Bonds with embedded options

Fixed Income

1 A story: the mortgage you can refinance

- Early repayment on a loan is an option
- The borrower owns it and uses it when rates fall
- An embedded option is built into the bond contract
- Always ask: who owns it, and when is it used?

IN THE LESSON

- Think about a home loan that lets you repay early without a penalty.
- Rates fall, so you take a new loan at the lower rate and pay off the old one.
- The bank is not happy, because it loses a loan that paid a good rate.
- That early repayment right is an option, and you own it.
- Many bonds work the same way.
- A company borrows by issuing a bond, and the bond contract contains a right that changes the payments.
- We call it an embedded option, because it is built into the bond rather than traded on its own.
- The first question is always the same.
- Who owns the right, and when will they use it?

2 The call: the issuer's right

- Callable: issuer may redeem early at the call price
- European: one date. American: any time after protection
- Bermudan: a set list of dates
- Issuer owns the call; the holder has sold it

IN THE LESSON

- The most common embedded option is the call.
- A callable bond lets the issuer buy the bond back early, at a price set in advance, called the call price.
- Issuers do this when rates have fallen, so they can borrow again more cheaply.
- The contract also says when a call is allowed.
- A European style call can be used on one single date only.
- An American style call can be used at any time after an initial protected period, called call protection.
- A Bermudan style call can be used on a list of set dates, such as every coupon date after year two.
- Notice the owner.
- The issuer has the right, and the bondholder has sold it.

3 The put, the extendible and the sinking fund

- Putable: holder may sell back, usually at par
- Extendible: holder keeps the coupon for longer
- Equivalent to a longer putable bond
- Sinking fund: issuer retires part each year, cheaper route

IN THE LESSON

- Now the options owned by the investor.
- A putable bond lets the holder sell the bond back to the issuer at a set price, usually par.
- That is valuable when rates have risen and the bond would otherwise trade below par.
- An extendible bond lets the holder extend the maturity at the same coupon.
- Declining to extend is the same as putting the bond back, so an extendible bond is equivalent to a longer putable bond.
- A sinking fund bond works differently.
- The issuer must retire part of the issue each year, either by calling bonds at par or by buying them in the market.
- That lowers credit risk, but the issuer picks the cheaper route, so holders face reinvestment risk.

4 Who owns what

Embedded option and its owner

Option	Owner
Call	Issuer
Put, extendible	Holder
Sinking fund buy-in	Issuer
Estate put, convertible	Holder

IN THE LESSON

- Let's collect these in one list, because the exam loves to ask who owns each right.
- The call belongs to the issuer.
- The put belongs to the holder.
- The extension right belongs to the holder, since it is a put in disguise.
- The sinking fund buy-in belongs to the issuer, who may buy bonds in the market when they trade below par.
- Complex bonds go further.
- An estate put lets the heirs of a holder who has died sell the bond back at par.
- A convertible lets the holder swap the bond for shares, so it is a holder option too.
- A simple rule helps.
- An option the issuer owns lowers the bond's value to you.
- An option you own raises it.

5 Example 1: why the issuer calls

Four-year 5% bond, callable at 100; new borrowing costs 3%

Coupon saved a year	5% − 3%	**2 points**
Straight value at 3% yield	5 a year, four years, plus 100	**107.43**
Holder is paid	the call price	**100**
Holder's loss	107.43 − 100	**7.43**

IN THE LESSON

- Here is a small case.
- An analyst, Maya Torres, studies a four year bond that pays a five per cent annual coupon and is callable at one hundred, which is par.
- Rates then fall, and new four year borrowing costs only three per cent.
- The issuer saves two points of coupon every year, so it calls the bond and refinances.
- Look at what the holder loses.
- A straight four year five per cent bond at a three per cent yield would be worth about one hundred and seven point four three.
- The holder must hand it back for one hundred.
- That is a loss of seven point four three per hundred of face value, transferred to the issuer.
- The call is exactly what limits the holder's gain.

6 A story: the price tag you cannot see

- Market quotes the bond with the option attached
- The straight bond is the same bond with no option
- Option value = the gap between the two prices
- Who owns it decides whether the gap adds or subtracts

IN THE LESSON

- Imagine buying a car with a price tag that includes a free spare key, and the same car without it for a bit less.
- The spare key is worth the difference between the two prices.
- You never see its price on its own.
- Embedded options are the same.
- The market quotes one price for the bond with the option attached.
- A model gives you the price of the same bond with the option removed, which we call the straight bond, or option-free bond.
- The option's value is the gap between the two.
- Whether the option adds to the bond or takes away from it depends on who owns it.
- So we always start from the straight bond, and measure the gap from there.

7 Callable and putable in value terms

Callable = Straight − Call
Putable = Straight + Put

Straight bond value	The anchor: no option
Call value (issuer owns)	Subtract it from the straight
Put value (holder owns)	Add it to the straight

IN THE LESSON

- Here are the two relationships.
- For a callable bond, the issuer owns the call.
- So the callable bond equals the straight bond minus the call.
- Turn it around, and the call equals the straight bond minus the callable bond.
- For a putable bond, the holder owns the put.
- So the putable bond equals the straight bond plus the put, and the put equals the putable bond minus the straight bond.
- Look at the terms.
- The straight bond value is the anchor, the same bond with no option.
- The call value is what the issuer's right is worth, and it is always zero or positive.
- The put value is what the holder's right is worth, also never negative.
- So a callable is never worth more than the straight bond, and a putable never less.

8 Three prices against yield

Chart: Price vs Yield

- Straight
- Callable
- Putable
- Call

IN THE LESSON

- Here is the picture.
- Yield runs along the bottom and price up the side.
- The green line is the straight bond.
- It falls as yields rise, and it bends, because it gains more when yields drop than it loses when yields rise.
- Now the callable, in red.
- On the right, where yields are high, a call is unlikely, so it follows the straight bond closely.
- On the left, where yields are low, the issuer would call, so the price levels off near the call price.
- The gap between green and red is the value of the call.
- The putable, in gold, is the mirror image.
- At high yields it levels off, held up by the right to sell back, and the gap above green is the put.

9 Example 2: a bond in three versions

Three-year 4% bond, call or put at 100 in years 1 and 2

Straight bond	discounted at spot rates	**102.85**
Callable bond	tree, 10% volatility	**101.96**
Putable bond	tree, 10% volatility	**103.06**
Call value	102.85 − 101.96	**0.89**
Put value	103.06 − 102.85	**0.21**

IN THE LESSON

- Let's price the same bond three ways.
- Maya Torres looks at a three year bond with a four per cent annual coupon.
- Spot rates are two point zero zero, two point five one and three point zero two per cent.
- The straight bond is simply discounted at those spot rates.
- Four over one point zero two, plus four over one point zero two five one squared, plus one hundred and four over one point zero three zero two cubed, comes to one hundred and two point eight five.
- A calibrated tree at ten per cent volatility then values the callable, exercisable at par in years one and two, at one hundred and one point nine six.
- The putable is worth one hundred and three point zero six.
- So the call is worth zero point eight nine, and the put zero point two one.

10 Floaters and the par trap

- Capped floater = straight floater − cap
- Floored floater = straight floater + floor
- Callable 101.96 is 1.96 over par; call is worth 0.89
- Trap: compare with the straight bond, not par

IN THE LESSON

- The same logic covers floating rate bonds.
- The issuer of a capped floater owns a cap that limits the coupon it must pay, so the capped floater equals the straight floater minus the cap.
- A floored floater gives the holder a floor, so it equals the straight floater plus the floor.
- Now the trap.
- Many candidates measure an option against par instead of against the straight bond.
- In our example the callable trades one point nine six above par, but that tells you nothing about the call.
- The call is worth zero point eight nine, found by comparing with the straight bond at one hundred and two point eight five.
- Always subtract from the straight bond, never from one hundred.

11 A story: deciding at every junction

- A tree of rates that prices the benchmark curve
- Volatility sets how far apart the rates sit
- Start at maturity, step backwards node by node
- At each node the option's owner decides

IN THE LESSON

- Imagine walking back along a path of forks, deciding at each fork what is best for you.
- You start from the end, where the answer is known, and work backwards to the start.
- That is how we value a bond whose payments depend on whether an option is used.
- We build a tree of possible interest rates.
- It is calibrated, which means that when we use it to price the benchmark bonds, it gives their market prices exactly.
- It also needs an assumed volatility, which sets how far apart the up and down rates sit.
- Then we move backwards from maturity.
- At each node we let the option's owner decide.
- This is called the arbitrage-free approach.

12 One node's work

Take the two later values → Average, discount at node rate → Apply the exercise rule → Add the coupon, step back

IN THE LESSON

- Here is the work done at a single node.
- You repeat it at every node, from maturity back to today.
- Follow the boxes from left to right.
- First, take the two values from the nodes that come after.
- Second, average them and discount for one period at the rate at that node.
- Third, apply the exercise rule.
- And only fourth, add the coupon that is paid at that node.
- The order matters.
- The exercise rule is applied to the value of what is still to come, before the coupon due on that date.
- The coupon is paid whether or not the bond is called, so the call price only buys back the remaining payments.
- Then step back one date and repeat.

13 The exercise rule

Callable node: V = lower of value and call price
Putable node: V = higher of value and put price

Value of what is still to come	Before the coupon is added
Call price (issuer pays)	Caps the callable's value
Put price (holder receives)	Floors the putable's value

IN THE LESSON

- Here is the exercise rule.
- At a node where the bond is callable, the value becomes the lower of the computed value and the call price.
- Why lower?
- Because the issuer calls when the bond is worth more than the call price, and then the holder only receives the call price.
- At a node where the bond is putable, the value becomes the higher of the computed value and the put price.
- Why higher?
- Because the holder puts when the bond is worth less than the put price, and then receives the put price.
- The computed value is the value of what is still to come at that node, before adding the coupon.
- Apply the rule at every date the option can be used, not only the last.

14 The tree we will use

Tree rates, 10% volatility, calibrated to the spot curve

Date	Rates at the nodes
Today	2.00%
Year 1	2.722% or 3.325%
Year 2	3.285% 4.012% 4.900%

IN THE LESSON

- Here is the tree for our bond.
- It is built at ten per cent volatility and calibrated to the spot curve of two point zero zero, two point five one and three point zero two per cent.
- Today's one year rate is two per cent.
- One year from now the rate is either two point seven two two or three point three two five per cent.
- Two years from now there are three possible rates: three point two eight five, four point zero one two or four point nine zero zero per cent.
- The rates are higher further out because the curve slopes upward.
- Each node leads to a higher node and a lower node with equal chance.
- For a risky issuer we would add one constant spread, the option-adjusted spread, to every one of these rates.

15 Example 3: callable, year two nodes

Year two: discount 104, apply the call, then add the coupon 4

Lowest-rate node, 3.285%	104 ÷ 1.03285 = 100.69	**called: 104.00**
Middle node, 4.012%	104 ÷ 1.04012 = 99.99	**103.99**
Highest-rate node, 4.900%	104 ÷ 1.049 = 99.14	**103.14**

IN THE LESSON

- Now value the callable, starting at maturity.
- At the end of year three, every node pays the face value plus the last coupon, one hundred and four.
- Step back to the three year two nodes.
- At the bottom node, with the lowest rate, one hundred and four discounted at three point two eight five per cent gives one hundred point six nine.
- That is above the call price, so the issuer calls, and the value becomes one hundred.
- Add the four coupon and the node is worth one hundred and four.
- The middle node computes to ninety nine point nine nine, which is below one hundred, so no call.
- With the coupon it is worth one hundred and three point nine nine.
- The top node gives ninety nine point one four, no call, and a value of one hundred and three point one four.

16 Example 3: year one, today, and the putable

Year one nodes, then today (callable bond)

Year 1, low-rate node	101.24 > 100: called	**104.00**
Year 1, high-rate node	100.23 > 100: called	**104.00**
Today	104 ÷ 1.02	**101.96**
Putable bond, same tree	higher of value and 100	**103.06**

IN THE LESSON

- Now step back to year one.
- The low-rate node averages one hundred and four and one hundred and three point nine nine, then discounts at two point seven two two per cent, giving one hundred and one point two four.
- That is above one hundred, so it is called, and with the coupon the node is worth one hundred and four.
- The high-rate node gives one hundred point two three, also called, so also one hundred and four.
- Today, one hundred and four divided by one point zero two is one hundred and one point nine six.
- That is the callable bond.
- For the putable, the same steps but taking the higher of the value and one hundred give one hundred and three point zero six, because the holder uses the put at the two higher-rate year-two nodes.

17 Spread, and where the rule goes wrong

- Risky issuer: add one constant spread to every node rate
- The spread can change which nodes are called
- Apply the rule at every exercise date
- Rule first, coupon second

IN THE LESSON

- Three practical points.
- First, the risky issuer.
- A government benchmark tree prices a safe bond.
- For a company, we add one constant spread to every rate in the tree.
- That spread, chosen so the model matches the bond's market price, is called the option-adjusted spread.
- Because it goes into every node, it can change the exercise decisions, not just the price.
- A node that looks callable on the safe tree may not be callable on the risky one.
- Second, apply the rule at every exercise date, including the early ones near today.
- Third, apply it before adding the coupon.
- Skipping any of these gives a wrong answer that still looks reasonable.

18 A story: the insurance seller

- Option value rises with volatility, whoever owns it
- Callable: holder sold the call, so price falls
- Putable: holder bought the put, so price rises
- The straight bond does not move

IN THE LESSON

- Think of insurance.
- When storms become harder to predict, insurance becomes more valuable, and the policy costs more.
- Whoever holds the policy is happier.
- An option is the same.
- Its value rises when outcomes are more spread out, because the owner collects the good cases and is protected from the bad ones.
- That holds for any option, whoever owns it.
- In bond terms, volatility measures how widely interest rates may move.
- A callable bondholder has sold a call, so a more valuable call hurts.
- A putable bondholder has bought a put, so a more valuable put helps.
- The straight bond itself does not change at all.

19 Value against volatility

Chart: Bond value vs Volatility, 0% to 30%

- Straight, flat
- Callable falls
- Putable rises

IN THE LESSON

- Here is the picture, drawn from our example.
- Volatility runs along the bottom, from zero to thirty per cent, and bond value up the side.
- Look at the flat green line first.
- That is the straight bond, worth one hundred and two point eight five at every volatility.
- The tree is rebuilt each time so that it still prices the benchmark bonds exactly, and a bond with fixed cash flows is just a bundle of those benchmarks.
- Now the red line, the callable.
- It starts flat, then falls as volatility rises, because the call grows more valuable.
- The gold line, the putable, climbs, because the put grows more valuable.
- Everything that moves is option value.

20 Example 4: volatility doubles

Same bond and spot curve; volatility 10% to 20%

Straight bond	no change	**102.85**
Callable	101.96 to 101.73	**−0.23**
Putable	103.06 to 103.26	**+0.20**
Call value	0.89 to 1.12	**+0.23**
Put value	0.21 to 0.41	**+0.20**

IN THE LESSON

- Now the numbers.
- The analyst revalues the three year bond when volatility doubles from ten to twenty per cent.
- The tree is rebuilt each time, and the spot curve is unchanged.
- The straight bond is worth one hundred and two point eight five at both.
- The callable falls from one hundred and one point nine six to one hundred and one point seven three, so the call rises from zero point eight nine to one point one two.
- The putable rises from one hundred and three point zero six to one hundred and three point two six, so the put rises from zero point two one to zero point four one.
- The holder of the callable lost zero point two three, and the holder of the putable gained zero point two.
- Both options gained value.
- Which bond rose and which fell depended on who owned the option.

21 Mistakes to avoid so far

- 1. Callable holder sold the option: volatility lowers price
- 2. Option value is vs the straight bond, not par
- 3. Exercise rule first, then add the coupon
- 4. Check every exercise date, not only the last
- 5. Straight bond value does not change with volatility

IN THE LESSON

- Mistakes that cost marks.
- One.
- Getting the callable direction wrong.
- The holder of a callable has sold the option, so higher volatility lowers its price.
- Two.
- Measuring an option against par, instead of against the straight bond.
- Three.
- Adding the coupon before applying the exercise rule, so the call price is compared with the wrong number.
- Four.
- Applying the rule only at the last exercise date, and forgetting the early ones.
- Five.
- Thinking the straight bond's value changes with volatility.
- On a calibrated tree it does not.
- The only things that move are the option values, and each moves the bond in the direction set by the owner.

22 A story: it is the whole road, not the first step

- Level: the whole curve moves up or down
- Shape: the slope, steeper or flatter
- The tree is built from the whole curve
- Do not look only at today's short rate

IN THE LESSON

- Suppose two hikers start at the same spot, but one road climbs steadily and the other levels out.
- Where they are now is identical, yet their futures differ.
- Interest rates work the same way.
- Many people think an option depends only on the level of rates.
- When rates fall, the call comes into the money.
- That is half the story.
- The tree is built from the whole yield curve, not from today's rate.
- A call that can be used in two years depends on the rate expected in two years, and the slope of the curve sets that.
- So we ask two questions.
- What happens when the level shifts?
- And what happens when the shape changes?

23 Example 5: the level shifts

Parallel shift of the curve by 1 point; 10% volatility

Straight bond	down 1: +2.94	**up 1: −2.83**
Callable bond	down 1: +1.01	**up 1: −1.94**
Putable bond	down 1: +2.73	**up 1: −1.91**

IN THE LESSON

- Let's shift the level.
- The curve moves down by one point at every maturity, then up by one point.
- First, rates fall by one point.
- The straight bond rises from one hundred and two point eight five to one hundred and five point seven nine, a gain of two point nine four.
- The callable rises only from one hundred and one point nine six to one hundred and two point nine seven, a gain of one point zero one.
- The call has moved into the money and swallowed most of the gain.
- The putable follows the straight bond.
- Now rates rise by one point.
- The straight bond loses two point eight three.
- The callable loses less than the straight bond, one point nine four, because the call has lost value, and the putable loses one point nine one, because the put now supports its price.

24 Call value as the curve flattens

- Steep: call 0.00
- Base: call 0.89
- Flat: call 3.81
- Inverted: call 6.78

IN THE LESSON

- Now the shape.
- Here are four curves that all start with the same one year rate of two per cent.
- Only the slope differs.
- Read the line from left to right, as the curve goes from steep, to our base curve, to flat, to inverted.
- On the steep curve, rates are expected to climb far above the coupon, so the call is worth nothing.
- On our base curve, it is worth zero point eight nine.
- On the flat curve, every future rate is expected to equal two per cent, well below the four per cent coupon, so the call is worth three point eight one.
- On the inverted curve, future rates are expected to fall, and the call is worth six point seven eight.
- Same short rate, very different options.

25 Example 6: the put goes the other way

Same one-year rate, four curve shapes

Steep	call 0.00	**put 3.17**
Base	call 0.89	**put 0.21**
Flat	call 3.81	**put 0.00**
Inverted	call 6.78	**put 0.00**

IN THE LESSON

- Now the same four curves for the put.
- On the steep curve, rates are expected to rise past the coupon, and the put is worth three point one seven.
- On the base curve it is worth zero point two one.
- On the flat and inverted curves, the put is worth nothing, because rates are expected to stay below the coupon.
- So a flatter or inverted curve with the same short rate raises the value of a call and lowers the value of a put.
- An upward sloping curve does the reverse.
- And do not look only at the current short rate.
- In all four cases it is two per cent, yet the answers run from a valuable put to a valuable call.

26 Recap: five ideas to carry

Embedded options in five ideas

Who owns the option	Call: issuer. Put: holder
Option value is the gap	Vs the straight, not par
Exercise rule at every node	Lower: callable. Higher: putable
Volatility	Callable falls, putable rises
Whole curve	Level and slope both count

IN THE LESSON

- Let's pull it together.
- First, who owns the option.
- The call is the issuer's, the put is the holder's, and the extendible is a put in disguise.
- Second, the option value is the gap between the bond with the option and the straight bond.
- Callable equals straight minus call, putable equals straight plus put.
- Third, the exercise rule is applied at every node, before the coupon, lower for a callable and higher for a putable.
- Fourth, higher volatility raises the value of every option, so it lowers a callable and raises a putable.
- Fifth, the whole curve matters.
- Falling rates and a flatter curve favour the call, rising rates and a steeper curve favour the put.

27 Try three quick questions

1. Straight 102.85, callable 101.96. What is the call worth?
   Answer: 102.85 − 101.96 = 0.89.
2. Volatility rises. What happens to each bond?
   Answer: Straight unchanged, callable falls, putable rises.
3. Curve flattens, same one-year rate. Effect on a call?
   Answer: Its value rises: expected rates at call dates fall.

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- A bond is worth one hundred and two point eight five as a straight bond and one hundred and one point nine six as a callable.
- What is the call worth?
- The answer is zero point eight nine, the straight value minus the callable value.
- Question two.
- Volatility rises, with the tree recalibrated.
- What happens to the straight, callable and putable bond?
- The straight bond is unchanged, the callable falls, and the putable rises.
- Question three.
- The curve flattens while the one year rate stays the same.
- What happens to the call?
- Its value rises, because the rates expected at the call dates fall.
- Well done.