- Second, easing into a slowdown.
- Short yields fall with the policy rate, long yields fall less.
- The gap widens: a bull steepening.
- Third, a flight to quality.
- Investors rush into government bonds, pushing government yields down and credit spreads out, since riskier borrowers must pay more.
- Fourth, inflation fears with doubt about the central bank.
- Long yields rise more than short yields: a bear steepening.

19 A credible tightening, drawn

Chart: Yield vs Maturity (short to long)

- Before
- After
- +50 bp
- +20 bp

IN THE LESSON

- Here is the first scenario as a picture.
- Maturity runs along the bottom, yield up the side.
- The grey line is the curve before.
- The green line is the curve after a credible tightening.
- At the short end it has climbed by fifty basis points.
- At the long end it has climbed by only twenty.
- Everything has moved up, which is why prices fall, but not evenly.
- The gap between short and long has narrowed from steep to shallow.
- That narrowing is the flattening.
- If Dana believes in this picture, she wants a position that gains from the change in slope, not just from the direction of rates, and the next example builds one.

20 Example 5: a duration-neutral flattener

Buy $20m ten-year (D 8.5); sell two-year (D 1.9), same money duration

Long ten-year, per bp	$20m × 8.5 × 0.01%	**$17,000**
Short two-year, per bp	$17,000 per bp, same	**$17,000**
Size of two-year short	$17,000 ÷ (1.9 × 0.01%)	**$89.47m**

Equal money duration: a parallel move nets about zero

IN THE LESSON

- Example five.
- Dana expects the flattening: two year yields up fifty basis points, ten year yields up twenty.
- Her position is a flattener.
- She buys twenty million dollars of a ten year bond with modified duration eight point five, and sells a two year bond with modified duration one point nine.
- The sizing is the clever part.
- The long ten year leg loses, per basis point, twenty million times eight point five times zero point zero one per cent, which is seventeen thousand dollars.
- To match that, the short leg needs a face amount of seventeen thousand divided by one point nine times zero point zero one per cent.
- That is about eighty nine point four seven million dollars of the two year.
- Now both legs have the same money duration.

21 Example 5: the result

2y +50 bp, 10y +20 bp: bear flattening

Short two-year gains	$17,000 × 50 bp	**+$850,000**
Long ten-year loses	$17,000 × 20 bp	**−$340,000**
Net	850,000 − 340,000	**+$510,000**

Rates rose everywhere, yet the flattener made money

IN THE LESSON

- Now the rates move as Dana expected.
- The two year yield rises fifty basis points, and since she is short, she gains.
- Eighty nine point four seven million times one point nine times zero point five per cent is eight hundred and fifty thousand dollars.
- Equivalently, seventeen thousand dollars a basis point times fifty.
- The ten year yield rises twenty basis points, and she is long, so she loses twenty million times eight point five times zero point two per cent, three hundred and forty thousand dollars.
- The net is a profit of five hundred and ten thousand dollars.
- Notice what happened.
- Every yield on the curve rose, and Dana still made money, because the position was built to profit from the change in slope.

22 Example 6: the scenario goes wrong

2y +20 bp, 10y +50 bp: bear steepening, same position

Short two-year gains	$17,000 × 20 bp	**+$340,000**
Long ten-year loses	$17,000 × 50 bp	**−$850,000**
Net	340,000 − 850,000	**−$510,000**

Check the scenario-to-curve link twice

IN THE LESSON

- Now the warning.
- Suppose the story was wrong.
- The market doubts the central bank's resolve, inflation fears build, and the term premium rises.
- The two year yield rises only twenty basis points, but the ten year rises fifty.
- That is a bear steepening.
- The same position now loses on the long leg: seventeen thousand dollars a basis point times fifty is eight hundred and fifty thousand.
- And it gains on the short leg only seventeen thousand times twenty, three hundred and forty thousand.
- The net is a loss of five hundred and ten thousand dollars.
- Same trade, same size, opposite outcome.
- The position is only as good as the link between your scenario and the part of the curve it moves most.

23 Mistakes to avoid so far

- 1. Rising yields = losses; check the sign
- 2. Duration neutral still has curve risk
- 3. Volatile yields ≠ volatile prices
- 4. Match the scenario to the right end of the curve
- 5. Never apply duration to an average yield move

IN THE LESSON

- Let's collect the mistakes that cost marks.
- One.
- The sign.
- Rising yields reduce value, so a key rate duration times a rise is a loss.
- Two.
- Treating duration neutral as safe.
- Matching total duration removes level risk, but steepness and curvature risk remain.
- Three.
- Concluding that the most volatile yields give the most volatile prices.
- Price risk is duration times yield volatility, so long bonds usually win.
- Four.
- Matching the wrong scenario to the wrong part of the curve, for example calling a credible tightening a steepener.
- Five.
- Averaging the yield changes and applying duration.
- That hides where the exposure sits.
- Keep this list in mind, because the next topic has its own favourite trap, the direction of the trade.

24 One price for one set of cash flows

- Same cash flows must mean the same price
- Arbitrage: riskless profit, no net investment
- Arbitrage-free value leaves none on the table
- A consistency rule, not a forecast

IN THE LESSON

- Now the foundation under all bond valuation.
- Imagine two fruit baskets.
- Each holds exactly the same fruit, picked on the same days.
- Would you expect one to cost twice the other?
- If it did, you would buy the cheap basket and sell the dear one, and the price gap would vanish quickly.
- That is the law of one price.
- For bonds, it says two instruments with identical cash flows must have the same price.
- Break that rule and someone can earn a riskless profit with no net investment.
- That profit is called an arbitrage.
- Arbitrage free valuation means finding the price that leaves no such profit available.
- It is a rule about consistency, not a prediction about the future.

25 Every bond is a bundle of strips

- A bond is a bundle of dated payments
- A strip pays one amount on one date
- Strip price = discount factor for that date
- Bond value = sum of payments at strip prices

IN THE LESSON

- How do we find the arbitrage free value of a coupon bond?
- Think of the bond as a bundle of separate payments.
- A three year bond paying a coupon each year is a payment in year one, a payment in year two, and a payment plus the principal in year three.
- Each of those payments can be bought on its own, as a zero coupon bond, often called a strip.
- A strip pays one amount on one date and nothing else.
- Its price per unit of face value is a discount factor for that date, and the yield behind it is the spot rate for that date.
- So the bond is worth the sum of its payments, each valued at its own strip price.

26 The arbitrage-free value formula

Value = $CF_1 \times Z_1 + CF_2 \times Z_2 + \ldots$
$Z = 1 \div (1 + s)^t$

Arbitrage-free value	The no-arbitrage price
Cash flow on a date	Coupon, or coupon + principal
Strip price for that date	Price of one unit of that date
Spot rate for that date	Rate for that date's maturity

IN THE LESSON

- Here is the formula.
- The arbitrage free value of a bond equals the sum of each cash flow multiplied by the strip price for its date.
- Take the terms.
- The cash flow is the payment on one date: the coupon, or in the last year the coupon plus the principal.
- The strip price is what one unit of that date's payment costs today, such as zero point nine seven for one year.
- It equals one divided by one plus the spot rate, raised to the power of the number of years.
- So multiplying by the strip price is the same as discounting at that date's own spot rate.
- Not one single yield for the whole bond, but a separate rate for each date.

27 Example 7: valuing a bond from strips

Strips: 0.9700, 0.9400, 0.8960. Bond: 3 years, 5% coupon, face 100

Year 1 coupon	5 × 0.9700	**4.85**
Year 2 coupon	5 × 0.9400	**4.70**
Year 3 coupon + principal	105 × 0.8960	**94.08**
Arbitrage-free value	4.85 + 4.70 + 94.08	**103.63**

IN THE LESSON

- Example seven.
- Dana sees strip prices of zero point nine seven zero zero, zero point nine four zero zero and zero point eight nine six zero per unit of face value for one, two and three years.
- A three year government bond has a five per cent annual coupon and a face value of one hundred.
- Year one: the coupon of five times zero point nine seven is four point eight five.
- Year two: five times zero point nine four is four point seven zero.
- Year three: coupon plus principal is one hundred and five, times zero point eight nine six, which is ninety four point zero eight.
- Add them: the arbitrage free value is one hundred and three point six three.

28 The bond trades below its strips: what to do

Bond cheap versus strips

Buy bond, pay 102.50 →

Sell strips, receive 103.63 → Keep 1.13 today

IN THE LESSON

- The bond is quoted at one hundred and two point five zero, but its strips are worth one hundred and three point six three.
- The bond is cheap.
- Here is the trade, as a flow.
- Step one, buy the bond, paying one hundred and two point five zero.
- Step two, sell the strips: sell five of the year one strip, five of the year two strip, and one hundred and five of the year three strip, which brings in one hundred and three point six three.
- Step three, keep the difference, one point one three.
- In the years that follow, the bond's coupons and principal are exactly what you owe on the strips you sold.
- So nothing more is paid or received, and the profit is locked in.

29 Example 7: why the trade needs no money and no risk

Buy bond at 102.50, sell the strips worth 103.63

Today	103.63 − 102.50	**+1.13**
Year 1	bond pays 5, strip owes 5	**0**
Year 2	bond pays 5, strip owes 5	**0**
Year 3	bond pays 105, strips owe 105	**0**

A gain today, nothing owed later

IN THE LESSON

- Let's check the trade year by year.
- Today: you receive one hundred and three point six three from the strips and pay one hundred and two point five zero for the bond, a net inflow of one point one three.
- Year one: the bond pays you a coupon of five, and you owe five on the year one strip.
- Net zero.
- Year two: the same, net zero.
- Year three: the bond pays one hundred and five, the strips you sold demand one hundred and five.
- Net zero.
- So you gain one point one three now, owe nothing later, and take no risk, because the payments match to the penny.
- That is what makes it an arbitrage, and not just a good bet.

30 Example 8: the other direction

Bond quoted at 104.80; strip value still 103.63

Sell the bond	dear: receive	**104.80**
Buy the strips	cheap: pay	**103.63**
Profit today	104.80 − 103.63	**+1.17**
Cash flows later	strips cover the short bond	**0**

Sell what is dear, buy what is cheap

IN THE LESSON

- Now suppose the bond is quoted at one hundred and four point eight zero, above the strip value of one hundred and three point six three.
- Now the bond is dear and the strips are cheap, so reverse everything.
- Sell the bond short, receiving one hundred and four point eight zero, and buy the strips, paying one hundred and three point six three.
- The difference is a profit of one point one seven today.
- The strips pay exactly what you owe on the short bond each year, so again nothing is left over later.
- The rule is simple: sell what is dear, buy what is cheap.
- Say it out loud before you answer an exam question, because getting the direction backwards turns a profit into a loss.

31 Why the gaps close, and why models are calibrated

- Arbitrage trading removes the gap
- Models are first set to match traded prices
- Implied spots: 3.09%, 3.14%, 3.73%
- Strip prices are the discount factors

IN THE LESSON

- What happens in real markets?
- Arbitrageurs notice the gap and trade.
- Their buying pushes the cheap bond's price up, and their selling pushes the dear strips' prices down, until the gap closes.
- This is why valuation models are calibrated to reproduce the prices of securities that trade.
- A model that did not would report an arbitrage in bonds that obviously have none.
- A last detail.
- The spot rates behind our strips are about three point zero nine per cent for one year, three point one four per cent for two years, and three point seven three per cent for three years.
- But the trade never needed them.
- The strip prices are the discount factors, and that is enough.

32 Recap: four ideas in one picture

Four ideas for valuing and positioning on the curve

Curve risk	Key rate durations, minus sign
Volatility	Duration × yield volatility
Rate views	Scenario → right part of curve
No arbitrage	Buy cheap, sell dear

IN THE LESSON

- Let's pull it together.
- Curve risk: yields move by level, steepness and curvature.
- Key rate durations measure exposure at each point, they add up to effective duration, and the change in value is minus the sum of each one times its yield change.
- Volatility: short yields are the most volatile, but price volatility is duration times yield volatility, so long bonds carry the price risk.
- Rate views: the policy rate anchors the short end, expected rates plus a term premium drive the long end, and a position should match the scenario to the right part of the curve.
- No arbitrage: a bond is worth the sum of its cash flows at strip prices, and if it is cheaper, buy the bond and sell the strips.

33 Try three quick questions

1. Durations 1, 2, 3, 2; yield moves −10, 0, +10, +20 bp. Value change?
   Answer: +0.1 − 0.3 − 0.4 = −0.6%. Rising yields cost.
2. Duration 2, vol 95 bp, or duration 17, vol 60 bp: higher price risk?
   Answer: The second: 10.2% against 1.9%. Duration × yield volatility.
3. A bond trades below the value of its strips. Which trade?
   Answer: Buy the bond, sell the strips; lock in the gap today.

IN THE LESSON

- Now try three quick questions.
- Pause after each one.
- Question one.
- Key rate durations are one, two, three and two, and the yields change by minus ten, zero, plus ten and plus twenty basis points.
- What is the change in value?
- Minus one times minus zero point one gives plus zero point one.
- Minus three times plus zero point one is minus zero point three.
- Minus two times plus zero point two is minus zero point four.
- The sum is minus zero point six per cent.
- Question two.
- Which bond has the higher price volatility, duration two and yield volatility ninety five basis points, or duration seventeen and sixty?
- The second, at ten point two per cent against one point nine.
- Question three.
- A bond trades below its strips.
- Buy the bond and sell the strips.