CHARTA LEARNING · LESSON NOTES

Bonds with options: value, spread and duration

Fixed Income

1 A bond with a trapdoor

- Callable: the issuer may repay early
- Putable: the investor may demand repayment
- Each right is an option with a price
- We price it with a tree of future rates

IN THE LESSON

- Picture lending a friend money for three years at a fixed rate of interest.
- Now suppose your friend says: I may pay you back early, whenever I like, for exactly what I borrowed.
- If interest rates fall, your friend will do just that, borrow more cheaply elsewhere, and leave you holding cash that now earns less.
- If rates rise, your friend keeps your cheap loan.
- That is a callable bond: the issuer owns an option to repay early.
- Now turn it around.
- A putable bond gives you, the lender, the right to demand your money back early.
- Each right has a price, and today we find it by working through a tree of possible interest rates.

2 The tree, and one new decision

- Start at maturity and work backward
- Callable: cap the value at the call price
- Putable: floor the value at the put price
- Then add the coupon, average, discount

IN THE LESSON

- A quick reminder from earlier study.
- An interest rate tree is a map of the one-year rates that might arrive, each step up or down being equally likely.
- A calibrated tree is built so that it prices the benchmark bonds exactly right.
- To value any bond on it, you start at maturity and work backward.
- At each node you add the coupon, take the average of the two values that follow, and discount by that node's rate.
- Now the one new idea.
- At every node where the option can be used, you stop and ask what its owner would do.
- The issuer calls when the bond is worth more than the call price, so the value is capped.
- The holder puts when the bond is worth less than the put price, so it is floored.

3 The rule at each node

Callable: V = smaller of V and call price
Putable: V = larger of V and put price
Node = [½(V↑ + C) + ½(V↓ + C)] ÷ (1 + r)

V: value before coupon	Value of the rest, ex-coupon
Exercise price	Call price or put price
C: coupon due	Paid whatever the owner does
r: node rate	Discounts the average

IN THE LESSON

- Here is the rule written down.
- Take the value of the bond at a node, before any coupon due on that date.
- Call it V.
- For a callable bond, replace V by the smaller of V and the call price.
- For a putable bond, replace V by the larger of V and the put price.
- Only after that decision do you add the coupon, which is paid whatever the option's owner decides.
- Then take the average of the two values that follow, and discount it at that node's one-year rate, written r.
- Do this at every exercise date, including the early ones.
- When you reach today, you have the value of the callable or putable bond, and the option itself is worth the difference from the straight bond.

4 Our made-up bond and its tree

3-year bond, 3.5% annual coupon, face 100

Node	One-year rate
Today	2.00%
Year one: up, down	3.47%, 2.57%
Year two: up, mid, down	5.37, 3.98, 2.95%
Straight bond value	101.42

IN THE LESSON

- Now the case.
- An analyst values a bond that pays a coupon of three point five per cent a year and matures in three years.
- Face value is one hundred.
- The calibrated tree is on the board.
- Today's one-year rate is two per cent.
- After a year, the rate is either three point four seven per cent or two point five seven per cent.
- After two years, it is five point three seven, three point nine eight or two point nine five per cent.
- On this tree, the plain straight bond with no option is worth one hundred and one point four two.
- We value two versions, one callable at one hundred and one putable at one hundred, each usable at the end of years one and two, straight after the coupon.

5 Callable bond, year two

Callable at 100, year-two nodes, before the coupon

Up node, rate 5.37%	103.5 ÷ 1.0537	**98.23**
Middle node, rate 3.98%	103.5 ÷ 1.0398	**99.54**
Down node, rate 2.95%	103.5 ÷ 1.0295	**100.54**
Down node is called	above 100, so capped	**100.00**

Carried back with the coupon: 100 + 3.5 = 103.50

IN THE LESSON

- We start at the right of the tree, with the callable bond, at the end of year two.
- The final payment of one hundred and three point five arrives a year later.
- At the top node, the rate is five point three seven per cent, and one hundred and three point five divided by one point zero five three seven is ninety-eight point two three.
- That is below one hundred, so no call.
- The middle node, at ninety-nine point five four, is also below.
- At the bottom node, the rate is lowest, and the value is one hundred point five four.
- That is above the call price of one hundred, so the issuer calls.
- The node is worth exactly one hundred, and with the coupon it carries back one hundred and three point five.

6 Callable bond, year one and today

Callable at 100, years one and zero

Year one, down node	(103.50 + 103.04) ÷ 2 ÷ 1.0257	**100.69**
Called, so capped	100 + 3.5 coupon	**103.50**
Year one, up node	(103.04 + 101.73) ÷ 2 ÷ 1.0347	**98.96**
Today: callable bond	(103.50 + 102.46) ÷ 2 ÷ 1.02	**100.96**
Value of the call	101.42 − 100.96	**0.46**

IN THE LESSON

- Now step back to year one.
- At the lower node, the rate is two point five seven per cent.
- The two values that follow are one hundred and three point five, from the called node, and one hundred and three point zero four.
- Their average, over one point zero two five seven, is one hundred point six nine.
- That is above one hundred, so the issuer calls here too.
- This is the step people miss.
- At the upper node, the rate is three point four seven per cent and the value is ninety-eight point nine six, below one hundred, so no call.
- With coupons, the year-one values are one hundred and three point five and one hundred and two point four six, and their average over one point zero two is one hundred point nine six.
- The callable bond is worth one hundred point nine six, and the call is worth one hundred and one point four two minus one hundred point nine six, which is zero point four six.

7 Putable bond, same tree

Putable at 100: floors at the high-rate nodes

Year two, up and middle	98.23 and 99.54, below 100	**100.00**
Year two, down node	100.54, no put	**100.54**
Year one, down node	(104.04 + 103.50) ÷ 2 ÷ 1.0257	**101.17**
Today: putable bond	(104.67 + 103.53) ÷ 2 ÷ 1.02	**102.06**
Value of the put	102.06 − 101.42	**0.64**

IN THE LESSON

- Now the putable bond, on the same tree, works the other way round.
- At the top year-two node, the value was ninety-eight point two three.
- That is below one hundred, so the holder puts the bond back and receives one hundred.
- The same happens at the middle node, at ninety-nine point five four.
- The bottom node, at one hundred point five four, is kept.
- Carry the values back.
- At the lower node in year one, the average of one hundred and four point zero four and one hundred and three point five, over one point zero two five seven, is one hundred and one point one seven.
- At the upper node it is one hundred point zero three, just above one hundred, so no put there.
- Today the putable bond is worth one hundred and two point zero six, and the put is worth that minus one hundred and one point four two, which is zero point six four.

8 What each option is worth

Option value (price points)

Call (issuer's)	**0.46**
Put (investor's)	**0.64**
Call if year one skipped	**0.13**

Callable = straight − call; putable = straight + put

IN THE LESSON

- Here is what we found, drawn as bars.
- The vertical axis is price points, per hundred of face value.
- The first bar is the call, worth zero point four six.
- The issuer owns it, so it comes off the straight bond's value of one hundred and one point four two, leaving one hundred point nine six for the callable.
- The second bar is the put, worth zero point six four.
- The investor owns it, so it is added, and the putable is worth one hundred and two point zero six.
- The third, short bar shows the error of forgetting to test the call in year one: a call worth only zero point one three.

9 Check every exercise date

- Test the option at every exercise date
- Compare the value before the coupon
- Callable below straight; putable above
- Option value = difference from straight

IN THE LESSON

- Let's pull out the traps.
- The first one is the date you forget.
- Our call can be used in year one and in year two.
- If you test only year two, the lower year-one node keeps its value of one hundred point six nine and you get one hundred and one point two nine for the callable bond, a call worth just zero point one three.
- That is too small.
- The second trap is the order of events.
- Compare the value before the coupon with the exercise price, then add the coupon, because the coupon is paid either way.
- The third trap is direction.
- A callable bond is worth less than a straight bond, and a putable bond is worth more.
- If your answer comes out the other way round, the arithmetic has gone wrong somewhere.

10 Two things inside one spread

- A spread pays for credit and liquidity
- A callable's spread also pays for the call
- Option-adjusted spread removes the option
- What is left says cheap or rich

IN THE LESSON

- Bond analysts compare bonds by their spread over the benchmark curve.
- But watch out.
- A callable bond's spread is paying for two things at once.
- Part of it rewards you for the issuer's credit risk and the bond's liquidity risk, the chance you cannot sell it easily.
- The other part is the price of the call you have sold to the issuer.
- Imagine two rental houses with the same rent, but the landlord of one can end your lease at will.
- Its rent hides a cost.
- The option-adjusted spread is the number that strips the option out, so what remains is credit and liquidity only.
- That is the part that says whether a bond looks cheap.

11 Defining the spread

Tree value (every rate + OAS) = market price
Option cost in spread = Z-spread − OAS

Tree value	From the calibrated tree
OAS	Same spread added at every node
Market price	What the bond trades at
Z-spread	Spot curve plus spread, no option

IN THE LESSON

- Here is the definition.
- The option-adjusted spread is the constant spread that, added to every rate in the calibrated tree, makes the model value of the bond with its options equal to its market price.
- On the board it is shortened.
- The tree value is what the model gives, the market price is what the bond actually trades at, and the spread is the single number that closes the gap.
- Next to it sits a cousin, the zero-volatility spread, or Z-spread.
- It is the spread added to the spot curve so that the promised cash flows match the price, ignoring the option entirely.
- The difference between the two is the option cost, measured in spread.

12 Searching for the spread

Bond priced at 101.50: try spreads until the tree matches

No spread: 102.45 → 50 bp: 101.90 → 75 bp: 101.43 → 71 bp: 101.50

IN THE LESSON

- The spread has no formula you can simply invert.
- The exercise decisions at each node can change as the spread changes, so you find it by trial and error.
- Take a three-year callable bond with a four point five per cent coupon, called at one hundred in years one and two, trading at one hundred and one point five zero.
- The tree is the one we already built.
- With no spread, the tree values the bond at one hundred and two point four five, above the price.
- At fifty basis points the value is one hundred and one point nine zero, still above.
- At seventy-five it is one hundred and one point four three, now below.
- So the answer sits between, and it turns out to be seventy-one basis points, where the value matches one hundred and one point five zero.

13 Spread and option cost, worked

Callable 4.5% bond at 101.50, 15% volatility

Tree value, no spread	above the price of 101.50	**102.45**
Option-adjusted spread	tree value falls to 101.50	**71 bp**
Z-spread, option ignored	same price, spot curve	**97 bp**
Option cost	97 − 71	**26 bp**

IN THE LESSON

- Now we compare the two spreads on that same bond.
- The tree gives the option-adjusted spread of seventy-one basis points, found as we saw.
- For the zero-volatility spread, we leave the tree behind.
- We discount the promised cash flows on the spot curve, as if the bond could never be called, and find the spread that matches the price.
- That comes to ninety-seven basis points.
- The Z-spread is larger, as it always is for a callable bond.
- The gap, ninety-seven minus seventy-one, is twenty-six basis points.
- That slice of the Z-spread is not compensation for credit or liquidity at all.
- It is the investor being paid for the call the issuer holds.

14 Which bond is really cheaper?

Spread (basis points)

A: Z-spread	**97**
A: OAS	**71**
B: Z-spread	**110**
B: OAS	**60**

B has the wider Z-spread but the narrower OAS

IN THE LESSON

- A comparison shows why this matters.
- The vertical axis is spread in basis points.
- Bond A is the callable we just worked: a Z-spread of ninety-seven and an option-adjusted spread of seventy-one.
- Bond B is another callable of similar credit quality, with a Z-spread of one hundred and ten and an option-adjusted spread of sixty, figures given in the example.
- Look at the Z-spread bars first.
- Bond B seems cheaper, with a bigger spread.
- Now look at the option-adjusted bars.
- Bond A is cheaper, seventy-one against sixty.
- Bond B's Z-spread was fattened by a more valuable call, fifty basis points of it, against twenty-six for Bond A.
- Rank on the option-adjusted spread, never the Z-spread.

15 Reading the spread: cheap or rich

- Callable: Z-spread above OAS
- Putable: OAS above Z-spread
- Similar credit: higher OAS means cheap
- Never compare a Z-spread with an OAS

IN THE LESSON

- So what do we do with the number?
- Three rules.
- First, the direction.
- For a callable bond, the Z-spread is above the option-adjusted spread, because the call has to be paid for.
- For a putable bond it is the other way round: the put is a gift to the investor, so the option-adjusted spread is above the Z-spread.
- Second, the verdict.
- Among bonds of similar credit quality, a higher option-adjusted spread means the bond is cheap, because you are paid more for the same risk.
- A lower one means it is rich, or expensive.
- Third, never mix measures.
- Do not compare one bond's Z-spread with another bond's option-adjusted spread.
- They answer different questions.

16 A putable bond, worked

Putable 3.5% bond at 100.60, put at 100

Tree value, no spread	above the price of 100.60	**102.06**
Option-adjusted spread	tree falls to 100.60	**90 bp**
Z-spread, put ignored	same price, spot curve	**29 bp**
OAS minus Z-spread	90 − 29	**61 bp**

IN THE LESSON

- Let's check the other direction with the putable bond from earlier.
- Suppose it trades at one hundred point six zero, below the straight bond's value, because this issuer is riskier than the benchmark.
- On the tree with no spread, the putable is worth one hundred and two point zero six, well above the price, so a spread is needed.
- Solving as before, the option-adjusted spread is ninety basis points.
- The Z-spread, which ignores the put, is only twenty-nine.
- So the option-adjusted spread is wider by sixty-one basis points.
- The put lifts the price, which squeezes the Z-spread, and once you take the put out the true credit and liquidity compensation is larger.

17 A spread that depends on an input

- OAS comes from a model, not the market
- One input is the volatility of rates
- Change the input, the OAS changes
- The market price does not move

IN THE LESSON

- An option-adjusted spread looks like something you simply read off the market, like a yield.
- It is not.
- It comes out of a model, and one input to that model is how volatile you think interest rates will be.
- Think of an insurance premium.
- The same car costs more to insure in a stormy region than in a calm one.
- The risk you assume changes the price you think is fair.
- In the same way, assume rates are wild, and the call option is worth a lot, assume them calm, and it is worth little.
- The bond's market price has not moved.
- Only your assumption has.
- So the answer you get for the spread moves with it.

18 The chain for a callable bond

Raise the volatility input

Rates spread wider → Call worth more → Model value falls → Smaller OAS

IN THE LESSON

- Follow the chain of boxes, left to right, for a callable bond.
- Start with a higher volatility assumption.
- In the tree, rates now spread out further at each step, so there are more low-rate paths where the issuer will happily call.
- That makes the call more valuable.
- Because the issuer owns the call, a more valuable call means the model value of the bond, before any spread is added, is lower.
- Now the market price has not changed.
- The tree value is closer to the price already, so a smaller spread is needed to close the gap.
- The option-adjusted spread falls.
- Lower volatility runs the whole chain the other way, and the spread rises.

19 The same bond, three assumptions

Callable at 101.50; peers average 70 bp at 15% volatility

Analyst A, volatility 10%	call valued low	**80 bp**
Analyst B, volatility 20%	call valued high	**63 bp**
Peers, volatility 15%	the like-for-like yardstick	**70 bp**
Same bond at 15%	fairly priced	**71 bp**

The bond looked cheap or rich only because of the input

IN THE LESSON

- Let's see it with numbers.
- The callable four point five per cent bond still trades at one hundred and one point five zero.
- Callable bonds of similar credit quality have an average option-adjusted spread of seventy basis points, computed at fifteen per cent volatility.
- Analyst A assumes ten per cent volatility.
- The call is valued low, so the model value is high, and a wide spread of eighty basis points is needed.
- Analyst B assumes twenty per cent.
- The call is worth more, and the spread is only sixty-three.
- Valued at the same fifteen per cent as its peers, the bond has seventy-one basis points.
- Against the peers' seventy, it is fairly priced, not cheap.

20 How the spread moves with volatility

Spread (basis points)

Bond at 10%	**80**
Bond at 15%	**71**
Bond at 20%	**63**
Peers at 15%	**70**

Same bond, same price: the spread depends on the input

IN THE LESSON

- Here are the same four figures as bars.
- The vertical axis is spread in basis points.
- The first three bars are our one bond, valued at ten, fifteen and twenty per cent volatility.
- The bar falls from eighty to seventy-one to sixty-three, though nothing about the bond has changed.
- The last bar is the peer average, seventy, computed at fifteen per cent.
- Compare it with the first bar, and the bond looks ten basis points cheap.
- Compare it with the third, and it looks seven basis points rich.
- Only the middle comparison, with the same volatility on both sides, is honest.
- That is the rule: spreads can be ranked only when every bond is run through the same volatility.

21 A putable bond goes the other way

Putable 3.5% bond priced at 100.60

Assumed volatility	Putable OAS
10%	88 bp
15%	90 bp
20%	95 bp

IN THE LESSON

- A putable bond runs the chain in the opposite direction.
- The put belongs to the investor, so a higher volatility makes the put more valuable, and the model value of the bond goes up.
- With the market price unchanged, the model value is now further above the price, and a larger spread is needed to bring it back down.
- Take our putable bond at one hundred point six zero.
- At ten per cent volatility, its option-adjusted spread is eighty-eight basis points.
- At fifteen per cent it is ninety, and at twenty per cent it is ninety-five.
- So for a putable bond, higher volatility raises the spread, and an analyst who understates volatility will understate the spread.
- For a callable, the same mistake overstates it.

22 Mistakes so far, and how to avoid them

- Skipping early exercise dates
- Adding the coupon before the option check
- Comparing a Z-spread with an OAS
- Ranking spreads built on different volatility
- Callable: more volatility, lower OAS

IN THE LESSON

- Before we move on, here are the mistakes that cost marks in the first three topics.
- One: testing the option only at the final date, and skipping the early dates like year one.
- Two: forgetting that the coupon is added after the option decision, not before.
- Three: comparing a Z-spread on one bond with an option-adjusted spread on another.
- Four: saying a wider spread always means a cheaper bond, without checking that the volatility assumption is the same.
- Five: getting the direction of volatility wrong.
- For a callable bond, higher volatility lowers the spread.
- For a putable bond, higher volatility raises it.
- If you hold on to those five, you have most of the topic.

23 Why modified duration fails here

- Callable cash flows change with rates
- Modified duration assumes fixed flows
- Effective duration shifts the curve and revalues
- Let the option decide each time

IN THE LESSON

- You already know duration as the measure of how far a bond's price moves when rates move.
- Now consider our callable bond.
- If rates fall, the issuer calls it, and your expected cash flows shorten.
- If rates rise, they stay long.
- The cash flows themselves depend on the rate.
- Modified duration assumes the coupons and principal are fixed, so it cannot see this.
- It would give a number close to the straight bond's, and overstate the callable's sensitivity.
- The answer is effective duration.
- Instead, you shift the whole benchmark curve down and up, rebuild the tree, revalue the bond each time, and let the option decide.

24 The effective duration formula

Effective duration = (PV down − PV up) ÷ (2 × shift × PV today)

PV down	Value after curve falls
PV up	Value after curve rises
Shift size	As a decimal, 25 bp = 0.0025
PV today	The market price today

IN THE LESSON

- Here is the formula.
- Effective duration equals the value after the curve moves down, minus the value after it moves up, divided by two times the size of the shift times today's value.
- Look at each term.
- The first is the present value when the benchmark curve falls by a small amount.
- The second is the present value when it rises by the same amount.
- The shift is a decimal, so twenty-five basis points is zero point zero zero two five.
- The last term is the bond's present value today, which is the market price.
- The two in the denominator is there because the gap covers a shift of twice that size.

25 Effective duration, worked

Callable at 101.50; curve shifted 25 bp each way, OAS 71 bp held

Curve down 25 bp	recalibrate, revalue	**101.988**
Curve up 25 bp	recalibrate, revalue	**100.992**
Effective duration	gap ÷ (2 × 0.0025 × 101.50)	**1.96**
Straight bond, same spread	call removed	**2.79**

Gain 0.488 on the fall, loss 0.508 on the rise

IN THE LESSON

- Now the example.
- Our callable four point five per cent bond trades at one hundred and one point five zero, with an option-adjusted spread of seventy-one basis points on a fifteen per cent volatility tree.
- We shift the benchmark par curve down by twenty-five basis points, recalibrate the tree, and revalue with the same spread held constant.
- The bond is worth one hundred and one point nine eight eight.
- Shift it up twenty-five, recalibrate again, and it is worth one hundred point nine nine two.
- The duration is one hundred and one point nine eight eight minus one hundred point nine nine two, divided by two times zero point zero zero two five times one hundred and one point five zero, which is one point nine six.
- The straight bond, same spread, has two point seven nine.

26 Callable against straight

Duration (years)

Straight bond	**2.79**
Callable bond	**1.96**
Forgot the 2	**3.93**
Years to maturity	**3**

Check: duration above maturity means the 2 is missing

IN THE LESSON

- Here are the results as bars.
- The vertical axis is duration, in years.
- The first bar is the straight bond at two point seven nine.
- The second bar is the callable, at one point nine six.
- The call caps the gain when rates fall, so the callable is less sensitive.
- The third bar is a warning.
- It shows the answer if you forget the two in the denominator: three point nine three, double the true figure.
- That is longer than the bond's whole life, the fourth bar, three years, which is impossible for a bond that pays coupons.
- If your duration comes out longer than the maturity, you probably left out the two.

27 Reading the number

- 1.96: about 1.96% change for a 1 point shift
- Shift the curve, recalibrate, revalue
- Hold the OAS constant while you do it
- Never drop the 2 in the denominator

IN THE LESSON

- So how do you read one point nine six?
- Effective duration is the approximate percentage change in value for a one percentage point shift in the benchmark curve.
- So a parallel fall of one point would raise the callable's value by about one point nine six per cent, and a rise would lower it by about the same.
- Three habits keep you safe.
- First, for a bond with options, get each value by shifting the curve, recalibrating the tree and revaluing.
- Do not just reprice the old tree.
- Second, hold the option-adjusted spread constant, so that you are measuring the effect of rates alone.
- Third, use the factor of two.
- Forget it and the answer doubles.

28 Why options shorten duration

Callable bond, rates fall

Call in the money → Issuer likely to redeem → Ends at call date → Duration drops

Putable bond, rates rise

Put in the money → Holder likely to put → Ends at put date → Duration drops

IN THE LESSON

- Why does a call or a put shorten duration?
- Look at the two rows.
- In the first row, a callable bond meets falling rates.
- As rates fall below the coupon, the call moves into the money.
- The issuer is likely to redeem at the first call date, so the bond behaves like a bond ending then, not at final maturity.
- A shorter bond has a shorter duration, so the duration drops.
- In the second row, a putable bond meets rising rates.
- As rates rise above the coupon, the put moves into the money.
- The holder is likely to hand the bond back at the put date, so the bond acts like a shorter bond again, and the duration drops.

29 Duration against the curve shift

Chart: Effective duration (years) vs Shift in benchmark curve

- Straight
- Callable
- Putable

IN THE LESSON

- Now the picture.
- Along the bottom is the shift in the benchmark curve, from minus two per cent on the left to plus three per cent on the right.
- Up the side is effective duration, in years.
- The top line is the straight bond.
- It barely moves, sitting near two point eight whatever the rates do.
- The next line is the callable.
- On the right, where rates are high and the call is far out of the money, it follows the straight bond.
- Moving left, as rates fall, it drops, reaching about one year when the call is nearly certain.
- The third line is the putable.
- It is the mirror image: on the left it follows the straight bond, and it falls towards one year on the right.

30 Three bonds, three durations

3-year, 3.5% coupon, options at 100 in years one and two

Effective duration	Straight / callable / putable
Today	2.83 / 2.04 / 1.86
Curve 1% lower	2.86 / 0.99 / 2.56
Curve 1.5% higher	2.79 / 2.79 / 0.97

IN THE LESSON

- Here are the exact numbers for three bonds from one issuer, each with three years to run and a three point five per cent coupon.
- One is straight, one callable at one hundred and one putable at one hundred, both options usable in years one and two.
- The tree is the same fifteen per cent volatility tree.
- Today the durations are two point eight three for the straight bond, two point zero four for the callable and one point eight six for the putable.
- With the curve one per cent lower, they are two point eight six, zero point nine nine and two point five six.
- With the curve one and a half per cent higher, they are two point seven nine, two point seven nine and zero point nine seven.
- The straight bond barely changes, and neither option bond is ever above it.

31 Which is shorter depends on distance

- Straight: duration changes little
- Callable: never above straight; drops as rates fall
- Putable: never above straight; drops as rates rise
- Lower of the two: whichever option is nearer exercise

IN THE LESSON

- Now you can state the rules.
- An option-free bond's effective duration changes little as rates move.
- A callable bond's effective duration is never higher than the straight bond's, because when rates fall the call moves into the money and the bond behaves like a shorter bond.
- A putable bond's is also never higher, because when rates rise the put moves into the money.
- When both options are far from exercise, all three durations are about the same.
- Which of the two option bonds has the lower duration depends on which option is nearer exercise.
- Today, forward rates rise past the coupon, so the put is closer, and the putable's duration, one point eight six, is below the callable's, two point zero four.

32 Recap

- Tree value: cap (call), floor (put), every date
- OAS: spread that matches tree value to price
- Callable: more volatility, lower OAS
- Effective duration = (PV down − PV up) ÷ (2 × shift × PV)
- Option bonds' duration is never above straight

IN THE LESSON

- Let's recap the five ideas.
- One: to value a callable or putable bond, work backward on a calibrated tree, cap the value at the call price or floor it at the put price at every exercise date, add the coupon, and the option is the difference from the straight bond.
- Two: the option-adjusted spread is the spread that makes the tree value equal the price, and it is the one to compare, with the Z-spread above it for a callable.
- Three: it depends on volatility.
- Four: effective duration comes from shifting the curve and revaluing, with the two in the denominator.
- Five: callable and putable bonds never have longer durations than the straight bond.

33 Try three quick questions

1. A callable bond's node is worth 101.20 before the coupon; call price 100; coupon 4. What value goes back?
   Answer: 100 + 4 = 104: the call caps it.
2. Callable bond: Z-spread 90 bp, option cost 28 bp. What is the OAS?
   Answer: 90 − 28 = 62 bp.
3. PV down 102.60, PV up 101.40, PV today 102.00, shift 50 bp. Effective duration?
   Answer: 1.2 ÷ (2 × 0.005 × 102) = 1.18

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- A callable bond's node is worth one hundred and one point two zero before the coupon.
- The call price is one hundred and the coupon is four.
- What value goes back?
- The call caps it at one hundred, plus the coupon, so one hundred and four.
- Question two.
- A callable bond has a Z-spread of ninety basis points and an option cost of twenty-eight.
- What is the option-adjusted spread?
- Sixty-two basis points.
- Question three.
- The value after the curve falls is one hundred and two point six zero, after it rises one hundred and one point four zero, and today's value is one hundred and two.
- The shift is fifty basis points.
- The gap of one point two, divided by two times zero point zero zero five times one hundred and two, gives one point one eight.